### **Probabilistic Topic Models**

#### David M. Blei

Department of Computer Science Princeton University

August 22, 2011

### Information overload



As more information becomes available, it becomes more difficult to find and discover what we need.

We need new tools to help us organize, search, and understand these vast amounts of information.

# **Topic modeling**



Topic modeling provides methods for automatically organizing, understanding, searching, and summarizing large electronic archives.

- Discover the hidden themes that pervade the collection.
- 2 Annotate the documents according to those themes.
- 3 Use annotations to organize, summarize, and search the texts.

#### **Discover topics from a corpus**

human genome dna genetic genes sequence gene molecular sequencing map information genetics mapping project sequences

evolution evolutionary species organisms life origin biology groups phylogenetic living diversity group new two common

disease host bacteria diseases resistance bacterial new strains control infectious malaria parasite parasites united tuberculosis

computer models information data computers system network systems model parallel methods networks software new simulations

#### Model the evolution of topics over time



"Neuroscience"



#### Model connections between topics



#### Find hierarchies of topics



## Annotate images



SKY WATER TREE MOUNTAIN PEOPLE



SCOTLAND WATER FLOWER HILLS TREE



SKY WATER BUILDING PEOPLE WATER







#### FISH WATER OCEAN TREE CORAL

PEOPLE MARKET PATTERN TEXTILE DISPLAY BIRDS NEST TREE BRANCH LEAVES

#### **Discover influential articles**



Derek E. Wildman et al., Implications of Natural Selection in Shaping 99.4% Nonsynonymous DNA Identity between Humans and Chimpanzees: Enlarging Genus Homo, PNAS (2003) [178 citations]

## **Predict links between articles**

Markov chain Monte Carlo convergence diagnostics: A comparative review			
Minorization conditions and convergence rates for Markov chain Monte Carlo			
Rates of convergence of the Hastings and Metropolis algorithms			
Possible biases induced by MCMC convergence diagnostics			
Bounding convergence time of the Gibbs sampler in Bayesian image restoration	RTM		
Self regenerative Markov chain Monte Carlo	$(\psi_e$		
Auxiliary variable methods for Markov chain Monte Carlo with applications	e)		
Rate of Convergence of the Gibbs Sampler by Gaussian Approximation			
Diagnosing convergence of Markov chain Monte Carlo algorithms			
Exact Bound for the Convergence of Metropolis Chains			
	LD		
Exact Bound for the Convergence of Metropolis Chains	LDA		
Exact Bound for the Convergence of Metropolis Chains Self regenerative Markov chain Monte Carlo	LDA +		
Exact Bound for the Convergence of Metropolis Chains Self regenerative Markov chain Monte Carlo Minorization conditions and convergence rates for Markov chain Monte Carlo	LDA + Re		
Exact Bound for the Convergence of Metropolis Chains Self regenerative Markov chain Monte Carlo Minorization conditions and convergence rates for Markov chain Monte Carlo Gibbs-markov models	LDA + Regre		
Exact Bound for the Convergence of Metropolis Chains Self regenerative Markov chain Monte Carlo Minorization conditions and convergence rates for Markov chain Monte Carlo Gibbs-markov models Auxiliary variable methods for Markov chain Monte Carlo with applications	LDA + Regressi		
Exact Bound for the Convergence of Metropolis Chains Self regenerative Markov chain Monte Carlo Minorization conditions and convergence rates for Markov chain Monte Carlo Gibbs-markov models Auxiliary variable methods for Markov chain Monte Carlo with applications Markov Chain Monte Carlo Model Determination for Hierarchical and Graphical Models	LDA + Regression		

## **Characterize political decisions**



- tax credit, budget authority, energy, outlays, tax
  - county,eligible,ballot,election,jurisdiction -
- bank,transfer,requires,holding company,industrial
  - housing,mortgage,loan,family,recipient -
  - energy,fuel,standard,administrator,lamp
    - student,loan,institution,lender,school -
  - medicare, medicaid, child, chip, coverage
  - defense, iraq, transfer, expense, chapter -
- business,administrator,bills,business concern,loan -
- transportation, rail, railroad, passenger, homeland security
  - cover,bills,bridge,transaction,following
    - bills,tax,subparagraph,loss,taxable -
    - loss,crop,producer,agriculture,trade -
    - head,start,child,technology,award -
    - computer,alien,bills,user,collection -
  - science, director, technology, mathematics, bills
  - coast guard, vessel, space, administrator, requires
    - child,center,poison,victim,abuse
      - land,site,bills,interior,river -
    - energy, bills, price, commodity, market -
    - surveillance, director, court, electronic, flood
      - child,fire,attorney,internet,bills -
      - drug,pediatric,product,device,medical -
    - human,vietnam,united nations,call,people
      - bills,iran,official,company,sudan -
    - coin,inspector,designee,automobile,lebanon
      - producer, eligible, crop, farm, subparagraph
        - people,woman,american,nation,school
          - veteran, veterans, bills, care, injury -
- dod,defense,defense and appropriation,military,subtitle -

#### Organize and browse large corpora



#### Stanley Kubrick



(film, series, show) (theory, work, human) (son, year, death) (black, white, people) (god, call, give) (math, energy, light)

#### Stanley Kubrick (b)/36, 1928 – Hen 7, 21999 was an Amrica fill mir entros writer, produces and pholographic with investigation of the method of the pholographic with investigation of the method of the structure of the structure of genera is neverified in method of nevering, the vrieter of genera is neverified in the school pholographic structure of genera is neverified and the Hollynoved presers, maintaining almost confines of the Hollynoved presers, maintaining almost complete artific control and misling more as according to the own which and dime constraints, bot with the rate method of the school pholographic structure of the school school and the school pholographic school is and endeerstructure.

Kubrick's films are characterized by a formal visual spie and meticulous attention to detail—his later films often have elements of surrealism and expressionism that eachews structured linear narrative. His films are repeatedly described as a low and methodical, and are often proceived as a reflection of his obsessive and perfectionist nazare.<sup>11</sup> A recurring theme in his films is much indumerito to man. While other weed as



#### {theory, work, human}

words	related documents	related topics	
theory	Meme	{work, book, publish}	
work	Intelligent design	(law, state, case)	
human	Immanuel Kant	(son, year, death)	
idea	Philosophy of mathematics	{woman, child, man}	
term	History of science	{god, call, give}	
study	Free will	{black, white, people}	
view	Truth	(film, series, show)	
science	Psychoanalysis	{war, force, army}	
concept	Charles Peirce	(language, word, form)	
form	Existentialism	{@card@, make, design}	
world	Deconstruction	(church, century, christian)	
argue	Social sciences	{rate, high, increase}	
social	Idealism	(company, market, business)	

- What are topic models?
- What kinds of things can they do?
- How do I compute with a topic model?
- What are some unsanswered questions in this field?
- How can I learn more?

# **Related subjects**

Topic modeling is a case study in modern machine learning with probabilistic models. It touches on

- Directed graphical models
- Conjugate priors and nonconjugate priors
- Time series modeling
- Modeling with graphs
- Hierarchical Bayesian methods
- Approximate posterior inference (MCMC, variational methods)
- Exploratory and descriptive data analysis
- Model selection and Bayesian nonparametric methods
- Mixed membership models
- Prediction from sparse and noisy inputs

### If you remember one picture...



# Organization

#### Introduction to topic modeling

- Latent Dirichlet allocation
- Open source implementations and tools

#### Beyond latent Dirichlet allocation

- Modeling richer assumptions
- Supervised topic modeling
- · Bayesian nonparametric topic modeling

#### Algorithms

- Gibbs sampling
- Variational inference
- Online variational inference

#### Discussion, open questions, and resources

# Introduction to Topic Modeling

- Data are assumed to be observed from a generative probabilistic process that includes hidden variables.
  - In text, the hidden variables are the thematic structure.
- 2 Infer the hidden structure using posterior inference
  - What are the topics that describe this collection?
- Situate new data into the estimated model.
  - How does a new document fit into the topic structure?

#### Latent Dirichlet allocation (LDA)

#### Seeking Life's Bare (Genetic) Necessities

Haemophilus

oenome

COLD SPRING HARBOR, NEW YORK— How many genes does an organism need to survive? Last week at the genome meeting here," two genome researchers with radically different approaches presented complementary views of the basic genes needed for life. One research team, using computer analyses to compare known genomes, concluded that today's organisms can be sustained with just 250 genes, and that the carliest life forms

required a mere 128 genes. The other researcher mapped genes in a simple parasite and estimated that for this organism, 800 genes are plenty to do the job—but that anything short of 100 wouldn't be enough.

Although the numbers don't match precisely, those predictions

\* Genome Mapping and Sequencing, Cold Spring Harbor, New York, May 8 to 12.

SCIENCE • VOL. 272 • 24 MAY 1996

"are not all that far apart," especially in comparison to the 75,000 genes in the human genome, notes Six Andersson of Uppsala University in Sweden, who arrived at the 800 number. But coming up with a consensus answer may be more than just a genetic number; game, particularly as more and note genomes are completely mapped and sequenced. "It may be a way of organizing any newly sequenced genome." explains Arcady Mushegian, a computational molecular biologist at the National Center for Biotechnology Information (NCBI) in Bothesda, Maryland. Comparing an



mate of the minimum modern and ancient genomes.

#### Simple intuition: Documents exhibit multiple topics.

# Generative model for LDA



- Each topic is a distribution over words
- Each document is a mixture of corpus-wide topics
- Each word is drawn from one of those topics

## The posterior distribution



- In reality, we only observe the documents
- The other structure are hidden variables

# The posterior distribution



- Our goal is to **infer** the hidden variables
- I.e., compute their distribution conditioned on the documents
  p(topics, proportions, assignments | documents)

# LDA as a graphical model



- Encodes our assumptions about the data
- Connects to algorithms for computing with data
- See Pattern Recognition and Machine Learning (Bishop, 2006).

# LDA as a graphical model



- Nodes are random variables; edges indicate dependence.
- Shaded nodes are observed.
- Plates indicate replicated variables.

### LDA as a graphical model



 $\prod_{i=1}^{K} p(\beta_i \mid \eta) \prod_{d=1}^{D} p(\theta_d \mid \alpha) \left( \prod_{n=1}^{N} p(z_{d,n} \mid \theta_d) p(w_{d,n} \mid \beta_{1:K}, z_{d,n}) \right)$ 



- This joint defines a posterior.
- From a collection of documents, infer
  - Per-word topic assignment z<sub>d,n</sub>
  - Per-document topic proportions θ<sub>d</sub>
  - Per-corpus topic distributions β<sub>k</sub>
- Then use posterior expectations to perform the task at hand, e.g., information retrieval, document similarity, exploration, ...



