CS 7180: Behavioral Modeling and Decision-making in AI

Situation Calculus & Temporal Logic

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September 19, 2012

Outline

- First-order logic (review of the review)
- Actions and events in FOL
    - Situation calculus
    - Frame problem
- Temporal Logic

Atomic sentences in FOL

- Term = $\mathit{function}(\mathit{term}_1,\ldots,\mathit{term}_n)$
  or constant or variable
- A term with no variables is a **ground term**
- Examples
  - Brother(KingJohn, RichardTheLionheart)
  - >(AgeOf(Richard), AgeOf(John))
  - Brother(AgeOf(Richard), AgeOf(John))

- Atomic sentence = $\mathit{predicate}(\mathit{term}_1,\ldots,\mathit{term}_n)$
  or $\mathit{term}_1 = \mathit{term}_2$
- An **atomic sentence** has value true or false

Complex and quantified sentences

- Complex sentences are made from atomic sentences using connectives
  - Connectives have same semantics as propositional logic
  - $\neg S, S_1 \land S_2, S_1 \lor S_2, S_1 \Rightarrow S_2, S_1 \Leftrightarrow S_2$
- Examples
  - *Sibling*(*John,Richard*) ⇔ *Sibling*(*Richard,John*)
  - $>(1,2) \land \leq(1,2)$
  - $>(1,2) \land \neg>(1,2)$

- A **quantified sentence** adds quantifiers ∀ and ∃
- A **well-formed formula (wff)** is contains no "free" variables—all variables are "bound" by quantifiers

  $(\forall x)P(x,y)$ has x bound as a universally quantified variable, but y is free

Substitution and Unification in FOL

- A **substitution** is a set of variable-term pairs: {x/term, y/term, . . . }
  - Often referred to using the symbol θ [theta]
  - No variable can occur more than once.

- Two formulas A and B **unify** if there is a substitution θ such that $A\theta = B\theta$
- If θ_1 is a unifier for formulas A and B, it is a **MOST GENERAL UNIFIER** (MGU) iff:

  There is no other unifier θ_2 for A and B s.t. $A\theta_2$ subsumes $A\theta_1$
- A formula F **subsumes** a formula G if there is a non-trivial substitution Π s.t. $F\Pi = G$

Review: Find a MGU for the following pairs (if one exists)

1. $Isa(Oliver, Dog)$
   $Isa(x, Dog)$

2. $Isa(x, Dog)$
   $Isa(y, z)$

3. $Likes(x, Owner(x))$
   $Likes(Joey, Owner(Oliver))$

4. $Likes(x, Owner(y))$
   $Likes(Joey, z)$

5. $Likes(x, Owner(y))$
   $Likes(Oliver, Sister(Amy))$

Review: Find a MGU for the following pairs (if one exists)

1.	$Isa(Oliver,Dog)$ $Isa(x, Dog)$	$\boldsymbol{\theta = \{x/Oliver\}}$
2.	$Isa(x,Dog)$ $Isa(y,z)$	$\boldsymbol{\theta = \{x/y, z/Dog\}}$
3.	$Likes(x,Owner(x))$ $Likes(Joey, Owner(Oliver))$	**None**
4.	$Likes(x,Owner(y))$ $Likes(Joey, z)$	$\boldsymbol{\theta = \{x/Joey, z/Owner(y)\}}$
5.	$Likes(x,Owner(y))$ $Likes(Oliver, Sister(Amy))$	**None**

Inference in FOL with Lifted Resolution

- **Rules of inference** same as in propositional logic in ground cases
- **Generalize (lift)** rules to FOL case with variables and quantifiers
  - Use **Existential** and **Universal Instantiation** rules to ground a *KB*

- First-order resolution rule:

$$\frac{l_1 \vee \ldots \vee l_k,\ m_1 \vee \ldots \vee m_k}{(l_1 \vee \cdots \vee l_{i-1} \vee l_{i+1} \vee \cdots \vee l_k \vee m_1 \vee \cdots \vee m_{j-1} \vee m_{j+1} \vee \cdots \vee m_n)\ \theta}$$

