CS 7180: Behavioral Modeling and Decision-making in AI

Temporal Logic, Part II

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Outline

- Temporal Logic (continued…)
  - Dividing instant problem
  - Event-based theories
  - Temporal incidence
  - Hybrid model of time

Theories of time

- Requirements for a formal theory of time
  - Represent **commonsense** notions of time
  - Represent events and changing values of fluents **without contradicting** intuitions about time

- What is going to be the **primitive** unit of time?
  - Instants (or points)—instants of time with no duration
  - Periods—intervals with positive duration
  - Events
  - Hybrid (instants AND intervals)

Instant-based theories of time

- Traditional structure for time adopted from classical physics
  - Time is **continuum** of consecutive instants

- Instant—**durationless** pieces of time (beginning and end not distinct)

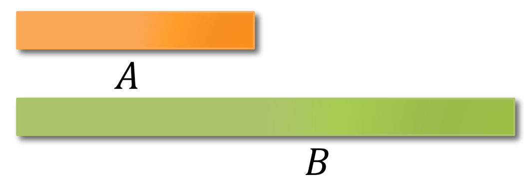
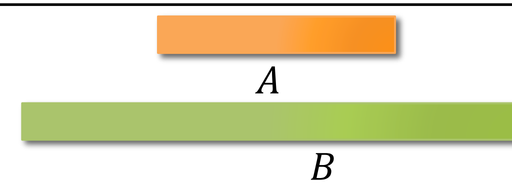
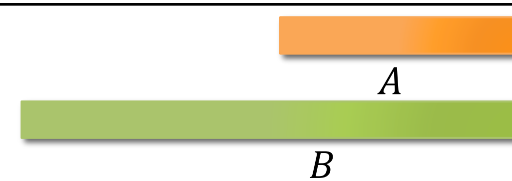
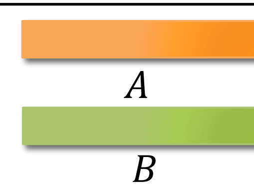
- Defined on a structure $(\mathcal{I}, \prec)$ where
  - $\mathcal{I}$ is a set of instants and
  - $\prec$ is an order(partial or total) over the instants

- Example models:
  - Real-numbers time $(\boldsymbol{R}, <)$
  - Integer-numbers time $(\boldsymbol{Z}, <)$

Period-based theories of time

- Periods (intervals) are associated with events that take time
  - Primitive time unit more related to our intuitive experiences
- Defined on a structure ($\mathcal{P}$, R) where
  - P is a set of **periods**
  - R is a set of **relationships** between periods
- Example models
  - Allen's Interval Theory ($\mathcal{P}$, AR)
  - Allen & Hayes Interval Theory ($\mathcal{P}$, *Meets*)

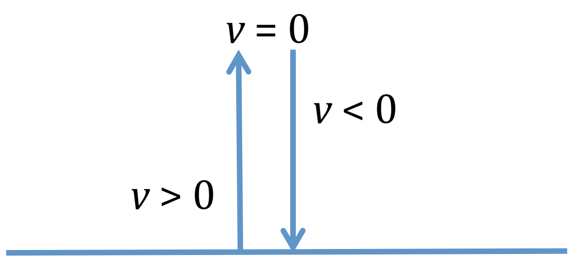
Qualitative interval relationships *AR*

A BEFORE *B*	*B* AFTER *A*	
A MEETS *B*	*B* MET_BY *A*	
A OVERLAPS *B*	*B* OVERLAPPED_BY *A*	
A STARTS *B*	*B* STARTED_BY *A*	
A DURING *B*	*B* CONTAINS *A*	
A FINISHES *B*	*B* FINISHED_BY *A*	
A EQUALS *B*	*B* EQUALS *A*	

Challenges caused by periods

- Instantaneous events
  - Some events naturally represented as instants rather than periods E.g., "shoot the gun", "start moving"
  - How do we represent sequences of instantaneous events when there is also continuous change—what happens "first"?
- Fluents that hold at an instant
  - E.g., "The patient's temperature was 99 degrees at 10:00am"
  - What is truth value at instant of change?