Approximate posterior inference algorithms

- Mean field variational methods (Blei et al., 2001, 2003)
- Expectation propagation (Minka and Lafferty, 2002)
- Collapsed Gibbs sampling (Griffiths and Steyvers, 2002)
- Collapsed variational inference (Teh et al., 2006)
- Online variational inference (Hoffman et al., 2010)

Also see Mukherjee and Blei (2009) and Asuncion et al. (2009).

### **Example inference**



- Data: The OCR'ed collection of Science from 1990-2000
  - 17K documents
  - 11M words
  - 20K unique terms (stop words and rare words removed)
- Model: 100-topic LDA model using variational inference.

#### **Example inference**

#### Seeking Life's Bare (Genetic) Necessities

COLD SPRING HARBOR, NEW YORK— How many genes does an organism need to survive? Last week at the genome meeting here," two genome researchers with radically different approaches presented complementary views of the basic genes needed for life. One research team, using computer analyses to compare known genomes, concluded that today's organisms can be sastained with host 255 genes. and that the entites life forms

required a mere 128 genes. The other researcher mapped genes in a simple parasite and estimated that for this organism, 800 genes are plenty to do the job—but that anything short of 100 wouldn't be enough.

Although the numbers don't match precisely, those predictions

\* Genome Mapping and Sequencing, Cold Spring Harbor, New York, May 8 to 12. "are not all that far apart," especially in comparison to the 75,000 genes in the human genome, notes Six Andresson of Uppsala University in Sweden, who arrived at the 800 numbers apart, who arrived at the about the second second second second numbers game, particularly as more and more genomes are completely mapped and sequenced. "It may be a way of organizing any newly sequenced genome," explains Arcady Mosherian a commensional mo-

Arcady Mushegian, a computational molecular biologist at the National Center for Biotechnology Information (NCBI) in Bethesda, Maryland. Comparing an





SCIENCE • VOL. 272 • 24 MAY 1996



#### **Example inference**

human genome dna genetic genes sequence gene molecular sequencing map information genetics mapping project sequences

evolution evolutionary species organisms life origin biology groups phylogenetic living diversity group new two common

disease host bacteria diseases resistance bacterial new strains control infectious malaria parasite parasites united tuberculosis

computer models information data computers system network systems model parallel methods networks software new simulations

#### Example inference (II)

#### Chaotic Beetles

Charles Godfray and Michael Hassell

Ecologists have known since the pioneering — convincing evidence to date of work of May in the mid-1970s (1) that the population dynamics of animals and plants can be exceedingly complex. This complexity arises from two sources: The tangled web of interactions that constitute any natural community provide a myriad of different pathways for species to interact, both directly and indirectly. And even in isolated populations the nonlinear feedback processes present in all natural populations can result in complex dynamic behavior. Natural populations can show persistent oscillatory dynamics and chaos, the latter characterized by extreme sensitivity to initial conditions. If such chaotic dynamics were common in nature, then this would have important ramifications for the management and conservation of natural resources. On page 389 of this issue, Costantino et al. (2) provide the most

complex dynamics and chaos in a biological population-of the flour beetle. Tribolium castaneum (see figure).

It has proven extremely difficult to demonstrate complex dynamics in populations in the field. By its very nature, a chaotically fluctuating population will superficially resemble a stable or cyclic population buffeted by the normal random perturbations experienced by all species. Given a long enough time series, diagnostic tools from nonlinear mathematics can be used to identify the telltale signatures of chaos. In phase space, chaotic trajectories come to lie on "strange attractors," curious geometric objects with fractal structure and hence noninteger dimension. As they into the future. The strength of the mixing that gives rise to the extreme sensitivity to initial conditions can be measured mathematically estimating the Liapunov expo-

Cannibalism and chaos. The flour beetle Tribolium castaneum, exhibits chaotic population dynamics when the amount of cannibalism is altered in a mathematical model.

1 mm

SCIENCE • VOL. 275 • 17 JANUARY 1997

nent, which is positive for chaotic dynamics and nonnositive otherwise. There have been many attempts to estimate attractor dimension and Lianunov exponents from time series data, and some candidate chaotic population have been identified (some insects, rodents, and most convincingly, human childhood diseases), but the statistical difficulties preclude any broad generalization (3).

move over the surface of the attractor, sets of

adjacent trajectories are pulled apart, then stretched and folded, so that it becomes impossible to predict exact population densities

> An alternative approach is to parameterize population models with data from natural populations and then compare their predictions with the dynamics in the field. This technique has been gaining popularity in recent years, helped by statistical advances in parameter estimation. Good ex-

> > 323

The authors are in the Department of Biology, Imperial College at Silwood Park, Ascot, Berks, SL5 7PZ UK, Email: m hassell@ic ac uk

### **Example inference (II)**

problem problems mathematical number new mathematics university two first numbers work time mathematicians chaos chaotic

model rate constant distribution time number size values value average rates data density measured models

selection male males females sex species female evolution populations population sexual behavior evolutionary genetic reproductive

species forest ecology fish ecological conservation diversity population natural ecosystems populations endangered tropical forests ecosystem

## Held out perplexity



perplexity = exp 
$$\left\{ \frac{-\sum_{d} \log p(\mathbf{w}_{d})}{\sum_{d} N_{d}} \right\}$$

#### Used to explore and browse document collections

#### Automatic Analysis, Theme Generation, and Summarization of Machine-Readable Texts

#### Gerard Salton, James Allan, Chris Buckley,

Vast amounts of text material are now available in machine-reac processing. Here, approaches are outlined for manipulating and ac subject areas in accordance with user needs. In particular, meth mining text themes, traversing texts selectively, and extracting as reflect text content.

Many kinds of texts are currently available in machine-readable form and are amenable to automatic processing. Because the available databases are large and cover many different subject areas, automatic aids must be provided to users interested in accessing the data. It has been suggested that links be placed between related pieces of text, connecting, for example, particular text paragraphs to other paragraphs covering related subject matter. Such a linked text structure, often called hypertext, makes it possible for the reader to start with particular text passages and use the linked structure to find related text elements (1). Unfortunately, until now, viable methods for automatically building large hypertext structures and for using such structures in a sophisticated way have not been available. Here we give methods for constructing text relation many and for using text relations to access and use text databases. In particular, we outline procedures for determining text themes, traversing texts selectively, and extracting summany statements that reflect text content.

#### Text Analysis and Retrieval: The Smart System

The Smart system is a sophisticated text retrieval tool, developed over the past 30 years, that is based on the vector space

The authors are in the Department of Computer Science, Cornel University, Ithaca, NY 14853-7501, USA. model of retrieval model, all informa sented by sets, or v is typically a word, associated with th ation. In principle chosen from a cont a thesaurus, but bec constructing such for unrestricted top to derive the terms under considerati terms assigned to a text content.

 $\Theta \Theta \Theta$ 

13 11

Refresh Home

Because the terr for content represent introduce a term-w signs high weights to and lower weights to A powerful term-w kind is the well-kn (term frequency frequency), which frequency (f,) in p with a low frequency (f.). Such terms dist which they occur fro When all texts sented by weighted  $D_i = (d_{i1}, d_{i2}, ...$ weight assigned to similarity measure tween pairs of ye similarity. Thus, a SCIENCE · VO "Automatic Analysis, Theme Generation, and Summarization of Machine-Readable Texts" (1994)

file:///Users/blei/doc.html

	TOPIC	PROB
	data computer system information network	0.30
1	nformation library text index libraries	0.19
1	wo three four different single	0.16
DOC	UMENT	SCOR
"Glo	bal Text Matching for Information Retrieval" (1991)	0.257
"Aut	comatic Text Analysis" (1970)	0.3110
	uging Similarity with n-Grams: Language-Independent agorization of Text" (1995)	0.3210
"Der	velopments in Automatic Text Retrieval" (1991)	0,548
"Sin (196	nple and Rapid Method for the Coding of Punched Cards" 2)	0.3610
"Dat	a Processing by Optical Coincidence" (1961)	0.429
"Pat	tern-Analyzing Memory" (1976)	0.4320
"Th	e Storing of Pamphlets" (1899)	0.444
Dev	unched-Card Technique for Computing Means, Standard iations, and the Product-Moment Correlation Coefficient for Listing Scattergrams" (1946)	0.455



Global Text Matching for Information Retrieval

by Professor Minot and has been adopted only after trial of several other methods.

# Aside: The Dirichlet distribution

 The Dirichlet distribution is an exponential family distribution over the simplex, i.e., positive vectors that sum to one

$$p(\theta \mid \vec{\alpha}) = \frac{\Gamma(\sum_{i} \alpha_{i})}{\prod_{i} \Gamma(\alpha_{i})} \prod_{i} \theta_{i}^{\alpha_{i}-1}$$

- It is conjugate to the multinomial. Given a multinomial observation, the posterior distribution of θ is a Dirichlet.
- The parameter  $\alpha$  controls the mean shape and sparsity of  $\theta$ .
- The topic proportions are a *K* dimensional Dirichlet. The topics are a *V* dimensional Dirichlet.












# $\alpha = 0.001$



Why does the LDA posterior put "topical" words together?

- Word probabilities are maximized by dividing the words among the topics. (More terms means more mass to be spread around.)
- In a mixture, this is enough to find clusters of co-occurring words.
- In LDA, the Dirichlet on the topic proportions can encourage sparsity, i.e., a document is penalized for using many topics.
- Loosely, this can be thought of as softening the strict definition of "co-occurrence" in a mixture model.
- This flexibility leads to sets of terms that more tightly co-occur.

# Summary of LDA



- LDA can
  - · visualize the hidden thematic structure in large corpora
  - generalize new data to fit into that structure
- Builds on Deerwester et al. (1990) and Hofmann (1999) It is a *mixed membership model* (Erosheva, 2004). Relates to *multinomial PCA* (Jakulin and Buntine, 2002)
- Was independently invented for genetics (Pritchard et al., 2000)

There are many available implementations of topic modeling-

LDA-C*	A C implementation of LDA
HDP*	A C implementation of the HDP ("infinite LDA")
Online LDA*	A python package for LDA on massive data
LDA in R <sup>*</sup>	Package in R for many topic models
LingPipe	Java toolkit for NLP and computational linguistics
Mallet	Java toolkit for statistical NLP
TMVE*	A python package to build browsers from topic models

\* available at www.cs.princeton.edu/~blei/

## Example: LDA in R (Jonathan Chang)

perspective identifying tumor suppressor genes in human... letters global warming report leslie roberts article global.... research news a small revolution gets under way the 1990s.... a continuing series the reign of trial and error draws to a close... making deep earthquakes in the laboratory lab experimenters... quick fix for freeways thanks to a team of fast working... feathers fly in grouse population dispute researchers...