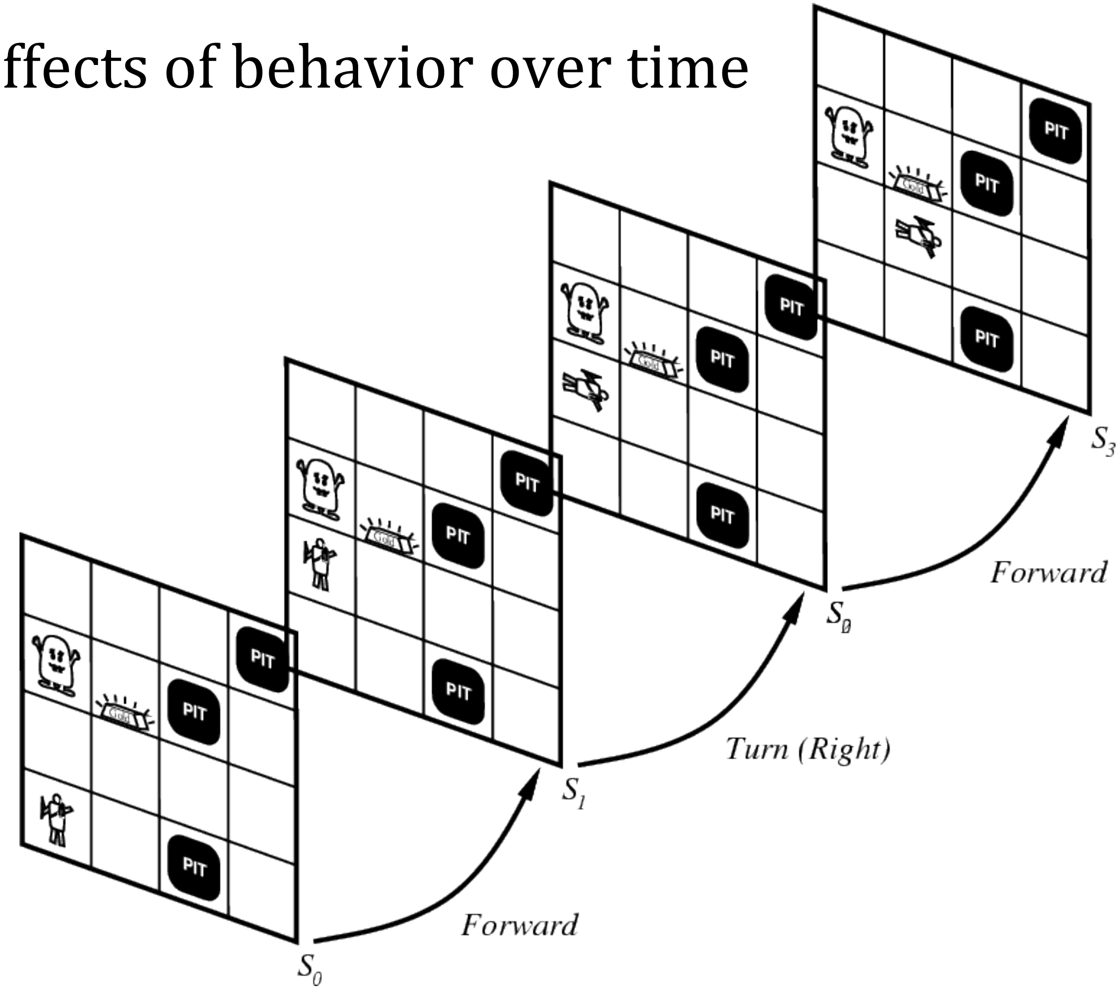
where $Unify(l_i, \neg m_j) = \theta$

Logic of events—the situation calculus

- We can model dynamic behavior with logic and resolution
  - FOL representation of the events over time
  - Resolution to find a solution
- **Key idea:** represent a snapshot of the world, called a "situation" explicitly
- **Fluents**—statements that are true or false in any given situation, e.g., "I am at home"
- Actions map situations to situations
  - When an agent performs action A in situation S_1, the result is a new situation S_2

Situations in the Wumpus world

- Each action leads to a new situation
- Models effects of behavior over time



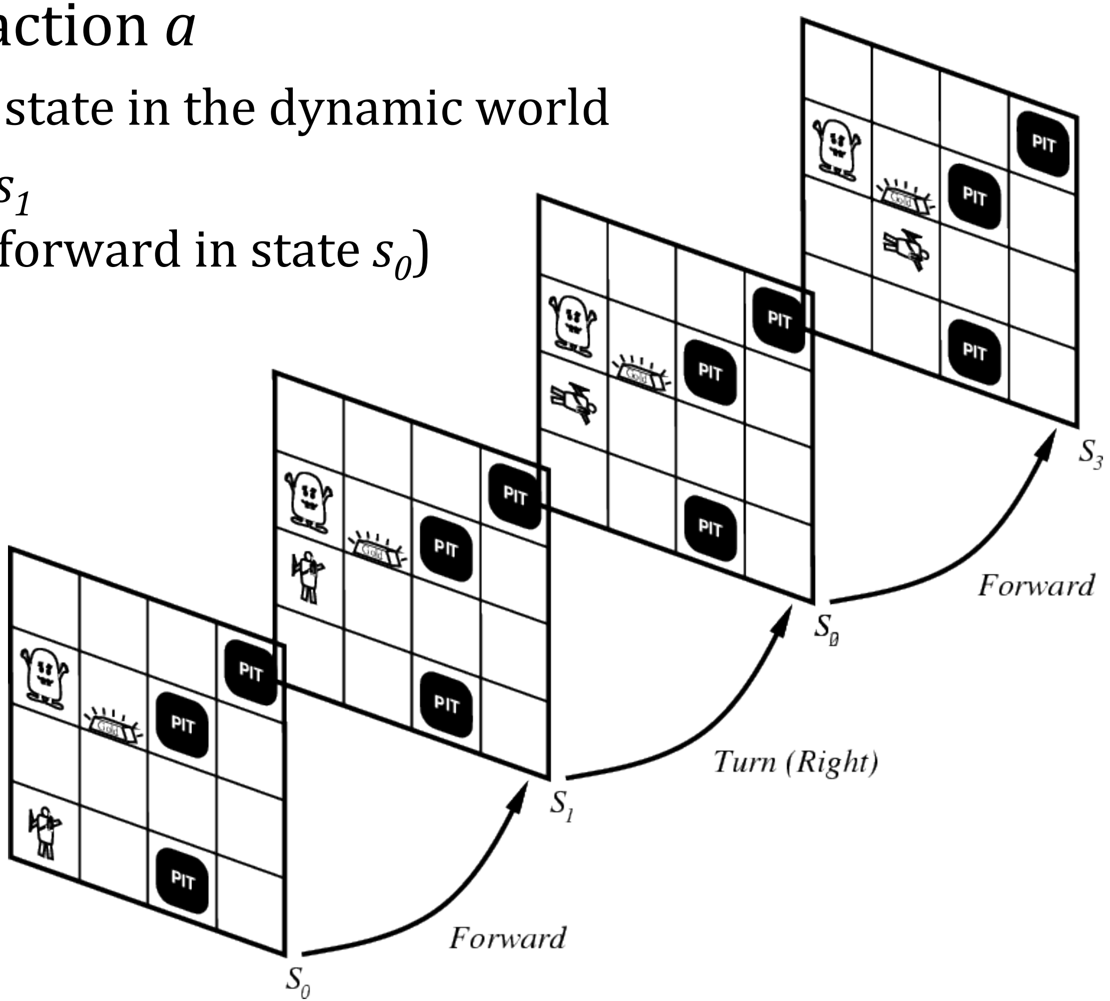
Describing actions and effects

- **Situation**—snapshot of the world at an interval of time where nothing changes
- Every true or false statement is made w.r.t. a particular situation
  - Add **situation variables** to every predicate
  - *At*(*Agent*,1,1) becomes *At*(*Agent*,1,1,*s0*)—At(*Agent*,1,1) is true in situation (i.e., state) *s0*

- Result function, ***Result*(*a*,*s*),** mapping situation *s* into a new situation as a result of performing action *a*

Advancing to the next situation

- *Result*(*a*,*s*) is the **successor state** (situation) that follows *s* after we execute action *a*
- Advances to the next state in the dynamic world
- *Result*(*forward*, s_0) = s_1
  (i.e. result of moving forward in state s_0)