Tossed ball scenario
$v > 0$ when ball moving up,
$v < 0$ when moving down,
$v = 0$ at point of change



  - How to represent the durationless instant where $v = 0$?

Dividing instant problem

- What if proposition holds for a period and is false in the next? How do we decide truth value at **instant in between**?
  - *A fire that has been burning is later burnt out. What happens at the intermediate instant between the two successive states of burning and being extinguished?*
  - *A light that has been off, and becomes on after it is switched on. Is the light on or off at the switching instant?*

- Period-based theories ignore question of whether instant is **member** of an interval
  - Ambiguity about the end points
  - Closed periods lead to **inconsistency** (both f and $\neg f$ true at meeting)
  - Open periods lead to **truth gap** (neither f nor $\neg f$ true at meeting)

Event-based theories of time

- Time is the relationship between **events** and processes
  - Similar to period-based theories
  - Events that happen at the same time not necessarily the same

- Defined on a structure $(\mathcal{E}, \prec, O)$ where
  - $\mathcal{E}$ is a set of events,
  - $\prec$ is a **precedence** relation, and
  - O is an **overlapping** relation

- Event theories based directly on perceived phenomena—events that take time
  - Do not distinguish between occurrences and their times of occurrence
  - Might be more intuitive to think of time as a separate entity

Hybrid theories with periods and instants

- Include instants and periods in a single theory of time
  - Real-world involves both instantaneous and durable events—**natural expression** for both
  - **Efficient computation**—use representation and relations that are most efficient for the particular event

- Defined on a structure ($\mathcal{I}$, $\mathcal{P}$, $\prec : \mathcal{I} \times \mathcal{I}$, *begin, end*: $\mathcal{I} \times \mathcal{P}$)where
  - $\mathcal{I}$ is an infinite set of instant symbols,
  - $\mathcal{P}$ is an infinite set of period symbols disjoint from $\mathcal{I}$,
  - $\prec$ is an order relation over the instants, and
  - the relations *begin, end* define the instants defining the beginning and ending of periods
- Periods are ordered pairs of instants

Axioms of a hybrid theory

- $\prec$ is a **strict linear order** over instants—single time line
  - $\neg(i \prec i)$
  - $i \prec i' \Rightarrow \neg(i' \prec i)$
  - $i \prec i' \wedge i' \prec i'' \Rightarrow i \prec i''$
  - $i \prec i' \vee i' \prec i \vee i = i'$
- Instants are **unbounded**
  - $\exists i' (i' \prec i), \exists i' (i \prec i')$
- End points of periods are ordered—no durationless periods
  - $begin(i,p) \wedge end(i',p) \Rightarrow i \prec i'$
- All periods have **unique beginning** and **ending** instants
  - $\exists i\ begin(i,p), \exists i\ end(i,p)$
  - $begin(i,p) \wedge begin(i',p) \Rightarrow i = i', end(i,p) \wedge end(i',p) \Rightarrow i = i'$
- All ordered pairs of instants define a **unique period**
  - $i \prec i' \Rightarrow \exists p\ (begin(i,p) \wedge end(i',p))$
  - $begin(i,p) \wedge end(i',p) \wedge begin(i,p') \wedge end(i',p') \Rightarrow p = p'$

Incorporating time into logic

- Temporal incidence—how do we represent and determine what is true at a given time?