245 1897:1 1467:1 1351:1 731:2 800:5 682:1 315:6 3668:1 14:1 260 4261:2 518:1 271:6 2734:1 2662:1 2432:1 683:2 1631:7 279 2724:1 107:3 518:1 141:3 3208:1 32:1 2444:1 182:1 250:1 266 2552:1 1993:1 116:1 539:1 1630:1 855:1 1422:1 182:3 2432:1 233 1372:1 1351:1 261:1 501:1 1938:1 32:1 14:1 4067:1 98:2 148 4384:1 1339:1 32:1 4107:1 2300:1 229:1 529:1 521:1 2231:1 193 569:1 3617:1 3781:2 14:1 98:1 3596:1 3037:1 1482:12 665:2

docs <- read.documents("mult.dat") K <- 20 alpha <- 1/20

. . . .

eta <- 0.001

....

model <- lda.collapsed.gibbs.sampler(documents, K, vocab, 1000, alpha, eta)</pre>

1	2	3	4	5
dna	protein	water	savs	mantle
gene	' cell	climate	researchers	high
sequence	cells	atmospheric	new	earth
genes	proteins	temperature	university	pressure
sequences	receptor	global	just	seismic
human	fig	surface	science	crust
genome	binding	ocean	like	temperature
genetic	activity	carbon	work	earths
analysis	activation	atmosphere	first	lower
two	kinase	changes	years	earthquakes
6	7	8	9	10
end	time	materials	dna	disease
article	data	surface	i rna	cancer
start	two	high	transcription	patients
science	model	structure	protein	human
readers	59	temperature	site	gene
service	ayatem	molecules	binding	medical
news	number	chemical	sequence	studies
card	different	molecular	proteins	drug
circle	mate	fo	specific	normal
letters		university	sequences	drugs
11	12	13	14	15
	species	protein	cells	space
years million	Species	protein structure	Cells	space solar
	species evolution population	protein structure proteins	Cells cell virus	space solar observations
years million ago age	species evolution population evolutionary	protein structure proteins two	Cells Cell virus hiv	space solar observations earth
years million ago age university	Species evolution population evolutionary university	protein structure proteins	CEIIS CEII virus hiv infection	space solar observations
Years million ago age university north	Species evolution population evolutionary university populations	protein structure proteins two amino binding	CellS Cell virus hiv infection immune	space solar observations earth stars university
years million ago age university	Species evolution population evolutionary university	protein structure proteins two amino binding acid	CEIIS CEII virus hiv infection	space solar observations earth stars
years million ago age university north early fig	Species evolution population evolutionary university populations	protein structure proteins two amino binding acid residues	Cells Cell virus hiv infection immune human antigen	Space solar observations earth stars university mass sun
Years million ago age university north early	Species evolution population evolutionary university populations natural studies genet:	protein structure proteins two amino binding acid residues molecular	Cells Cell virus hiv infection immune human antigen infected	space solar observations earth stars university mass
years million ago age university north early fig	Species evolution population evolutionary university populations natural studies	protein structure proteins two amino binding acid residues	Cells Cell virus hiv infection immune human antigen	Space solar observations earth stars university mass sun
years million ago age university north early fig evidence record 16	Species evolution population evolutionary university populations natural studies genetic heap 17	protein structure proteins two amino binding acid residues molecular structural	Cells Cells virus hiv infection immune human antigen infected viral	Space solar observations earth stars university mas sun attronomers telescope 20
years million ago age university north early fig evidence record 16 fax	Species evolution population evolutionary university populations natural studes genet: teap 17 Cells	protein structure proteins two amino binding acid residues molecular structural	CEIS CEIS Virus hiv infection immune human antigen infected viral 19 research	Space solar observations earth stars university mass sun astronomers telescope 20 neurons
years million ago age university north early fig evidence record 16 fax manager	species evolution population evolutionary university populations studies genetic tearry 17 Cells Cells	protein structure proteins two amino binding acid residues molecular structural 18 energy electron	Cells Cells Virus hiv infection human antigen infected viral 19 research science	space solar observations earth stars university astroamers stelecope 20 neurons brain
Years million ago age university north early fig evidence record 16 fax manager science	species evolution population evolutionary university populations studes studes studes studes cell cell gene	protein structure proteins two amino binding acid residues molecular structural 18 energy electron state	Cells Cell Virus hiv infection immune human antigen infected viral 19 research science national	space solar observations earth stars university mass son autoromers telerope 20 PeurOns brain oelis
Yearts million age university early early fax manager science aaas	Species by output population evolutionary university populations and states states states and states states st	protein structure proteins two amino binding acid residues molecular stockars thocka	CEIS CEI Virus Infection Immune human antigen infected viral 19 research science national scientific	space solar observations earth stars stars astronome astronome astronome astronome astronome starsope 20 neurons brain cells activity
Years million ago age university north early fig evidence record 16 fax manager science aaas advertising	species evolution population evolutionary university populations studes studes exerts tates tates genes genes genes genes genes genes	protein structure proteins two amino binding acid residues residues residues two tural 18 energy electron state light quantum	Cells Cell Virus hiv infection immune human antigen infected viral 19 research science national	space solar observations earth stars university mas autoromes telecope 20 Peurons brain cells activity fig
Vearts million age university early early record 16 fax manager science aaas advertising sales	Species brouten population evolutionary university populations man states states states states states states states states states cells cells cell genes genes expression development	protein structure proteins two amino acid residues molecular acid energy electron state light quantum physics	CEIS CEIS VITUS Infection immune human antigen infected viral 19 research science national scientific science	space solar observations earth stars sars wan advocmes wan advocmes advocmes advocmes advocmes advocmes brain cells activity fig chamets
Vears million ago age university north early fig evidence record 16 16 16 16 16 18 advertising sales member	species evolution population evolutionary university populations atudes evolutionary atudes evolutions cell genes genes expression development mutent	protein structure proteins two amino binding acid residues molecular atructural atructural 18 energy electron state light quantum physics electrons	Cells Cell Virus hiv infection immune human antigen infected vrai 19 research science national scientis scientis	space solar observations earth stars unversity mas autonomes beaution 20 20 20 20 20 20 20 20 20 20 20 20 20
Vears million age undersity north early ervicence record 16 fax manager science aaas advertising sales member recruitment	Species brouten population evolutionary university populations man states states states states states states states states states cells cells cell genes genes expression development	protein structure proteins hwo amino binding acid residues molecular molecular state energy electron state light quantum physics electrons high	CEIS CEIS VITUS Infection immune human antigen infected viral 19 research science national scientific science	space solar observations earth stars sars advoumes wan advoumes wan advoumes wan advoumes gan brain colis activity fig chamels university colex
Vears million ago age university north early fig evidence record 16 16 16 16 16 18 advertising sales member	species evolution population evolutionary university populations atudes evolutionary atudes evolutions cell genes genes expression development mutent	protein structure proteins two amino binding acid residues molecular atructural atructural 18 energy electron state light quantum physics electrons	Cells Cell Virus hiv infection immune human antigen infected vrai 19 research science national scientis scientis scientis	space solar observations earth stars university mean 20 20 20 20 20 20 20 20 20 20 20 20 20

### Open source document browser (with Allison Chaney)



#### Stanley Kubrick



related topics (film, series, show) {theory, work, human} {son, year, death} {black, white, people} (god, call, give) {math, energy, light}

#### Standarg Kuberick (bit) 26, 1929 – Putch 7, 1999 was an American IIIm Greeness writese produces and photographic who hold in Eighted during most of the last comparison of the standard during most of the last complexes are write which he closes his subjects. In a low method of warding, the warley of games he worked in the schemical performance and the schemistary of the complexe are strict concerns, and in reclusions about 18 complexe are the complexe write complexe are the complexe are strict correst and moles (more as complexes and ha own while and time constraints, but with the rare schemers).

Kubrick's films are characterized by a formal visual spie and meticulous attendint to detail—his later films often have elements of surrealism and expressionism that eachews structured linear narratives. His films are repeatedly described as a low and methodical, and are often proceived as a reflection of his obsessive and perfectionist nazare.<sup>11</sup> A recurring theme in his films is much indumerito to man. While other weed as



#### {theory, work, human}

words	related documents	related topics
theory	Meme	{work, book, publish}
work	Intelligent design	(law, state, case)
human	Immanuel Kant	{son, year, death}
idea	Philosophy of mathematics	{woman, child, man}
term	History of science	(god, call, give)
study	Free will	{black, white, people}
view	Truth	(film, series, show)
science	Psychoanalysis	{war, force, army}
concept	Charles Peirce	(language, word, form)
form	Existentialism	(@card@, make, design)
world	Deconstruction	(church, century, christian)
argue	Social sciences	{rate, high, increase}
social	Idealism	(company, market, business)

# Why develop these kinds of models?



- Organizing and finding patterns in data has become important in the sciences, humanties, industry, and culture.
- LDA can be embedded in more complicated models that capture richer assumptions about the data.
- Algorithmic improvements let us fit models to massive data.

# **Bigger Picture: Probabilistic modeling**



- Research in modeling separates these basic activities
- Though linked, we can work on each piece separately

# **Beyond Latent Dirichlet Allocation**

# So far...



- LDA is a simple topic model
- Can be used to find topics that describe a corpus
- Each document exhibits multiple topics
- · How can we build on this simple model of text?

# LDA is extendible



- LDA can be **embedded in more complicated models**, embodying further intuitions about the structure of the texts.
- LDA models can include syntax, authorship, word sense, dynamics, correlation, hierarchies, ...

# LDA is extendible



- The data generating distribution can be changed.
- LDA models can be built for images, social networks, music, purchase histories, computer code, genetic data, click-through-data, neural spike trains, ...

# LDA is extendible



- The LDA posterior can be used in creative ways
- It can be used for information retrieval, collaborative filtering, document similarity, visualization, ...