Advancing to the next situation

- *Result*(*a*,*s*) is the **successor state** (situation) that follows *s* after we execute action *a*
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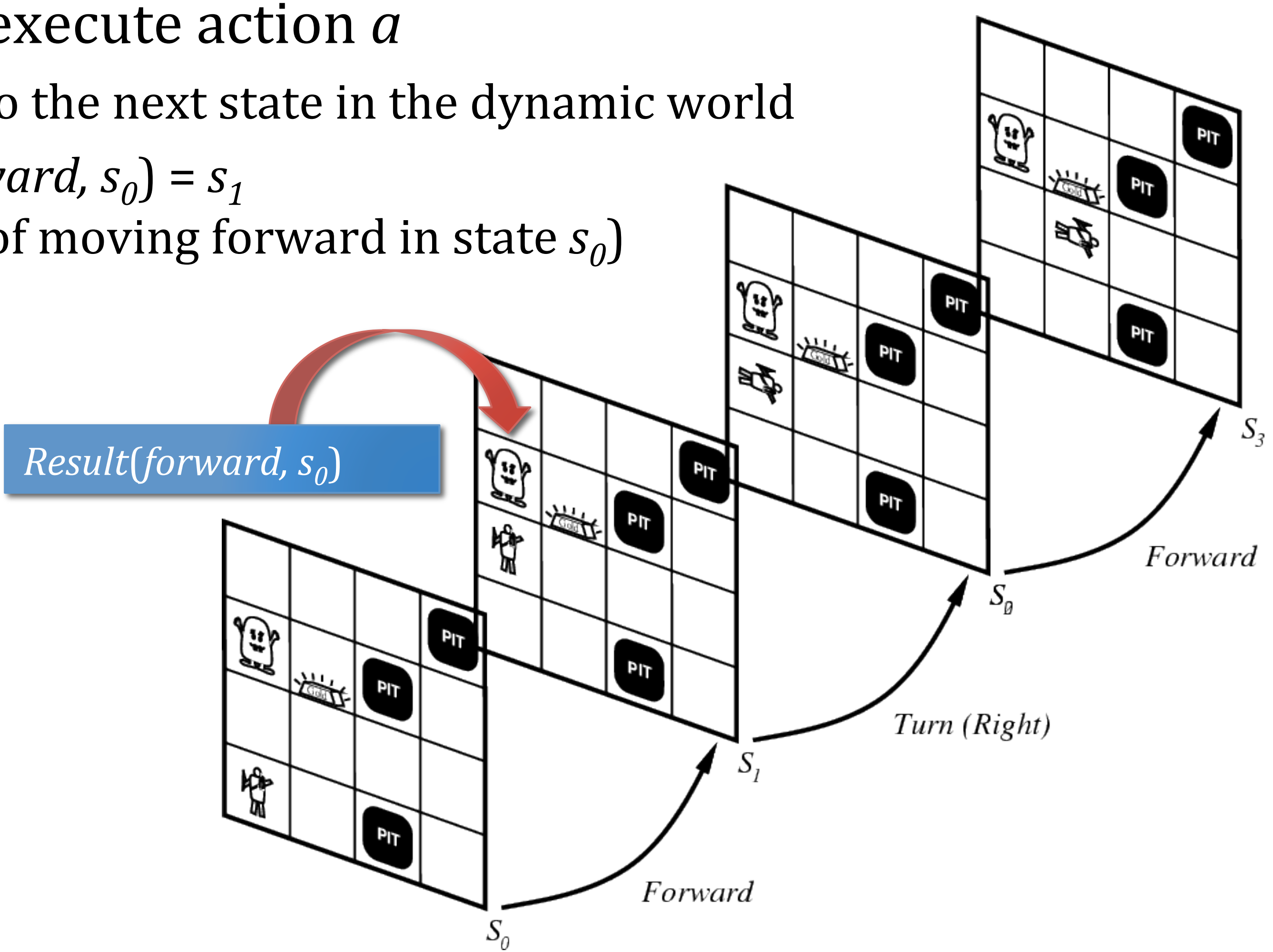

$Result(forward, s_0)$

Advancing to the next situation

- *Result*(a,s) is the **successor state** (situation) that follows s after we execute action a
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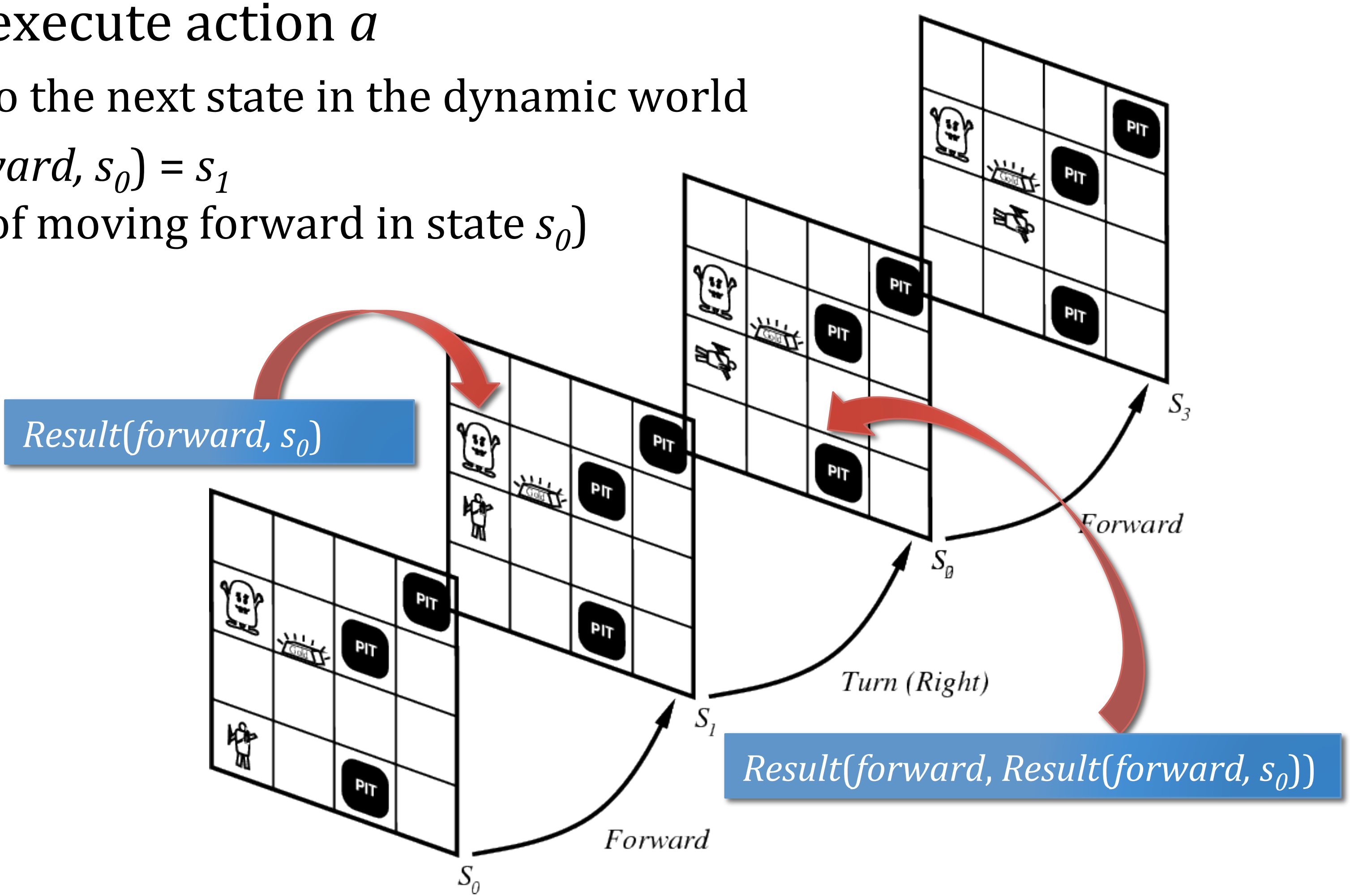