- Several ways of incorporating time into logic s.t. inference is still sound and complete
  - Temporal arguments
  - Modal temporal logics
  - Reified temporal logics

Temporal arguments in FOL

- Adds time as **argument** to FOL functions and predicates
  - Similar to adding state to predicates in the Situation Calculus

"The patient's temperature is 99 degrees at 10:00 on Sept. 18" as *Temperature*(*P*, 99, 09/18/2012 10:00am)

- No special status for time
  - Not expressive enough for general temporal assertions
  - Cannot represent commonsense notions of time e.g., "effects cannot precede their causes"

Modal temporal logics

- Extend propositional logic with additional modal operators
  - Referred to as **tense logic** in philosophy
  - Indicate temporal relationships between propositions

- Example modal operators

  $F\phi$: formula ϕ "will be true"
  $P\phi$: formula ϕ "was true"

  $G\phi$: formula ϕ "will always be true"
  $H\phi$: formula ϕ "was always true"

  "The patient's temperature was 99 degrees at 10:00 on Sept. 18" as $P(Temperature\_99)$

Reified temporal logics

- **Reify** standard propositional or first-order logic
  - Use "meta-predicates" such as *Holds*, *Occur*, *Cause*, etc.
  - Relate predicates to temporal information

"The patient's temperature is 99 degrees at 10:00 on Sept. 18" as *Holds*(*Temperature*(*P*, 99), 09/18/2012 10:00am)

- Accords special status to time
- Describe different types of temporal occurrence
- Negation easy to represent
- Represent **causal relationships** between events and their effects

Reified hybrid temporal model ($\mathcal{CD}$)

- Hybrid structure ($\mathcal{I}, \mathcal{P}, \prec : \mathcal{I} \times \mathcal{I}, begin, end : \mathcal{I} \times \mathcal{P}$)
  - Temporal **primitives**—instants and periods
    - Define instant-period (e.g., within) and period-period relations (e.g., MEETS) using $\prec$, *begin, end*
  - Instants and periods are both primitives
    - Avoid dividing instant problem because can represent interior of periods and the instants at end points in separate predicates
- Temporal propositions for **continuous fluents**, **discrete fluents**, and **events**
  - Fluents classified by whether change they represent is continuous (e.g., charging battery) or discrete (i.e., light on or off)
- Reified **temporal incidence** relations for each combination of temporal proposition and temporal primitive

$\mathcal{CD}$ Temporal incidence predicates

- Reify each possible relationship between primitives (instants and periods) and predicates (fluents and events)

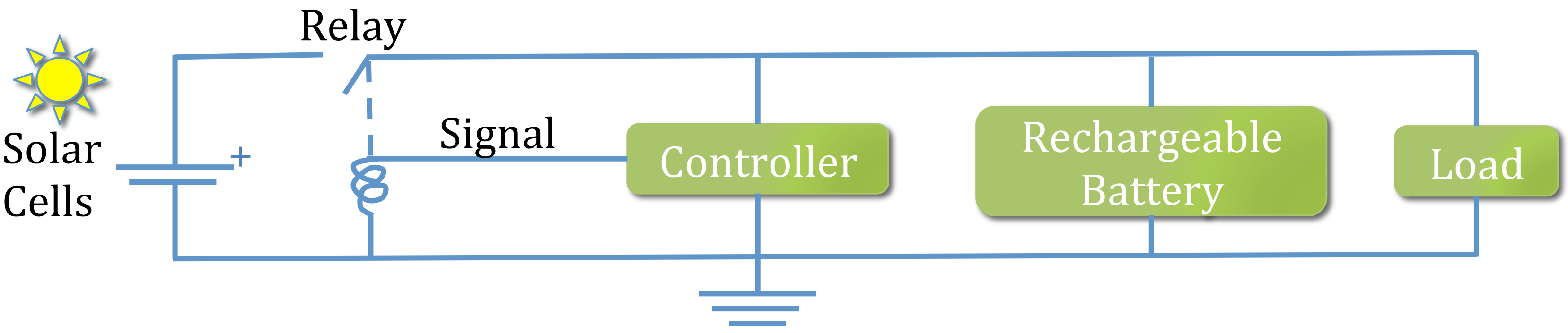
- $HOLDS_{on}(f,p)$ The continuous fluent *f* holds throughout period *p*
- $HOLDS_{on}(f,p)$ The discrete fluent *f* holds throughout period *p*
- $HOLDS_{at}(f,i)$ The continuous fluent *f* holds at instant *i*
- $HOLDS_{at}(f,i)$ The discrete fluent *f* holds at instant *i*
- $OCCURS_{on}(e,p)$ The event *e* occurs on the period *p*
- $OCCURS_{at}(e,i)$ The event *e* occurs at the instant *I*