# **Beyond latent Dirichlet allocation**

- Modeling richer assumptions
  - Correlated topic models
  - Dynamic topic models
  - Measuring scholarly impact
- Supervised topic models
  - Supervised LDA
  - Relational topic models
  - Ideal point topic models
- Bayesian nonparametric topic models

# Modeling richer assumptions

- Correlated topic models
- Dynamic topic models
- Measuring scholarly impact

# The hidden assumptions of the Dirichlet



- The Dirichlet is an exponential family distribution on the *simplex*, positive vectors that sum to one.
- However, the near independence of components makes it a poor choice for modeling topic proportions.
- An article about *fossil fuels* is more likely to also be about *geology* than about *genetics*.

# The logistic normal distribution



- The logistic normal is a distribution on the simplex that can model dependence between components (Aitchison, 1980).
- The natural parameters of the multinomial are drawn from a multivariate Gaussian distribution.

$$egin{array}{rcl} X & \sim & \mathcal{N}_{K-1}(\mu, \Sigma) \ heta_i & \propto & \exp\{x_i\} \end{array}$$

# The correlated topic model (CTM) (Blei and Lafferty, 2007)



- Draw topic proportions from a logistic normal, where topic occurrences can exhibit correlation.
- Use for:
  - Providing a "map" of topics and how they are related
  - Better prediction via correlated topics

# Held out log probability in a CTM



- Analyzed held-out log probability on Science, 1960.
- CTM supports more topics and provides a better fit than LDA.



## Dynamic topic models (Blei and Lafferty, 2006)

1789



Inaugural addresses

2009



AMONG the vicissitudes incident to life no event could have filled me with greater anxieties than that of which the notification was transmitted by your order...

My fellow citizens: I stand here today humbled by the task before us, grateful for the trust you have bestowed, mindful of the sacrifices borne by our ancestors...

- LDA assumes that the order of documents does not matter.
- Not appropriate for corpora that span hundreds of years
- · We may want to track how language changes over time.

## **Dynamic topic models**



Topics drifting in time

# Modeling evolving topics



- Use a logistic normal distribution to model topics evolving over time (Aitchison, 1980)
- A state-space model on the natural parameter of the topic multinomial (West and Harrison, 1997)

$$\begin{array}{lll} \beta_{t,k} \mid \beta_{t-1,k} & \sim & \mathcal{N}(\beta_{t-1,k}, I\sigma^2) \\ p(w \mid \beta_{t,k}) & \propto & \exp\left\{\beta_{t,k}\right\} \end{array}$$

### Analyzing a document

#### **Original article**

#### **Topic proportions**

TECHNIEW: DNA SEQUENCING

#### Sequencing the Genome, Fast

James C. Hulikin and America A. Holbarray

Gregorie sequencing projects reveal for generic making of an experience by making of the sequence of the DNA bases, which encodes all of the infor-mation necessary for the kile of the arganclostides-adenine, thymidine, gameeine, and cytesine-which are lasked together last two decades, astornated DNA se-corrects have made the process of obtains ing the baso-by-baso sequence of DNA casiar. By application of an electric field across a gel matrix, these sequencers sepa-rate fluorescently labeled DNA reclevales that differ in size by one base. As the can be automatically recorded. The latest sequencer to be launched in

interest in the ABI 3700 has been generat-ed because Craig Venter of Celera Ge-nomics Corporation anticipates that -200 party to produce now inquiries for the ch-ties 3 gigabases (Gb) of the human prooms in 3 years. The specifications of the ABI 3700 machine say that, with less that 1 bour of human there per day, is en an-peares 148 anaptic per also, Anamig tata acha suspite per ana a wennge of 400 human there are a set for an an energy of 400 human the formation of the set of the set of the set of the read traiter and the spectre data of the set of the read traiter and set of the read traiter and set of the read traiter of the set of the s and solidal patients (Cortacy an Jeron age of 10 overlapping independent reads (2), the 75 exilino samples that Colora many pocesses will require -100,000 ABI 3700 machine days. With ~230 machines, that weeks out to bes that 2 years in shoul 454 days, which affords some margin of or-

rat for anotpected developments. At the Saraper Center, we have finished 146 Mb of groomic sequence from a vari-

The authors are at The Sangar Centre, Melloom Tool General Campus, Hissian, Carris, CB10 108 UK Frank Scridbargus, CA10

EXTERNAL DESIGNATION OF A STATE OF A ST ples from the plates into wells that open inshort of the design questification of four %-well plans in 12 hoars. The main increasion of the AIM 3300 is the use of a shorth their flavorance data-

ety of genomes, including 81 Mb of se-quence from the kuman genome, the lanust amount of any center so far 171 We largent amount of any center so far (3). We are atming to sequence 1 Gb of human so-quence in sough-dash farm by 2001, with a finished vention by 2005. Our sequenc-ing equipment includes 44 ABI 373XL, 61 ABI 373XL, and 31 ABI 373XL, 46 slab gel sequences from Parkis-Elmer plus 6 Molecular Dynamics MegaBACE 1000 capillary sequences, allowing a maximum detected with a spectral OCD (charge-cea-pled device) detectar. This arrangement standard and second standard and second standard and second standard second standard second standard second masse that there are no reoving parts in the detection system, other than a shutter in front of the CCD detector. We have evaluated these ma-



Inside per rand where the predicted ever rate is benchmore. With either type of system, regard to 10% (0.2.20), the "phene" () value was reads based to each type of rate. the aim is to read as many bases as people for a given sample of DNA-that is, long read lengths

to the stagger claims in locationer (1996-aux is our Research and Divergence do-partment for evaluations. Thus, the ABI 300 well attimum by a data to an proto-cut oparity to reach our good. The ABI 200 DNA supportent is the assore than a some than a source that and the assore that an assore than a source that assore that assore than a source that assore that assore that assore than the assore that the assore that assore that assore that assore that ABI 200 DNA supportent is that assore that assore that assore that assore that that assore that as or that as or that as or that assore tha The ABI 3700 DNA sequence is buil-into a floor-standing cabient, which con-tains in its base all the rangents required for its operation. The rangent containers are readily accessible for replanishment, which is required overy day under high-throughbling many short once. So, read length is

new sequencing technologies. We have directly compared the AIH 3700 sequencer to the ABI 377XL slab ard sequencer by evaluating the sequence dol obtained from both machines with huma DNA samples. These samples were sub minuter chemistry.

www.sciencemag.org SCIINCE VOL283 19 MARCH 1998

a no an in an an



### Analyzing a document

#### **Original article**

TECHNIEW: DNA SEQUENCING

Sequencing the Genome, Fast

James C. Haddin and America & Helbarres

Gregorie sequencing projects reveal for generic making of an experience by making of the sequence of the DNA bases, which encodes all of the infor-mation necessary for the kile of the argan last two decades, astornated DNA se-corrects have made the process of obtain ing the base-by-base sequence of DNA casiar. By application of an electric field across a gel matrix, these sequencers sepa-rate fluorescenily labeled DNA moleculer that differ in size by one base. As the

of these machines (7) will made the com-pany to produce raw sequence for the co-tise 3 gigsbases (Ob) of the human persons in 3 years. The specifications of the ABI 3700 machine my that, with less than 1 hour of human labor per day, it can se-quence 748 samples per day. Assuming that such sample gives an average of 400 huse pairs (bp) of tashle sequence data (its read length) and any socion from the on-tion human geneme is covered by an averand solidal patients (Cortacy an Jeron age of 10 overlapping independent reads (2), the 75 exilino samples that Colora many pocesses will require -100,000 ABI 3700 machine days. With ~230 machines, that weeks out to bes that 2 years in shoul 454 days, which affords some margin of or-

rat for anotpected developments. At the Saraper Center, we have finished 146 Mb of groomic sequence from a vari-

The authors are at The Sangar Centre, Melloom Tool General Campus, Hissian, Carris, CB10 108 UK Frank Scridbargus, CA10

short of the design questification of four %-well plans in 12 hoars. The main increasion of the AIM 3300 is the use of a shorth their flavorance dataety of genomes, including 81 Mb of se-quence from the kuman genome, the are attring to sequence 1 Gb of human si-quence in sough-dash fami by 2001, with a finished vension by 2005. Our sequenc-ing equipment includes 44 ABU 3773XL 46 ABU 3773XL, and 31 ABU 3773XL-46 shab-ed assusement from Parkie-Elmer abu 6 Molecular Dynamics MegaBACE 1000 capillary sequences, allowing a maximum

CA . TECH SIGHT ples from the plates into wells that open in-



tris into a capillary tinternal di the AB 1770. No sail per reachine the capitary inachine under-performs the slab gel machine by about 200 heres. Both sets of reads are from sure with AB Bg Dye Termine-ter chemistries. Read length is computed as the number of Lases per read where the predicted error rate is less than or regard to  $100, 1/2 \ge 20$ , the "phone" (2 value was result based for each type of real. With either type of syntax, the sim is to read as many bases as pendide for a phone sample of to the Sanoer Centre in December 1988-

are in our Research and Development de-rartment for evaluation. Thus, the ABI read twice as many bases but at half the second of another system is profesable, if partment for evaluation. Thus, the visit 3700 will ultimately be added to our prot-out capacity to reach our god. The ABI 3700 DNA sequences is built both systems cost the same. This is be cause assembling relatively fewer long-se The ABI 3200 DNA sequencier is total into a floor-standing cabinet, which con-rains in in base all the rangents required for its operation. The reagest containers are readily accessible for replanishman, which is required every day made high-throughbling many short case. So, read leasth is

new sequencing technologies. We have directly compared the ABI 3700 sequences to the ABI 377XL slab gel sequencer by evaluating the sequence dol obtained from both machines with huma DNA samples. These samples were sub

www.sciencemag.org SCIINCE VOL283 19 MARCH 1998

dna

sequence genome aenes sequences human gene sequencing chromosome regions analysis data genomic number

devices device materials current high gate light silicon material technology electrical fiber power based

data information network web computer language networks time software system words algorithm number internet

#### Most likely words from top topics

# Analyzing a topic



## Visualizing trends within a topic

"Theoretical Physics"

"Neuroscience"



## Evaluating the DTM on all of Science



See the browser at http://topics.cs.princeton.edu/Science/

# **Time-corrected document similarity**

Consider the expected Hellinger distance between the topic proportions of two documents,

$$d_{ij} = \mathrm{E}\left[\sum_{k=1}^{K} (\sqrt{\theta_{i,k}} - \sqrt{\theta_{j,k}})^2 \,|\, \mathbf{w}_i, \mathbf{w}_j\right]$$

- · Uses the latent structure to define similarity
- Time has been factored out because the topics associated to the components are different from year to year.
- Similarity based only on topic proportions

### **Time-corrected document similarity**

#### The Brain of the Orang (1880)

#### 5

#### Tribute in these many, which more solutiond in the authors as the gh of Decomber last the correction or to prefixe; to collection being and we prefer them in source sources. After publication by beaute a quanti and writes that the experts under the same are rain and being witherers.

and Provision 1. E. Bligged are proparing more clubes reports of their important papers, and promise them its early day.