Result(forward, s_0)

Result(forward, Result(forward, s_0))

Advancing to the next situation

- *Result*(*a*,*s*) is the **successor state** (situation) that follows *s* after we execute action *a*
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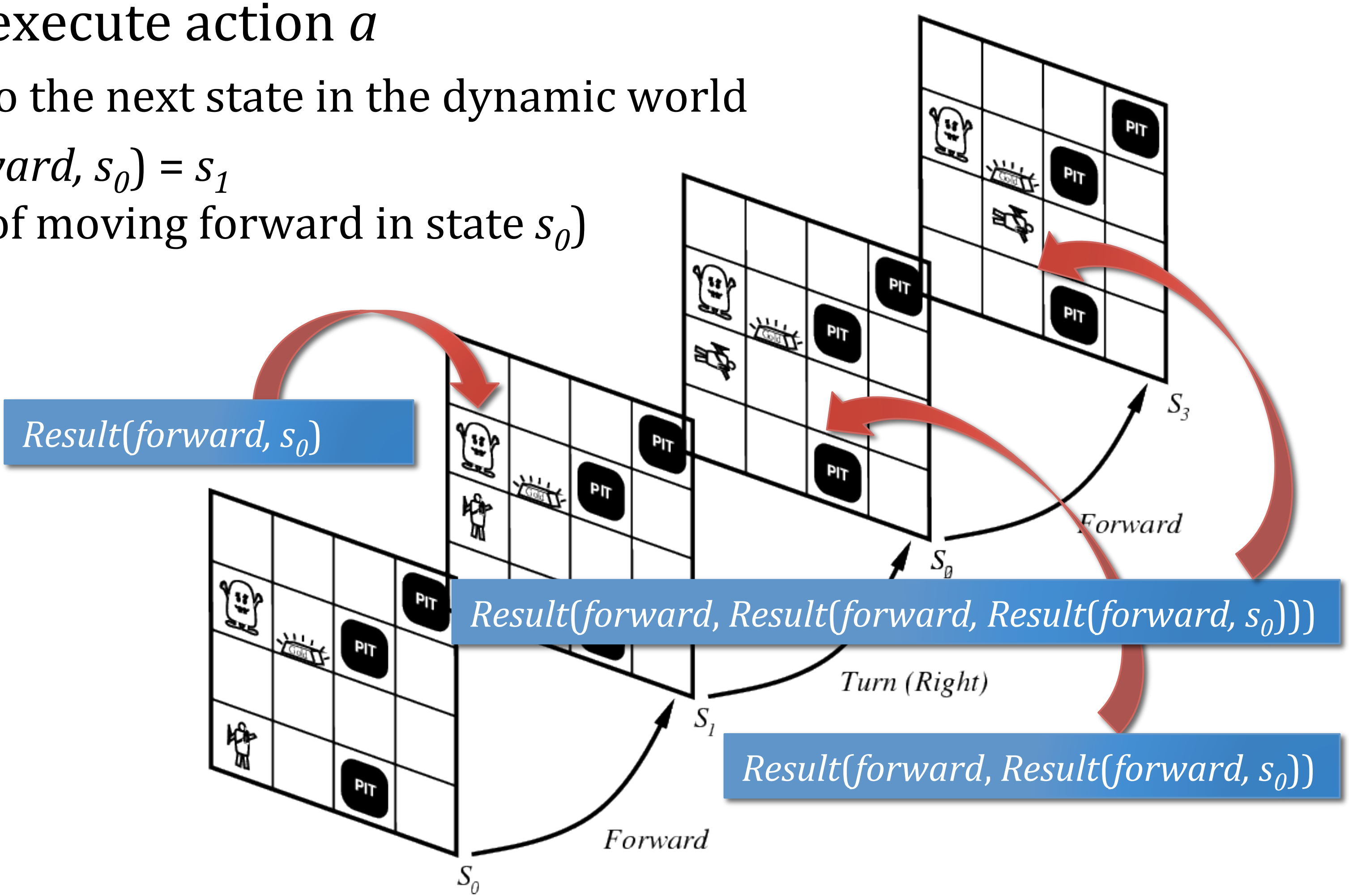


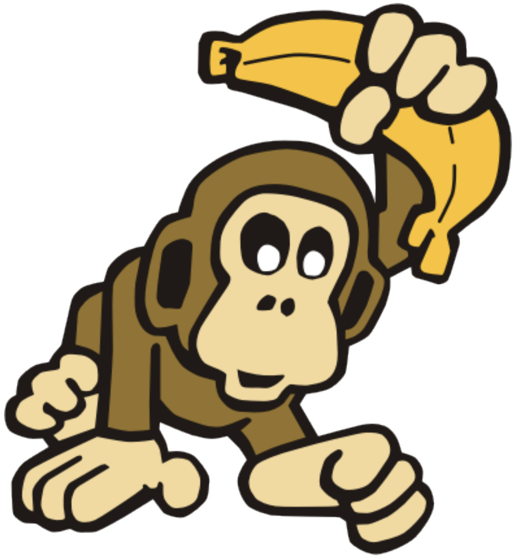
$Result(forward, s_0)$

$Result(forward, Result(forward, Result(forward, s_0)))$

$Result(forward, Result(forward, s_0))$

Monkeys and bananas problem

- The **monkey-and-bananas problem** is faced by a **monkey** standing under some **bananas** which are hanging out of reach from the ceiling. There is a box in the corner of the room that will enable the **monkey** to reach the **bananas** if he climbs on it.



- Use situation calculus to represent this problem
- Once we have a representation in FOL, we can solve it using resolution

Representation of Monkey/banana problem

Fluents

$At(x, loc, s)$

$On(x, y, s)$

$Reachable(x, Bananas, s)$

$Has(x, y, s)$

Other Predicates

$Moveable(x)$

$Climbable(x)$

$Can\text{-}move(x)$

Constants

BANANAS

MONKEY

BOX

$S0$

Corner

UNDER-BANANAS

Actions

$Climb\text{-}on(x, y)$	$Move(x, loc)$
$Reach(x, y)$	$Push(x, y, loc)$

Describe axioms of the world in FOL

1	**If a person (or monkey!) can reach the bananas ,then the result of reaching them is to have them**
2	**If a box is under the bananas and the monkey is on the box then the monkey can reach the bananas**
3	**The result of moving to a location is to be at that location**