- Use the *begin* and *end* relations in a functional form $i = begin(p)$

$\mathcal{CD}$ Axioms and properties

- Fluent holds during a period iff it holds at its **inner instants**
  - $HOLDS_{on}(f,p) \Leftrightarrow (Within(i,p) \Rightarrow HOLDS_{at}(f,i))$
- Discrete fluents **cannot** hold at an **isolated instant**
  - Has some value for one period, change event occurs at the instant, and then has another durable value w.r.t. a period
  - $HOLDS_{at}(f,i) \Rightarrow \exists p\ (HOLDS_{on}(f,p) \wedge (Within(i,p) \vee begin(i,p) \vee end(i,p)))$
- No special rules for events in general—**application specific** (e.g., a war can occur at multiple periods, but only if they meet; represent an election as instantaneous or durable)
- Non-atomic fluents—$\mathcal{CD}$ is **compositional**
  - $HOLDS_{at}(\neg f,i) \Leftrightarrow \neg HOLDS_{at}(f,i)$
  - $HOLDS_{at}(f \wedge f',i) \Leftrightarrow HOLDS_{at}(f,i) \wedge HOLDS_{at}(f',i)$
  - $HOLDS_{at}(f \vee f',i) \Leftrightarrow HOLDS_{at}(f,i) \vee HOLDS_{at}(f',i)$

Modeling a (green!) circuit with *CD*



- Description of a simple solar powered circuit
  1. If the sun is shining and the relay is closed, then the solar array acts as a constant current source and the battery accumulates charge
  2. If the relay is closed, when the signal from the controller goes high the relay opens.
  3. If the relay is open, when the signal from the controller goes low the relay closes.
  4. If the signal is low, when the controller detects that the charge level in the battery has reached threshold *q2* the controller turns on the signal to the relay

The challenge of sequences of events

- **Initial state**: *signal* = *low*, *relay* = *closed*, sun is shining
- How can we model the temporal incidence of the following:

“The signal goes high and immediately after the relay opens.”

using the environment rules from the previous slide?

State	Time	Q_{BA} *(battery charge)*	*signal*	*relay*
s0	(*t1*, *t2*)	< *q2*	*low*	*closed*
s1	*t2*	= *q2*	*low*	*closed*
s1.1	*t2.1*	?	*high*	*closed*
s1.2	*t2.2*	?	*high*	*open*
s2	(*t2.2*,__)	< *q2*	*high*	*open*

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s1.1	*t2.1*	?	*high*	*closed*
s1.2	*t2.2*	?	*high*	*open*
s2	(*t2.2*,__)	< *q2*	*high*	*open*

signal goes high (→ *s1.1*)

relay opens (→ *open*, row *s1.2*)

dividing instant! (→ the two "?" cells)

Model instantaneous and continuous change

- What we intuitively want to model is much simpler

State	Time	Q_{BA} *(battery charge)*	*signal*	*relay*
s0	$(t1, t2)$	$< q2$	*low*	*closed*
s1	*t2*	$= q2$	*?*	*?*
s2	(*t2*,__)	$< q2$	*high*	*open*

- Can formalize this with $\mathcal{CD}$

State	$\mathcal{CD}$ Logic Representation
s0	$HOLDS_{on}(Q_{BA} < q2, p1)$ $HOLDS_{on}(signal = low, p2)$ $HOLDS_{on}(relay = closed, p3)$
s1	$HOLDS_{at}(Q_{BA} = q2, end(p1))$ $OCCURS_{at}(turn\_on(signal), end(p1))$ $end(p1) = end(p2)$ $HOLDS_{on}(signal = on, p4)$ $MEETS(p2,p4)$
s2	$OCCURS_{at}(open(relay), end(p3))$ $end(p2) = end(p3)$ $HOLDS_{on}(relay = open, p5)$ $MEETS(p3,p5)$