THE BRAIN OF THE ORANG.

PERSONAL PROPERTY AND INCOME.

The lends of the Cong has been figured by Tubers, Benefits, belowdraw at or Lob and Work, Gandah, Liblanan, etc. Da answare, however, of the displant, Lobalan and the sensors, however, of the answare the sensor of any Dang's balance of the sensor answare the sensor of any Dang's balance figures to assume the sensor of any Dang's balance between the sensors of the sensors of the lab hashed and the sensors of the sensors of the lab hashed and the sensors of the sensors of the lab hashed and the sensors of the labor of the lab hashed and the sensors of the labor of the labor of the labor and the sensors of the labor of the labor of the good condition. It is mighted analytics measure, the of these first of the labor of the laboration of the sensors of the labor of the laboration of the of the laboration of the laboration of the laboration of the of the laboration of the laboration of the laboration of the of the laboration of the laboration of the laboration of the of the laboration of the laboration of the laboration of the of the laboration of the laboration of the laboration of the of the laboration of the laboration of the laboration of the of the laboration of the laboration of the laboration of the laboration of the of the laboration of the laboration of the laboration of the laboration of the of the laboration of the laboratio

beam of the Orang is in potent contour reactibility of of man more than those of whether of the Orangana which Eccumberd. In these the brain was more cluated. The general character of the toble and frequention.



f the Osser, Chimasure, and

builty, these any cortain lines' advection, however, inthe Orang runs and doesn the population between pabuilty of the orange of the state of the state of the builty of the state of the state of the state of the toward is may be parted in the target of the state of the toward is the Orang than is man. It is dimensions the borned is the Orang than is man. It is dimensioned by the state of the orange of the state of the state of the toward is the Orang than is man. It is dimensioned by the state of the orange of the state of the state of the toward is the Orang the state of the toward is the orange of the state of the state of the state of the the state of t

Encode Proceedings of the Academy of National Sciences, Fields, eth.

b and the second sec

SCIENCE.

1. Set a second seco



To a spind favor: cerembly k is continuous with the colial lobe, as the first, isotopical grows, networky is to smooth two the popurities compared concludes many memory in the population of the spin state of the hote entropic favore. Then is in the Cong Jaka a meritaning population with the population poldre the upper partial block into issue and south the result. The prevention, etc. In poper on the needed


### **Time-corrected document similarity**

Representation of the Visual Field on the Medial Wall of Occipital-Parietal Cortex in the Owl Monkey (1976)



### Measuring scholarly impact (Gerrish and Blei, 2009)

Einstein's Theory of Relativity

> Relativity paper #1 Relativity paper #3 Relativity paper #2 Relativity paper #4

My crackpot theory

History of Science

- Influential articles reflect future changes in language use.
- The "influence" of an article is a latent variable.
- Influential articles affect the drift of the topics that they discuss.
- The posterior gives a retrospective estimate of influential articles.

# Measuring scholarly impact



# Measuring scholarly impact



- Each document has an influence score *I<sub>d</sub>*.
- Each topic drifts in a way that is biased towards the documents with high influence.
- The posterior of *I*<sub>1:D</sub> can be examined to retrospectively find articles that best explain future changes in language.

# Measuring scholarly impact



- This measure of impact only uses the words of the documents. It correlates strongly with citation counts.
- High impact, high citation: "The Mathematics of Statistical Machine Translation: Parameter Estimation" (Brown et al., 1993)
- "Low" impact, high citation: "Building a large annotated corpus of English: the Penn Treebank" (Marcus et al., 1993)

# Measuring scholarly impact at large scale

(with S. Gerrish, A. Chaney and D. Mimno)



- PNAS, Science, and Nature from 1880–2005
- 350,000 Articles
- 163M observations
- Year-corrected correlation is 0.166



# Summary: Modeling richer assumptions

- The Dirichlet assumptions on topics and topic proportions makes strong conditional independence assumptions about the data.
- The **correlated topic model** uses a logistic normal on the topic proportions to find patterns in how topics tend to co-occur.
  - See also Li and McCallum (2007) for another approach.
  - See http://www.cs.princeton.edu/~blei/ for code.
- The **dynamic topic model** uses a logistic normal in a linear dynamic model to capture how topics change over time.
  - Documents can exhibit sequential structure.
  - Opens the door to a citation-free model of scholarly impact.
  - See also Wang and Blei (2010) for a continuous time variant
- What's the catch? The Dirichlet is easier to compute with than the logistic normal. (Stay tuned.)

- Supervised LDA
- Relational topic models
- Ideal point topic models

- LDA is an unsupervised model. How can we build a topic model that is good at the task we care about?
- Many data are paired with **response variables**.
  - User reviews paired with a number of stars
  - Web pages paired with a number of "likes"
  - Documents paired with links to other documents
  - Images paired with a category
- **Supervised topic models** are topic models of documents and responses, fit to find topics predictive of the response.

# Supervised LDA



- **1** Draw topic proportions  $\theta \mid \alpha \sim \text{Dir}(\alpha)$ .
- 2 For each word
  - Draw topic assignment  $z_n | \theta \sim Mult(\theta)$ .
  - Draw word  $w_n | z_n, \beta_{1:K} \sim \text{Mult}(\beta_{z_n})$ .
- ③ Draw response variable  $y | z_{1:N}, \eta, \sigma^2 \sim N(\eta^{\top} \bar{z}, \sigma^2)$ , where

$$\bar{z} = (1/N) \sum_{n=1}^{N} z_n$$

# Supervised LDA



- The response variable *y* is drawn *after* the document because it depends on *z*<sub>1:*N*</sub>, an assumption of **partial exchangeability**.
- Consequently, y is necessarily conditioned on the words.
- In a sense, this blends generative and discriminative modeling.

- Fit sLDA parameters to documents and responses. This gives:
  - topics β<sub>1:K</sub>
  - coefficients η<sub>1:K</sub>
- We have a new document  $w_{1:N}$  with unknown response value.
- We predict y using the SLDA expected value:

$$\mathbf{E}\left[\boldsymbol{Y} \mid \boldsymbol{w}_{1:N}, \boldsymbol{\alpha}, \boldsymbol{\beta}_{1:K}, \boldsymbol{\eta}, \boldsymbol{\sigma}^{2}\right] = \boldsymbol{\eta}^{\top} \mathbf{E}\left[\bar{\boldsymbol{Z}} \mid \boldsymbol{w}_{1:N}\right]$$

## **Example: Movie reviews**



- 10-topic sLDA model on movie reviews (Pang and Lee, 2005).
- Response: number of stars associated with each review
- Each component of coefficient vector η is associated with a topic.

## Held out correlation



## Diverse response types with GLMs

- Want to work with response variables that don't live in the reals.
  - binary / multiclass classification
  - count data
  - waiting time
- Model the response response with a generalized linear model

$$p(y \mid \zeta, \delta) = h(y, \delta) \exp\left\{rac{\zeta y - A(\zeta)}{\delta}
ight\} \; ,$$

where  $\zeta = \eta^{\top} \overline{z}$ .

• Complicates inference, but allows for flexible modeling.

# Image classification and annotation (Wang et al., 2009)



- Uses GLM sLDA for multiclass classification.
- Uses ideas from Blei and Jordan (2004) for annotation.

## Supervised topic models



- SLDA enables model-based regression where the predictor "variable" is a text document.
- It can easily be used wherever LDA is used in an unsupervised fashion (e.g., images, genes, music).
- SLDA is a supervised dimension-reduction technique, whereas LDA performs unsupervised dimension reduction.

### Relational topic models (Chang and Blei, 2010)



- Many data sets contain connected observations.
- For example:
  - Citation networks of documents
  - Hyperlinked networks of web-pages.
  - Friend-connected social network profiles

### Relational topic models (Chang and Blei, 2010)



- Research has focused on finding communities and patterns in the link-structure of these networks.
- We adapt sLDA to pairwise response variables.
   This adaptation leads to a model of content and connection.
- RTMs find related hidden structure in both types of data.

## **Relational topic models**



- Adapt fitting algorithm for sLDA with binary GLM response
- RTMs allow predictions about new and unlinked data. These predictions are out of reach for traditional network models.

# Predicting links from documents

Markov chain Monte Carlo convergence diagnostics: A comparative review	
Minorization conditions and convergence rates for Markov chain Monte Carlo	
Rates of convergence of the Hastings and Metropolis algorithms	
Possible biases induced by MCMC convergence diagnostics	RTM
Bounding convergence time of the Gibbs sampler in Bayesian image restoration	M
Self regenerative Markov chain Monte Carlo	$(\psi_e)$
Auxiliary variable methods for Markov chain Monte Carlo with applications	<sup>5</sup> e)
Rate of Convergence of the Gibbs Sampler by Gaussian Approximation	
Diagnosing convergence of Markov chain Monte Carlo algorithms	
Exact Bound for the Convergence of Metropolis Chains	
	LD
Exact Bound for the Convergence of Metropolis Chains	LDA
Exact Bound for the Convergence of Metropolis Chains Self regenerative Markov chain Monte Carlo	+
Exact Bound for the Convergence of Metropolis Chains Self regenerative Markov chain Monte Carlo Minorization conditions and convergence rates for Markov chain Monte Carlo	+
Exact Bound for the Convergence of Metropolis Chains Self regenerative Markov chain Monte Carlo Minorization conditions and convergence rates for Markov chain Monte Carlo Gibbs-markov models	+
Exact Bound for the Convergence of Metropolis Chains Self regenerative Markov chain Monte Carlo Minorization conditions and convergence rates for Markov chain Monte Carlo Gibbs-markov models Auxiliary variable methods for Markov chain Monte Carlo with applications	+
Exact Bound for the Convergence of Metropolis Chains Self regenerative Markov chain Monte Carlo Minorization conditions and convergence rates for Markov chain Monte Carlo Gibbs-markov models Auxiliary variable methods for Markov chain Monte Carlo with applications Markov Chain Monte Carlo Model Determination for Hierarchical and Graphical Models	

#### Given a new document, which documents is it likely to link to?

# **Predicting links from documents**

Commutation and the second	
Competitive environments evolve better solutions for complex tasks	
Coevolving High Level Representations	
A Survey of Evolutionary Strategies	
Genetic Algorithms in Search, Optimization and Machine Learning	RI
Strongly typed genetic programming in evolving cooperation strategies	RTM $(\psi_e)$
Solving combinatorial problems using evolutionary algorithms	(y
A promising genetic algorithm approach to job-shop scheduling	9 <sub>e</sub> )
Evolutionary Module Acquisition	
An Empirical Investigation of Multi-Parent Recombination Operators	
A New Algorithm for DNA Sequence Assembly	LI
Identification of protein coding regions in genomic DNA	LDA
Solving combinatorial problems using evolutionary algorithms	+
A promising genetic algorithm approach to job-shop scheduling	R
A genetic algorithm for passive management	eg
The Performance of a Genetic Algorithm on a Chaotic Objective Function	res
Adaptive global optimization with local search	Regression
Mutation rates as adaptations	5

#### Given a new document, which documents is it likely to link to?

### Predictive performance of each type



Cora corpus (McCallum et al., 2000)

### Predictive performance of each type



WebKB corpus (Craven et al., 1998)

### Predictive performance of each type



PNAS corpus (courtesy of JSTOR)

# **Spatially consistent topics**



- · For exploratory tasks, RTMs can be used to "guide" the topics
- Documents are geographically-tagged news articles from Yahoo! Links are the adjacency matrix of states
- RTM finds spatially consistent topics.