Describe axioms of the world in FOL

1	**If a person (or monkey!) can reach the bananas ,then the result of reaching them is to have them**
	$\forall x1, s1\ [\ Reachable(x1, BANANAS, s1) \Rightarrow$ $Has(x1, BANANAS, Result(Reach(x1, BANANAS), s1))\ ]$
2	**If a box is under the bananas and the monkey is on the box then the monkey can reach the bananas**
	$\forall s2\ [At(BOX, UNDER\text{-}BANANAS, s2) \wedge On(MONKEY, BOX, s2) \Rightarrow$ $Reachable(MONKEY, BANANAS, s2)$
3	**The result of moving to a location is to be at that location**
	$\forall x3,loc3,s3\ [\ Can\text{-}move(x3) \Rightarrow$ $At(x3, loc, Result(Move(x3, loc3), s3))\ ]$

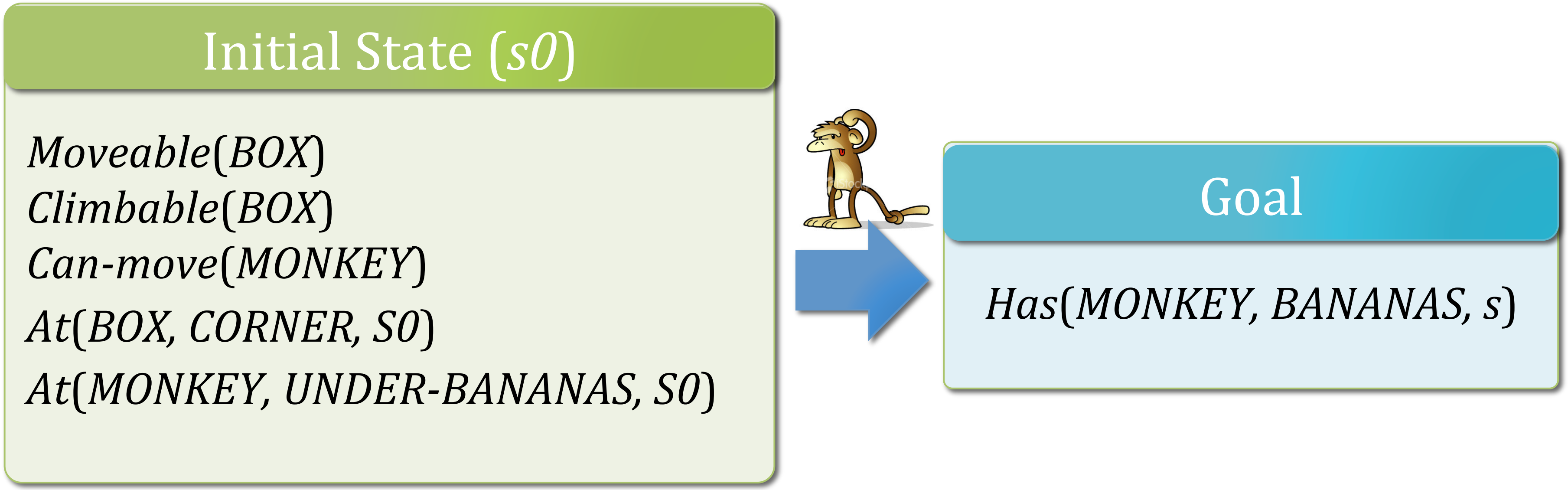
Describe axioms of the world in FOL

4	**The result of climbing on an object is to be on the object**
5	**The result of x pushing y to a location is y is at that location**
6	**The result of x pushing y to a location is x is at that location**

Describe axioms of the world in FOL

4	**The result of climbing on an object is to be on the object**
	$\forall x4, y4, s4\ [\exists loc4\ [At(x4, loc4, s4) \wedge At(y4, loc4, s4)] \wedge Climbable(y4) \Rightarrow On(x4, y4, Result(Climb\text{-}on(x4, y4), s4))]$
5	**The result of x pushing y to a location is y is at that location**
	$\forall x5, y5, loc5, s5\ [\exists loc\ [At(x5, loc0, s5) \wedge At(y5, loc0, s5)\ ] \wedge Moveable(y5) \Rightarrow At(y5, loc5, Result(Push(x5, y5, loc5), s5))$
6	**The result of x pushing y to a location is x is at that location**
	$<same> \Rightarrow At(x6, loc6, Result(Push(x6, y6, loc6), s6))\ ]$

How does the monkey get the bananas?



Initial State (*s0*)

Moveable(*BOX*)
Climbable(*BOX*)
Can-move(*MONKEY*)
At(*BOX, CORNER, S0*)
At(*MONKEY, UNDER-BANANAS, S0*)

Goal

Has(*MONKEY, BANANAS, s*)

- Prove *Has*(*MONKEY, BANANAS,* ***Result(Reach(. . .), Result(. .) . .), S0)***
  - Can use resolution
  - Will give the actions from s0 to goal in reverse order
- For the proof to work, we need two additional **frame axioms**…

Frame axioms for monkey/bananas problem

- Frame axioms keep track of what **does not** change from one situation to the next

7	**The location of an object does not change as a result of someone moving to the same location**
8	**The location of an object does not change as a result of someone climbing on it**

Frame axioms for monkey/bananas problem

- Frame axioms keep track of what **does not** change from one situation to the next

7	**The location of an object does not change as a result of someone moving to the same location**
	$\forall x, y, loc, s\ [\ At(x, loc, s) \Rightarrow At(x, loc, Result(Move(y, loc), s))\ ]$
8	**The location of an object does not change as a result of someone climbing on it**
	$\forall x, y, loc, s\ [\ At(x, loc, s) \Rightarrow At(x, loc, Result(Climb\text{-}on(y, x), s))\ ]$

Limitation of situation calculus

- **Frame problem** (it's back again!!)
- I go from home to the store—new situation *S'*
- In *S'*
  - The store still sells chips
  - My age is still the same
  - Los Angeles is still the largest city in California…

- How can we efficiently represent everything that doesn't change?
  Successor state axioms…

Successor state axioms

- Normally, things stay true from one state to the next—unless an action **changes** them

$$At(p, loc, Result(a, s)) \text{ iff } a = Go(p, x) \\ \text{or } [At(p, loc, s) \text{ and } a \mathrel{!=} Go(p, y)]$$

- We need one or more of these for every fluent
- Now we can infer an agent's course of action
  - Still not very practical
- Need for **effective heuristics** for situation calculus

Temporal reasoning in dynamic domains

- Situation calculus describes actions and events
  - Resolution can provide a **sequence**, but no real temporal data

- Time is crucial part of common sense and fundamental to natural phenomena
  - Behavioral modeling involves **dynamic** information
  - Changing environments require **temporal reasoning**

- Formal **theory of time** for reasoning about temporal information, changes over time, and knowledge about how these changes occur

Components of a temporal representation

- Theory of time
  - Structure of the temporal representation—primitive units
  - Expressiveness of the language and what we can reason about

- Temporal incidence
  - Domain independent properties for determining truth-value
    - How we determine what propositions are true or false relative to time (i.e., P and $\neg P$ cannot both be true in the same instant)