## **Relational topic models**



- RTMs let us analyze connected documents, modeling both content and connections.
- Most network models cannot predict with new and unlinked data.
- RTMs allow for such predictions
  - links given the new words of a document
  - words given the links of a new document

## The ideal point model



- A model devised to uncover voting patterns (Clinton et al., 2004).
- We observe roll call data v<sub>ij</sub>.
- Bills attached to discrimination parameters *a<sub>j</sub>*. Senators attached to ideal points *x<sub>i</sub>*.

# The ideal point model



- · Posterior inference reveals the political spectrum of senators
- Widely used in quantitative political science.

# The ideal point model is limited for prediction



- We can predict a missing vote.
- But we cannot predict all the missing votes from a bill.
- Cf. the limitations of collaborative filtering

### Ideal point topic models (Gerrish and Blei, 2010)



Use supervised topic modeling assumptions as a predictive mechanism from bill texts to bill discrimination.

### Ideal point topic models



# **Ideal point topics**



In addition to senators and bills, IPTM places **topics** on the spectrum.

### Prediction on completely held-out votes



Versus the LASSO, the IPTM correctly predicted 126,000 more votes.

## Summary: Supervised topic models

- Many of documents are associated with response variables.
- **Supervised LDA** embeds LDA in a generalized linear model that is conditioned on the latent topic assignments.
- **Relational topic models** use sLDA assumptions with pair-wise responses to model networks of documents.
- **Ideal point topic models** demonstrates how the response variables can themselves be latent variables. In this case, they are used downstream in a model of legislative behavior.
- Note that sLDA and the RTM (and others) are implemented in Jonathan Chang's excellent R package "Ida."
## Still other ways to build on LDA

#### New applications—

- Syntactic topic models (Boyd-Graber and Blei 2009)
- Topic models on images (Fei-fei and Perona 2005 and others)
- Topic models on social network data (Airoldi et al. 2008)
- Topic models on music data (Hoffman et al. 2008)
- Topic models for user recommendation (Wang and Blei, 2011)

#### Testing and relaxing assumptions—

- Spike and slab priors (Wang and Blei 2009 and Williamson et al. 2010)
- Models of word contagion (Elkan 2006)
- N-gram topic models (Wallach 2006)

#### Bayesian nonparametric topic models

- Why Bayesian nonparametric models?
- The Chinese restaurant process
- Chinese restaurant process mixture models
- The Chinese restaurant franchise
- Bayesian nonparametric topic models

## Why Bayesian nonparametric models?

- Topic models assume that the number of topics is fixed.
- It can be determined by cross validation and other model selection techniques.
- Bayesian nonparametric methods skirt model selection—
  - The data determine the number of topics during inference
  - Future data can exhibit new topics
- This is really a field unto itself, but it has found wide application in topic modeling.

### The Chinese restaurant process (CRP)



 N customers arrive to an infinite-table restaurant. Each sits down according to how many people are sitting at each table,

$$p(z_i = k \mid z_{1:(i-1)}, \alpha) \propto \begin{cases} n_k & \text{for } k \leq K \\ \alpha & \text{for } k = K+1. \end{cases}$$

- The resulting seating plan provides a partition
- This distribution is **exchangeable**: Seating plan probabilities are the same regardless of the order of customers (Pitman, 2002).

#### **CRP** mixture models



- Associate each table with a topic (β\*).
   Associate each customer with a data point (grey node).
- The number of clusters is infinite a priori; the data determines the number of clusters in the posterior.
- Further: the next data point might sit at new table.
- Exchangeability makes inference easy (see Neal, 2000).

#### The CRP is not a mixed-membership model



- Mixture models draw each data point from one component.
- The advantage of LDA is that it's a mixed membership model.
- This is addressed by the Chinese restaurant franchise.

## The Chinese restaurant franchise (Teh et al., 2006)

 $\beta_3^*$ 

#### **Corpus level restaurant**



#### **Document level restaurants**

At the corpus level, topics are drawn from a prior.



Each document-level table is associated with a customer at the corpus level restaurant.







Each word is associated with a customer at the document's restuarant. It is drawn from the topic that it's table is associated with

#### The CRF selects the "right" number of topics



#### Extended to find hierarchies (Blei et al., 2010)



#### BNP correlated topic model (Paisley et al., 2011)

{president party elect} (military army armed) (colony <b>finte-friateerfancehina</b> anu ion) (law convention international) {film award director} (willaw lord early (kill prisoner arrest) {County home population}	
(son fathele <b>rbpitherj</b> oman territory) (Post centre tostall) (publish story publication) (emperor reign imperial) ((sland <b>b3t)/pisk/f3t</b> ) (claum acces musica)	
student uffWersity igyestigation)  {album song music} (jersey york uniform)	
{church catholic roman} (ateriar month holday) {law legat/vootuth}ce award) (film scene movie) (ateriar ward) {company car engine) (sate town war) {company car engine)	
{language culture spanish} {art paipting addistly capitalism)	
(universe destroy series) (population female male) (political society argue) <b>(weapգանչիցԱցիցմեթնին</b> ին՝ örgänizätion) (god greek andiëtifty <sup>wall</sup> design)	
(music instrum <sup>6</sup> 8At <sup>t</sup> Af0560 <sup>10</sup> ) <sup>n event</sup> )	
(language letter, soup 4social theory cultural) (heat pressure mechanical)	
Глея рессие полянову	
(mot <b>(Aratik, plagidy,</b> solar) {water sub metal) (mathematician numeral decimal) (wave light field)	
{math function define}	

#### Summary: Bayesian nonparametrics

- Bayesian nonparametrics is a growing field (Hjort et al., 2011).
- BNP methods can define priors over combinatorial structures.
- In the posterior, the documents determine the particular form of the structure that is best for the corpus at hand.
- These models are also interpretable as **random distribution models**, such as the Dirichlet process (Fergusen 1973, Antoniak 1974).
- Recent innovations:
  - Improved inference methods (Blei and Jordan, 2005)
  - Dependent models, such as time series models (MacEachern 1999, Dunson 2010)
  - Models for predictions (Hannah et al. 2011)
  - Models for matrix factorization and other non-mixtures (Griffiths and Ghahramani, 2011)

# Algorithms

#### So far...



- We can express many kinds of assumptions about a corpus.
- Next: How can we analyze it under those assumptions?

## **Posterior inference**



- Posterior inference is the main computational problem.
- Inference links observed data to statistical assumptions.
- Inference on large data is crucial for topic modeling applications.

## **Posterior inference**



 Our goal is to compute the distribution of the hidden variables conditioned on the documents

*p*(topics, proportions, assignments | documents)

#### **Posterior inference for LDA**



· The joint distribution of the latent variables and documents is

$$\prod_{i=1}^{K} p(\beta_i \mid \eta) \prod_{d=1}^{D} p(\theta_d \mid \alpha) \left( \prod_{n=1}^{N} p(z_{d,n} \mid \theta_d) p(w_{d,n} \mid \beta_{1:K}, z_{d,n}) \right)$$

· The posterior of the latent variables given the documents is

$$p(\beta_{1:K}, \theta_{1:D}, Z_{1:D,1:N} | W_{1:D,1:N}).$$

#### **Posterior inference for LDA**



This is equal to

$$\frac{p(\beta_{1:K}, \theta_{1:D}, \mathbf{z}_{1:D}, \mathbf{w}_{1:D})}{\int_{\beta_{1:K}} \int_{\theta_{1:D}} \sum_{\mathbf{z}_{1:D}} p(\beta_{1:K}, \theta_{1:D}, \mathbf{z}_{1:D}, \mathbf{w}_{1:D})}.$$

- We can't compute the denominator, the marginal p(w<sub>1:D</sub>).
- This is the crux of the inference problem.

#### **Posterior inference for LDA**



- There is a large literature on approximating the posterior.
- We will focus on
  - Gibbs sampling
  - Mean-field variational methods (batch and online)

- Construct a **Markov chain** on the hidden variables, whose limiting distribution is the posterior.
- Collect independent samples from that distribution; approximate the posterior with them
- In **Gibbs sampling** the chain is defined by the conditional distribution of each hidden variable given observations and the current setting of the other hidden variables.

## Local and global variables



- · Local variables are local to each document
  - Topic proportions  $\theta_d$
  - Topic assignemnts z<sub>d,n</sub>
- Global variables are shared by the corpus
  - Topics  $\beta_k$

#### Local and global variables



- Assume the topics are fixed.
- Even "local inference" is intractable,

$$p(\theta, z_{1:N} \mid w_{1:N}, \beta_{1:K}) = \frac{p(\theta) \prod_{n=1}^{N} p(z_n \mid \theta) p(w_n \mid \beta_{z_n})}{\int_{\theta} p(\theta) \prod_{n=1}^{N} \sum_{z_n} p(z_n \mid \theta) p(w_n \mid \beta_{z_n})}$$

## Local Gibbs sampling for LDA



- We observe words  $\mathbf{w} = w_{1:N}$ . The Markov chain is defined on  $\{\theta, z_{1:N}\}$ , the topic proportions and topic assignments.
- Some notation—

$$n(z_{1:N}) = \sum_{n=1}^{N} z_n$$
  
$$m_k(\mathbf{z}_{1:D}, \mathbf{W}) = \sum_{d=1}^{D} \sum_{n=1}^{N} z_{d,n}^k w_{d,n}.$$

n(z<sub>1:N</sub>) are topic counts;
 m<sub>k</sub>(z<sub>1:N</sub>, W) are within-topic word counts.