- Temporal propositions
  - **Fluents**—propositions that can change over time
    “the light is on”, “the ball is moving at speed v”, “the battery is charging”
  - **Events/Actions**—occurrences that happen in the world and can change its state

Theories of time

- Requirements for a formal theory of time
  - Represent **commonsense** notions of time
  - Represent events and changing values of fluents **without contradicting** intuitions about time

- What is going to be the **primitive** unit of time?
  - Instants (or points)—instants of time with no duration
  - Periods—intervals with positive duration
  - Events
  - Hybrid (instants AND intervals)

Instant-based theories of time

- Traditional structure for time adopted from classical physics
  - Time is **continuum** of consecutive instants

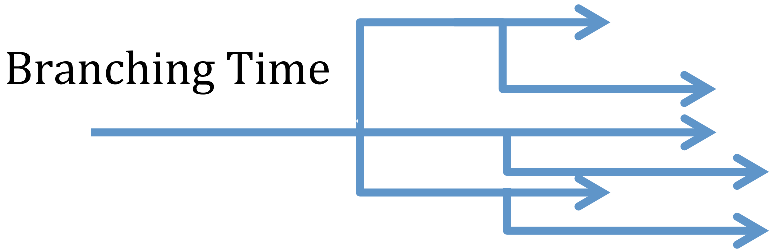
- Instant—**durationless** pieces of time (beginning and end not distinct)

- Defined on a structure $(\mathcal{I}, \prec)$ where
  - $\mathcal{I}$ is a set of instants and
  - $\prec$ is an order(partial or total) over the instants

- Example models:
  - Real-numbers time $(\boldsymbol{R}, <)$
  - Integer-numbers time $(\boldsymbol{Z}, <)$

Application-specific properties of instant theory

- Linear vs. Non-linear (parallel, circular, etc.)
  - **Linear** time has only one possible time line
    - $(i \prec i') \vee (i = i') \vee (i' \prec i)$
    - Can restrict linearity to only the past (i.e., left-linear time)
  - **Branching** time allows multiple possible time lines when uncertain



- Boundedness vs. infinite set of possible instants
  - Bounds allow for more efficient computation

- Discrete vs. Dense
  - **Discrete**—instants are adjacent, but not continuous
    - Each instant has a single previous and next instant
  - **Dense**—there is an instant between any two instants
    - $\forall i, i' \, (i \prec i' \Rightarrow \exists i'' \, (\mathrm{i} \prec i'' \prec i'))$

Period-based theories of time

- Periods (intervals) are associated with events that take time
  - Primitive time unit more related to our intuitive experiences

- Defined on a structure ($\mathcal{P}$, R) where
  - P is a set of **periods**
  - R is a set of **relationships** between periods

- Example models
  - Allen's Interval Theory ($\mathcal{P}$, AR)
  - Allen & Hayes Interval Theory ($\mathcal{P}$, $Meets$)

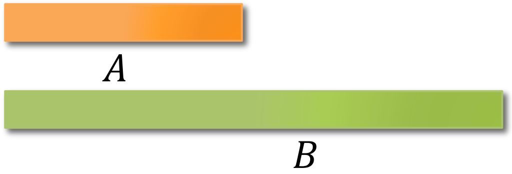
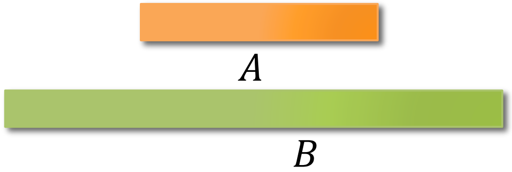
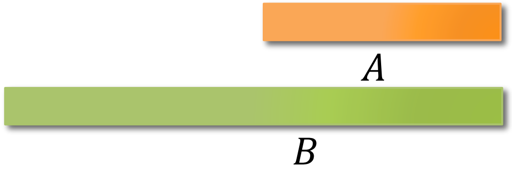
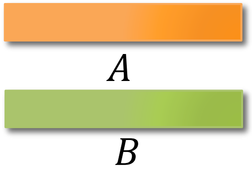
Allen’s Interval Theory

James F. Allen: *Maintaining knowledge about temporal intervals*. In: *Communications of the ACM*. 26 November 1983. ACM Press. pp. 832–843.)

- Structure ($\mathcal{P}$, *AR*) where
  - *AR* is the set of 13 **primitive period relationships** corresponding to all possible qualitative relationships between two intervals

Axioms for Allen’s Interval Theory	
A1	Given any period, there is another period related to it by each relationship in *AR*
	$\forall P \in P, R \in AR\ \ \exists P'\ R(P, P')$
A2	The relationships in *AR* are **mutually exclusive**
	$\forall P,P' \in P, R \in AR\ \forall R' \in AR - R,\ \ (R(P, P') \Rightarrow \neg R'(P, P'))$
A3	The relationships in *AR* have **transitive** behavior (e.g., *p1* BEFORE *p2* and *p2* MEETS *p3* then *p1* BEFORE *p3*)

Qualitative interval relationships *AR*

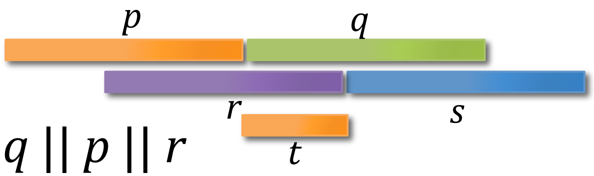
A BEFORE *B*	*B* AFTER *A*	
A MEETS *B*	*B* MET_BY *A*	
A OVERLAPS *B*	*B* OVERLAPPED_BY *A*	
A STARTS *B*	*B* STARTED_BY *A*	
A DURING *B*	*B* CONTAINS *A*	
A FINISHES *B*	*B* FINISHED_BY *A*	
A EQUALS *B*	*B* EQUALS *A*	

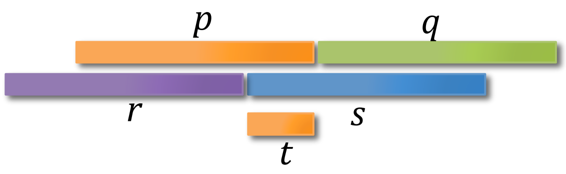
Allen-Hayes Interval Theory

- Structure ($\mathcal{P}$, *Meets*) where
  - Intuitive meaning of *Meets*(*p1*,*p2*) is that *p1* is one of the **immediate predecessors** (though not necessarily the unique one) of *p2*
  - All other 12 relations can be represented by *Meets*

- Axioms (|| indicates *Meets* and is xor)
  - $\forall p, q, r, s\ (p \,||\, q \land p \,||\, s \land \mathrm{r} \,||\, q \Rightarrow r \,||\, s)$