#### Local Gibbs sampling for LDA

(



A simple Gibbs sampler is

$$\theta \,|\, \mathbf{W}, \mathbf{Z}_{1:N} \sim \operatorname{Dir}(\gamma)$$
 $\mathbf{Z}_n \,|\, \theta, \mathbf{W} \sim \operatorname{Mult}(\phi_n)$ 

where

$$\gamma = \alpha + n(z_{1:N})$$
  
$$\phi_n \propto \theta \cdot p(w_n | \beta_{1:K}).$$

#### **Collapsed local Gibbs sampling**



• The topic proportions  $\theta$  can be integrated out,

$$p(z_n | z_{-n}, \mathbf{w}) = p(w_n | \beta_{1:K}) \cdot \int_{\theta} p(z_n | \theta) p(\theta | z_{-n}) d\theta$$

A collapsed Gibbs sampler constructs a chain on z<sub>1:N</sub>

$$z_n | z_{-n}, \mathbf{W} \sim \operatorname{Mult}(\phi_n),$$

where  $\phi_n \propto p(w_n | \beta_{1:K})(n(z_{-n}) + \alpha)$ .

#### **Example inference**

#### Seeking Life's Bare (Genetic) Necessities

COLD SPRING HARBOR, NEW YORK— How many genes does an organism need to survive? Last week at the genome meeting here," two genome researchers with radically different approaches presented complementary views of the basic genes needed for life. One research team, using computer analyses to compare known genomes, concluded that today's organisms can be sastained with host 255 genes. and that the entites life forms

required a mere 128 genes. The other researcher mapped genes in a simple parasite and estimated that for this organism, 800 genes are plenty to do the job—but that anything short of 100 wouldn't be enough.

Although the numbers don't match precisely, those predictions

\* Genome Mapping and Sequencing, Cold Spring Harbor, New York, May 8 to 12. "are not all that far apart," especially in comparison to the 75,000 genes in the human genome, notes Six Andresson of Uppsala University in Sweden, who arrived at the 800 numbers apart, who arrived at the about the second second second second numbers game, particularly as more and more genomes are completely mapped and sequenced. "It may be a way of organizing any newly sequenced genome," explains Arcady Mosherian a commensional mo-

Arcady Mushegian, a computational molecular biologist at the National Center for Biotechnology Information (NCBI) in Bethesda, Maryland. Comparing an





SCIENCE • VOL. 272 • 24 MAY 1996



## Sampling the topics



- We observe the corpus **W** = **w**<sub>1:D</sub>.
- We define the chain on  $\{\mathbf{z}_{1:D}, \theta_{1:D}, \beta_{1:K}\}$ .
- First, sample latent variables  $(\mathbf{z}_d, \theta_d)$  for each document.
- Then, sample each topic from

$$\beta_k | \mathbf{z}_{1:D}, \mathbf{W} \sim \operatorname{Dir}(\lambda_k),$$

where

$$\lambda_k := \eta + m_k(\mathbf{z}_{1:D}, \mathbf{W}).$$

Recall  $m_k(\mathbf{z}_{1:D}, \mathbf{W})$  are words counts for topic *k*.

#### **Collapsed Gibbs sampling with topics**



- We can integrate out the topics  $\beta_{1:K}$  too.
- The sampler is defined on the topic assignments z<sub>1:D</sub>

$$\mathcal{D}(z_{n,d} = k \,|\, \mathbf{z}_{-(n,d)}, \mathbf{W}) \propto \left( \frac{m_k(\mathbf{z}_{-(n,d)}, \mathbf{W}) + \eta}{\sum_v m_k^v(\mathbf{z}_{-(n,d)}) + V\eta} 
ight) (n_k(z_{-i}) + \alpha)$$

 This is an excellent Gibbs sampler for LDA. It was developed by Giffiths and Steyvers (2002) and is widely used.

#### **Example topic inference**

human genome dna genetic genes sequence gene molecular sequencing map information genetics mapping project sequences

evolution evolutionary species organisms life origin biology groups phylogenetic living diversity group new two common

disease host bacteria diseases resistance bacterial new strains control infectious malaria parasite parasites united tuberculosis

computer models information data computers system network systems model parallel methods networks software new simulations

## Gibbs sampling for LDA in practice

#### In practice:

- Obtain a corpus of documents W
- 2 Run the Gibbs sampler for some number of iterations.
- 3 Store states at some lag, or store the MAP state.
- Look at counts like m<sub>k</sub>(z<sub>1:D</sub>, W) to investigate the topics; look at n(z<sub>d</sub>) to investigate how each document exhibits them.

#### • A good habit: Assess the convergence of the chain.

- Monitor the log probability of the state & observations. (Its exponential is proportional to the posterior.)
- Do something fancier, e.g., Raftery and Lewis (1992).

#### Assessing convergence example



#### Assessing convergence example



- Simple algorithm for sampling from a complex distribution.
- Works well in practice. Is the best first algorithm to try.
- However
  - Can be slow for very large data sets
  - It is difficult to handle nonconjugacy; it is hard to generalize to the dynamic topic model and correlated topic model.

## Variational inference

- Variational inference replaces sampling with optimization.
- The main idea—
  - Place a distribution over the hidden variables with free parameters, called **variational parameters**.
  - Optimize the variational parameters to make the distribution close (in KL divergence) to the true posterior
- In some settings, variational inference is faster than MCMC.
- It is easier to handle nonconjugate pairs of distributions with variational inference. (This is important in the CTM, DTM, etc.)

#### A useful picture (from Wainwright and Jordan, 2008)



## Variational inference (in general)

- Let x = x<sub>1:N</sub> be observed variables; let z = z<sub>1:M</sub> be the latent variables.
- Our goal is to compute the posterior distribution

$$p(z \mid x) = \frac{p(z, x)}{\int p(z, x) dz}$$

 For many interesting distributions, the marginal likelihood of the observations is difficult to efficiently compute.

## Variational inference

- Introduce a distribution over the latent variables q<sub>ν</sub>(z), parameterized by variational parameters ν.
- Use Jensen's inequality to bound the log probability of the observations, (Jordan et al., 1999)

$$\begin{array}{lll} \log p(x) &=& \log \int p(z,x) dz \\ &=& \log \int p(z,x) \frac{q_{\nu}(z)}{q_{\nu}(z)} dz \\ &\geq& \operatorname{E}_{q_{\nu}}[\log p(Z,x)] - \operatorname{E}_{q_{\nu}}[\log q_{\nu}(Z)] \end{array}$$

(J. McAuliffe calls this the evidence lower bound, or ELBO.)

- Optimize the variational parameters to tighten this bound.
- This is the same as finding the member of the family q<sub>ν</sub> that is closest in KL divergence to p(z | x).
#### Mean-field variational inference

- Complexity is determined by the factorization of *q<sub>ν</sub>*
- In mean field variational inference  $q_{\nu}$  is fully factored

$$q_{\nu}(z)=\prod_{m=1}^M q_{\nu_m}(z_m).$$

- Each latent variable is independently governed by its own variational parameter ν<sub>m</sub>.
- In the true posterior they can exhibit dependence.
  (Often, this is what makes exact inference difficult.)

## Variational inference for LDA



- The *mean field distribution* places a variational parameter on each hidden variable.
- Optimize these with coordinate ascent, iteratively optimizing each parameter while holding the others fixed.

#### Variational inference for LDA



 In the "local step" we iteratively update the parameters for each document, holding the topic parameters fixed.

$$\begin{aligned} \gamma^{(t+1)} &= \alpha + \sum_{n=1}^{N} \phi_n^{(t)} \\ \phi_n^{(t+1)} &\propto \exp\{\mathbb{E}_q[\log \theta] + \mathbb{E}_q[\log \beta_{.,w_n}]\}. \end{aligned}$$

#### **Example inference**

#### Seeking Life's Bare (Genetic) Necessities

COLD SPRING HARBOR, NEW YORK— How many genes does an organism need to survive? Last week at the genome meeting here," two genome researchers with radically different approaches presented complementary views of the basic genes needed for life. One research team, using computer analyses to compare known genomes, concluded that today's organisms can be sastained with host 255 genes. and that the entites life forms

required a mere 128 genes. The other researcher mapped genes in a simple parasite and estimated that for this organism, 800 genes are plenty to do the job—but that anything short of 100 wouldn't be enough.

Although the numbers don't match precisely, those predictions

\* Genome Mapping and Sequencing, Cold Spring Harbor, New York, May 8 to 12. "are not all that far apart," especially in comparison to the 75,000 genes in the human genome, notes Six Andresson of Uppsala University in Sweden, who arrived at the 800 numbers apart, who arrived at the about the second second second second numbers game, particularly as more and more genomes are completely mapped and sequenced. "It may be a way of organizing any newly sequenced genome," explains Arcady Mosherian a commensional mo-

Arcady Mushegian, a computational molecular biologist at the National Center for Biotechnology Information (NCBI) in Bethesda, Maryland. Comparing an





SCIENCE • VOL. 272 • 24 MAY 1996



#### Variational inference for LDA



• In the "global step" we aggregate the parameters computed from the local step and update the parameters for the topics,

$$\lambda_{k} = \eta + \sum_{d} \sum_{n} \mathbf{w}_{d,n} \phi_{d,n}.$$

#### **Example topic inference**

human genome dna genetic genes sequence gene molecular sequencing map information genetics mapping project sequences

evolution evolutionary species organisms life origin biology groups phylogenetic living diversity group new two common

disease host bacteria diseases resistance bacterial new strains control infectious malaria parasite parasites united tuberculosis

computer models information data computers system network systems model parallel methods networks software new simulations

## Variational inference for LDA (sketch)

- 1: Initialize topics randomly.
- 2: repeat
- 3: for each document do
- 4: repeat
- 5: Update the topic assignment variational parameters.
- 6: Update the topic proportions variational parameters.
- 7: **until** document objective converges
- 8: end for
- 9: Update the topics from aggregated per-document parameters.
- 10: until corpus objective converges.