  - $\forall p, q, r, s\ (p \,||\, q \land \mathrm{r} \,||\, s \Rightarrow \exists t\ (p \,||\, t \,||\, s) \oplus p \,||\, s \oplus \exists t\ (r \,||\, t \,||\, q))$



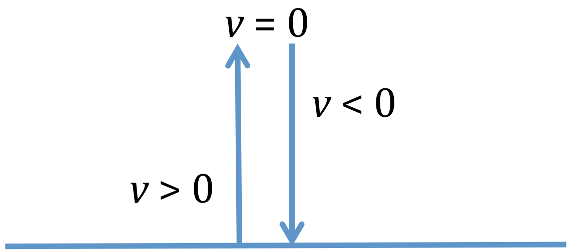


  - $\forall p\ \exists q, r\ \ q \,||\, p \,||\, r$
    Every period has a direct previous and next period
  - $\forall p, q, r, s\ (p \,||\, q \,||\, s \land p \,||\, r \,||\, s \Rightarrow q = r)$
    Periods with the same begin and end are equal
  - $\forall p, q\ (p \,||\, q \Rightarrow \exists r, s, t\ (r \,||\, p \,||\, q \,||\, s \land\ r \,||\, t \,||\, s)$
    Periods can be concatenated to form a single period

Challenges caused by periods

- Instantaneous events
  - Some events naturally represented as instants rather than periods E.g., “shoot the gun”, “start moving”
  - How do we represent sequences of instantaneous events when there is also continuous change—what happens “first”?
- Fluents that hold at an instant
  - E.g., “The patient’s temperature was 99 degrees at 10:00am”
  - What is truth value at instant of change?

Tossed ball scenario
$v > 0$ when ball moving up,
$v < 0$ when moving down,
$v = 0$ at point of change



  - How to represent the durationless instant where $v = 0$?

Dividing instant problem

- What if proposition holds for a period and is false in the next? How do we decide truth value at **instant in between**?
  - *A fire that has been burning is later burnt out. What happens at the intermediate instant between the two successive states of burning and being extinguished?*
  - *A light that has been off, and becomes on after it is switched on. Is the light on or off at the switching instant?*

- Period-based theories ignore question of whether instant is **member** of an interval
  - Ambiguity about the end points
  - Closed periods lead to **inconsistency** (both f and $\neg f$ true at meeting)
  - Open periods lead to **truth gap** (neither f nor $\neg f$ true at meeting)

Event-based theories of time

- Time is the relationship between **events** and processes
  - Similar to period-based theories
  - Events that happen at the same time not necessarily the same

- Defined on a structure $(\mathcal{E}, \prec, O)$ where
  - $\mathcal{E}$ is a set of events,
  - $\prec$ is a **precedence** relation, and
  - O is an **overlapping** relation

- Event theories based directly on perceived phenomena—events that take time
  - Do not distinguish between occurrences and their times of occurrence
  - Might be more intuitive to think of time as a separate entity

Hybrid theories with periods and instants

- Include instants and periods in a single theory of time
  - Real-world involves both instantaneous and durable events—**natural expression** for both
  - **Efficient computation**—use representation and relations that are most efficient for the particular event

- Defined on a structure ($\mathcal{I}, \mathcal{P}, \prec : \mathcal{I} \times \mathcal{I}, begin, end : \mathcal{I} \times \mathcal{P}$)where
  - $\mathcal{I}$ is an infinite set of instant symbols,
  - $\mathcal{P}$ is an infinite set of period symbols disjoint from $\mathcal{I}$,
  - $\prec$ is an order relation over the instants, and
  - the relations *begin, end* define the instants defining the beginning and ending of periods
- Periods are ordered pairs of instants

Axioms of a hybrid theory

- $\prec$ is a **strict linear order** over instants—single time line
  - $\neg(i \prec i)$
  - $i \prec i' \Rightarrow \neg(i' \prec i)$
  - $i \prec i' \wedge i' \prec i'' \Rightarrow i \prec i''$
  - $i \prec i' \vee i' \prec i \vee i = i'$
- Instants are **unbounded**
  - $\exists i' \, (i' \prec i), \exists i' \, (i \prec i')$
- End points of periods are ordered—no durationless periods
  - $begin(i,p) \wedge end(i',p) \Rightarrow i \prec i'$
- All periods have **unique beginning** and **ending** instants
  - $\exists i \; begin(i,p), \exists i \; end(i,p)$
  - $begin(i,p) \wedge begin(i',p) \Rightarrow i = i', end(i,p) \wedge end(i',p) \Rightarrow i = i'$
- All ordered pairs of instants define a **unique period**
  - $i \prec i' \Rightarrow \exists p \, (begin(i,p) \wedge end(i',p))$
  - $begin(i,p) \wedge end(i',p) \wedge begin(i,p') \wedge end(i',p') \Rightarrow p = p'$

Incorporating time into logic

- Temporal incidence—how do we represent and determine what is true at a given time?

- Several ways of incorporating time into logic s.t. inference is still sound and complete
  - Temporal arguments
  - Modal temporal logics
  - Reified temporal logics

Temporal arguments in FOL

- Adds time as **argument** to FOL functions and predicates
  - Similar to adding state to predicates in the Situation Calculus

“The patient’s temperature is 99 degrees at 10:00 on Sept. 18” as
Temperature(*P*, 99, 09/18/2012 10:00am)

- No special status for time
  - Not expressive enough for general temporal assertions
  - Cannot represent commonsense notions of time e.g., “effects cannot precede their causes”

Modal temporal logics

- Extend propositional logic with additional modal operators
  - Referred to as **tense logic** in philosophy
  - Indicate temporal relationships between propositions

- Example modal operators

$F\phi$: formula ϕ "will be true"
$P\phi$: formula ϕ "was true"

$G\phi$: formula ϕ "will always be true"
$H\phi$: formula ϕ "was always true"

"The patient's temperature is 99 degrees at 10:00 on Sept. 18" as
Temperature(*P*, 99, 09/18/2012 10:00am)

Reified temporal logics

- **Reify** standard propositional or first-order logic
  - Use “meta-predicates” such as *Holds*, *Occur*, *Cause*, etc.
  - Relate predicates to temporal information

“The patient’s temperature is 99 degrees at 10:00 on Sept. 18” as *Holds*(*Temperature*(*P*, 99), 09/18/2012 10:00am)

- Accords special status to time
- Describe different types of temporal occurrence
- Negation easy to represent
- Represent **causal relationships** between events and their effects

Reified hybrid temporal model ($\mathcal{CD}$)

- Hybrid structure ($\mathcal{I}, \mathcal{P}, \prec : \mathcal{I} \times \mathcal{I}, begin, end : \mathcal{I} \times \mathcal{P}$)
  - Temporal **primitives**—instants and periods
    - Define instant-period (e.g., within) and period-period relations (e.g., MEETS) using $\prec$, *begin, end*
  - Instants and periods are both primitives
    - Avoid dividing instant problem because can represent interior of periods and the instants at end points in separate predicates
- Temporal propositions for **continuous fluents**, **discrete fluents**, and **events**
  - Fluents classified by whether change they represent is continuous (e.g., charging battery) or discrete (i.e., light on or off)
- Reified **temporal incidence** relations for each combination of temporal proposition and temporal primitive

$\mathcal{CD}$ Temporal incidence predicates

- Reify each possible relationship between primitives (instants and periods) and predicates (fluents and events)

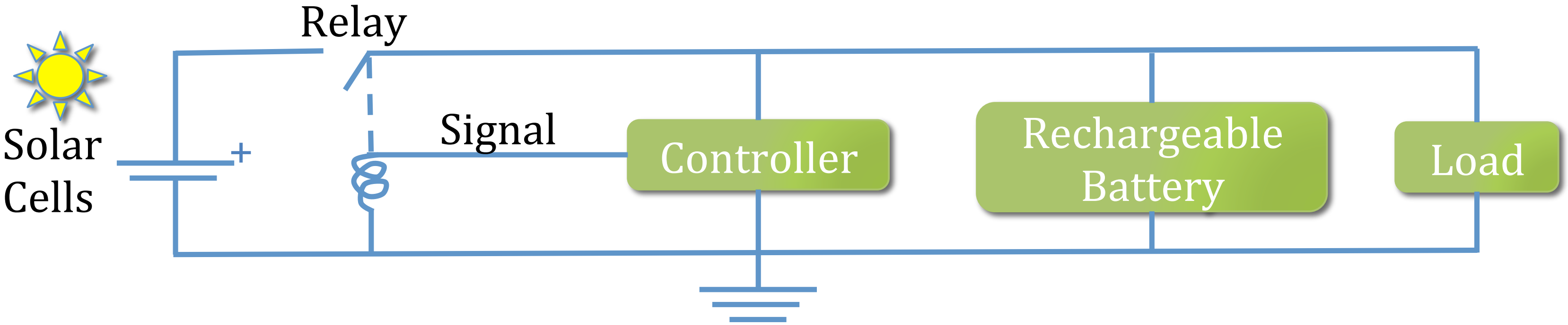
$HOLDS_{on}(f,p)$	The continuous fluent f holds throughout period p
$HOLDS_{on}(f,p)$	The discrete fluent f holds throughout period p
$HOLDS_{at}(f,i)$	The continuous fluent f holds at instant i
$HOLDS_{at}(f,i)$	The discrete fluent f holds at instant i
$OCCURS_{on}(e,p)$	The event e occurs on the period p
$OCCURS_{at}(e,i)$	The event e occurs at the instant I

- Use the *begin* and *end* relations in a functional form $i = begin(p)$

$\mathcal{CD}$ Axioms and properties

- Fluent holds during a period iff it holds at its **inner instants**
  - $HOLDS_{on}(f,p) \Leftrightarrow (Within(i,p) \Rightarrow HOLDS_{at}(f,i))$
- Discrete fluents **cannot** hold at an **isolated instant**
  - Has some value for one period, change event occurs at the instant, and then has another durable value w.r.t. a period
  - $HOLDS_{at}(f,i) \Rightarrow \exists p\ (HOLDS_{on}(f,p) \wedge (Within(i,p) \vee begin(i,p) \vee end(i,p)))$
- No special rules for events in general—**application specific** (e.g., a war can occur at multiple periods, but only if they meet; represent an election as instantaneous or durable)
- Non-atomic fluents—$\mathcal{CD}$ is **compositional**
  - $HOLDS_{at}(\neg f,i) \Leftrightarrow \neg HOLDS_{at}(f,i)$
  - $HOLDS_{at}(f \wedge f',i) \Leftrightarrow HOLDS_{at}(f,i) \wedge HOLDS_{at}(f',i)$
  - $HOLDS_{at}(f \vee f',i) \Leftrightarrow HOLDS_{at}(f,i) \vee HOLDS_{at}(f',i)$

Modeling a (green!) circuit with *CD*



- Description of a simple solar powered circuit
    1. If the sun is shining and the relay is closed, then the solar array acts as a constant current source and the battery accumulates charge
    2. If the relay is closed, when the signal from the controller goes high the relay opens.
    3. If the relay is open, when the signal from the controller goes low the relay closes.
    4. If the signal is low, when the controller detects that the charge level in the battery has reached threshold *q2* the controllers turns on the signal to the relay

The challenge of sequences of events

- **Initial state**: *signal* = *low*, *relay* = *closed*, sun is shining
- How can we model the temporal incidence of the following:

  “The signal goes high and immediately after the relay opens.”

  using the environment rules from the previous slide?

State	Time	Q_{BA} *(battery charge)*	*signal*	*relay*
s0	$(t1, t2)$	$< q2$	*low*	*closed*
s1	$t2$	$= q2$	*low*	*closed*
s1.1	$t2.1$	?	*high*	*closed*
s1.2	$t2.2$	?	*high*	*open*
s2	(*t2.2,*__)	$< q2$	*high*	*open*

The challenge of sequences of events

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s0	(*t1*, *t2*)	< *q2*	*low*	*closed*
s1	*t2*	= *q2*	*low*	*closed*
s1.1	*t2.1*	?	*high*	*closed*
s1.2	*t2.2*	?	*high*	*open*
s2	(*t2.2*,__)	< *q2*	*high*	*open*

signal goes high

relay opens

dividing instant!

Model instantaneous and continuous change

- What we intuitively want to model is much simpler

State	Time	Q_{BA} *(battery charge)*	*signal*	*relay*
s0	*(t1, t2)*	$< q2$	*low*	*closed*
s1	*t2*	$= q2$	*?*	*?*
s2	*(t2.2,__)*	$< q2$	*high*	*open*

- Can formalize this with $\mathcal{CD}$

State	$\mathcal{CD}$ Logic Representation
s0	$\text{HOLDS}_{on}(Q_{BA} < q2, p1)$ $\text{HOLDS}_{on}(\text{signal} = \text{low}, p2)$ $\text{HOLDS}_{on}(\text{relay} = \text{closed}, p3)$
s1	$\text{HOLDS}_{at}(Q_{BA} = q2, \text{end}(p1))$ $\text{OCCURS}_{at}(\text{turn\_on}(\text{signal}), \text{end}(p1))$ $\text{HOLDS}_{on}(\text{signal} = \text{on}, p4)$ $\text{end}(p1) = \text{end}(p2)$ $\text{MEETS}(p2,p4)$
s2	$\text{OCCURS}_{at}(\text{open}(\text{relay}), \text{end}(p3))$ $\text{HOLDS}_{on}(\text{relay} = \text{open}, p5)$ $\text{end}(p2) = \text{end}(p3)$ $\text{MEETS}(p3,p5)$