## Variational inference for LDA

- 1: Initialize topics  $\lambda_{1:K}$  randomly.
- 2: while relative improvement in  $\mathcal{L}(\mathbf{w}, \phi, \gamma, \lambda) > \epsilon$  do
- 3: **for** d = 1 to *D* **do**
- 4: Initialize  $\gamma_{d,k} = 1$ .
- 5: repeat
- 6: Set  $\phi_{d,n} \propto \exp\{\mathbb{E}_q[\log \theta_d] + \mathbb{E}_q[\log \beta_{\cdot,w_n}]\}$

7: Set 
$$\gamma_d = \alpha + \sum_n \phi_{d,n}$$

8: **until** 
$$\frac{1}{K} \sum_{k} |\text{change in } \gamma_{d,k}| < \epsilon$$

9: end for

10: Set 
$$\lambda_k = \eta + \sum_d \sum_n W_{d,n} \phi_{d,n}$$

11: end while

## "E step"

- 1: Initialize topics  $\lambda_{1:K}$  randomly.
- 2: while relative improvement in  $\mathcal{L}(\mathbf{w}, \phi, \gamma, \lambda) > \epsilon$  do
- 3: **for** d = 1 to *D* **do**
- 4: Initialize  $\gamma_{d,k} = 1$ .

#### 5: repeat

- 6: Set  $\phi_{d,n} \propto \exp\{\mathbb{E}_q[\log \theta_d] + \mathbb{E}_q[\log \beta_{\cdot,w_n}]\}$
- 7: Set  $\gamma_d = \alpha + \sum_n \phi_{d,n}$
- 8: **until**  $\frac{1}{K} \sum_{k} |\text{change in } \gamma_{d,k}| < \epsilon$
- 9: end for
- 10: Set  $\lambda_k = \eta + \sum_d \sum_n W_{d,n} \phi_{d,n}$
- 11: end while

Do variational inference for each document.

## "M step"

- 1: Initialize topics  $\lambda_{1:K}$  randomly.
- 2: while relative improvement in  $\mathcal{L}(\mathbf{w}, \phi, \gamma, \lambda) > \epsilon$  do
- 3: **for** *d* = 1 to *D* **do**
- 4: Initialize  $\gamma_{d,k} = 1$ .

#### 5: repeat

6: Set  $\phi_{d,n} \propto \exp\{\mathbb{E}_q[\log \theta_d] + \mathbb{E}_q[\log \beta_{\cdot,w_n}]\}$ 

7: Set 
$$\gamma_d = \alpha + \sum_n \phi_{d,n}$$

- 8: **until**  $\frac{1}{K} \sum_{k} |$  change in  $\gamma_{d,k}| < \epsilon$
- 9: end for

10: Set 
$$\lambda_k = \eta + \sum_d \sum_n W_{d,n} \phi_{d,n}$$

11: end while

Update the posterior estimates of the topics based on the "E step."

#### Online inference for LDA (with M. Hoffman and F. Bach)



- Our goal is to use this (and related) models for analyzing massive collections of millions of documents.
- But, in the first step of batch inference we estimate the posterior for *every document* based on randomly initialized topics.

#### Online inference for LDA (with M. Hoffman and F. Bach)



- Online variational inference is much more efficient.
- It allows us to easily analyze millions of documents.
- It lets us develop topic models on streaming collections.

## **Online inference for LDA**



- Randomly pick a document.
- Perform local variational inference with the current topics.
- Sorm "fake" topics, treating the sampled document as though it were the only document in the collection.
- Update the topics to be a weighted average of the fake topics and current topics.

## Online variational inference for LDA (sketch)

- 1: Define an appropriate sequence of weights.
- 2: Initialize topics randomly.
- 3: for ever do
- 4: Choose a random document *d*.
- 5: repeat
- 6: Update the topic assignment variational parameters.
- 7: Update the topic proportions variational parameters.
- 8: **until** document objective converges
- 9: Compute topics as though *d* is the only document.
- 10: Set the topics to a weighted average of the current topics and the topics from step 9.
- 11: end for

#### **On-line variational inference for LDA**

- 1: Define  $\rho_t \triangleq (\tau_0 + t)^{-\kappa}$
- 2: Initialize  $\lambda$  randomly.
- 3: for t = 0 to  $\infty$  do
- 4: Choose a random document *w*<sub>t</sub>
- 5: Initialize  $\gamma_{tk} = 1$ . (The constant 1 is arbitrary.)
- 6: repeat
- 7: Set  $\phi_{t,n} \propto \exp\{\mathbb{E}_q[\log \theta_t] + \mathbb{E}_q[\log \beta_{\cdot,w_n}]\}$

8: Set 
$$\gamma_t = \alpha + \sum_n \phi_{t,n}$$

9: **until** 
$$\frac{1}{K} \sum_{k} |\text{change in } \gamma_{t,k}| < \epsilon$$

- 10: Compute  $\tilde{\lambda}_k = \eta + D \sum_{n} w_{t,n} \phi_{t,n}$
- 11: Set  $\lambda_k = (1 \rho_t)\lambda_k + \rho_t \tilde{\lambda}_k$ .

12: end for

#### Analyzing 3.3M articles from Wikipedia



analyzed	2048	4096	8192	12288	16384	32768	49152	65536
Top eight words	systems road made service announced national west language	systems health communication service billion language care road	service systems health companies market communication company billion	service systems companies business company billion health industry	service companies systems business company industry market billion	business service companies industry company management systems services	business service companies industry services company management public	business industry service companies services company management public

#### Why does this work?

#### A STOCHASTIC APPROXIMATION METHOD<sup>1</sup>

By Herbert Robbins and Sutton Monro

University of North Carolina

**1.** Summary. Let M(x) denote the expected value at level x of the response to a certain experiment. M(x) is assumed to be a monotone function of x but is unknown to the experimenter, and it is desired to find the solution  $x = \theta$  of the equation  $M(x) = a_c$  where  $\alpha$  is a given constant. We give a method for making successive experiments at levels  $x_1, x_2, \cdots$  in such a way that  $x_n$  will tend to  $\theta$  in probability.

- Why waste time with the real gradient, when a cheaper noisy estimate of the gradient will do (Robbins and Monro, 1951)?
- Idea: Follow a noisy estimate of the gradient with a step-size.
- By decreasing the step-size according to a certain schedule, we guarantee convergence to a local optimum.
- See Hoffman et al. (2010) and Sato (2001).

## Online inference is promising, in general

- Stochastic variational methods are a general way to approximate the posterior for massive/streaming data.
- No need to process the whole data set in advance; can easily link to web APIs and other data sources
- Powerful algorithm for topic modeling, and can be adapted hierarchical models for many types of data.
- Software and papers: www.cs.princeton.edu/~blei/

## Latent Dirichlet allocation (flashback)



- This joint defines a posterior.
- From a collection of documents, infer
  - Per-word topic assignment z<sub>d,n</sub>
  - Per-document topic proportions θ<sub>d</sub>
  - Per-corpus topic distributions β<sub>k</sub>
- Then use posterior expectations to perform the task at hand, e.g., information retrieval, document similarity, exploration, ...

## Latent Dirichlet allocation (flashback)



Approximate posterior inference algorithms

- Mean field variational methods (Blei et al., 2001, 2003)
- Expectation propagation (Minka and Lafferty, 2002)
- Collapsed Gibbs sampling (Griffiths and Steyvers, 2002)
- Collapsed variational inference (Teh et al., 2006)
- Online variational inference (Hoffman et al., 2010)

Also see Mukherjee and Blei (2009) and Asuncion et al. (2009).

# Discussion

- What are topic models?
- What kinds of things can they do?
- How do I compute with a topic model?
- What are some unsanswered questions in this field?
- How can I learn more?

## Introduction to topic modeling



- LDA assumes that there are K topics shared by the collection.
- Each document exhibits the topics with different proportions.
- Each word is drawn from one topic.
- We discover the structure that best explain a corpus.

## **Extensions of LDA**



- Topic models can be adapted to many settings
- Bayesian nonparametric topic models let the corpus determine the number of topics (or more complicated topic structure).

## **Posterior inference**



- Posterior inference is the central computational problem.
- We discussed three algorithms
  - MCMC based on collapsed Gibbs sampling
  - Mean-field variational inference
  - Online variational inference

## Some open issues

- Model interpretation and model checking Which model should I choose for which task? (Chang et al. 2009, Ramadge et al. 2009, Newman et al. 2010, Mimno and Blei 2011, Mimno et al. 2011)
- Incorporating corpus, discourse, or linguistic structure How can our knowledge of language help us build and use exploratory models of text?
- Interfaces and "downstream" applications of topic modeling What can I do with an annotated corpus? How can I incorporate latent variables into a user interface?
- Theoretical understanding of approximate inference What do we know about variational inference from either the statistical or learning perspective?

#### Interpretation I: Human studies of topic models



(see Chang et al. 2009 and Newman et al. 2010)

### Interpretation II: Labelled LDA on JSTOR



(see Ramadge et al. 2009 and Ramadge et al. 2011)

## Interptetation III: Mutual information discrepancy

4	10	3	13		
ax	labor	women	contract		
come workers		sexual	liability		
axation employees		men	parties		
xes union		sex	contracts		
evenue	employer	child	party		
estate	employers	family	creditors		
subsidies	employment	children	agreement		
exemption	work	gender	breach		
organizations	employee	woman	contractual		
year	job	marriage	terms		
treasury	bargaining	discrimination	bargaining		
consumption	unions	male	contracting		
taxpayers	worker	social	debt		
earnings	collective	female	exchange		
tunds	industrial	parents	limited		
6	15	1	16		
jury	speech	firms	constitutional		
trial	free	price	political		
crime	amendment	corporate	constitution		
defendant	freedom	firm	government		
defendants	expression	value	justice		
sentencing	protected	market	amendment		
judges	culture	cost	history		
punishment	context	capital	people		
judge	equality	shareholders	legislative		
crimes	values	stock	opinion		
evidence	conduct	insurance	fourteenth		
sentence	ideas	efficient	article		
jurors	information	assets	majority		
offense	protect	offer	citzens		
guilty	content	share	republican		

(see Mimno and Blei 2011)

- The topic modeling mailing list is a good discussion group.
- Bibliography: http://www.cs.princeton.edi/~mimno/
- Software and papers: http://www.cs.princeton.edu/~blei/

#### If you remember one picture...



"We should seek out unfamiliar summaries of observational material, and establish their useful properties... And still more novelty can come from finding, and evading, still deeper lying constraints." (J. Tukey, *The Future of Data Analysis*, 1962)