

Formal Semantics

Natural Language Processing
CS 6120—Spring 2014
Northeastern University

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some slides from Jason Eisner

Language as Structure

- So far, we've talked about **structure**
- What structures are **more probable?**
 - Language modeling: Good sequences of words/characters
 - Text classification: Good sequences in defined contexts
- How can we recover **hidden structure?**
 - Tagging: hidden word classes
 - Parsing: hidden word relations

What Does It All Mean?

- Studying phonology, morphology, syntax, etc. independent of meaning is methodologically very useful
- We can study the structure of languages we don't understand
- We can use HMMs and CFGs to study protein structure and music, which don't bear meaning in the same way as language

What Does It All Mean?

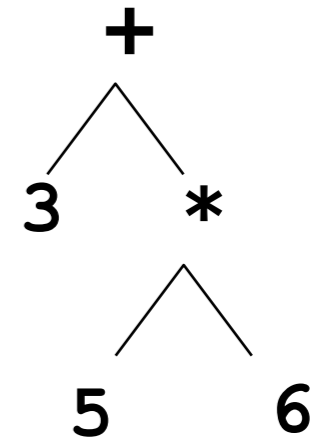
- How would you know if a computer “understood” the “meaning” of an (English) utterance (even in some weak “scare-quoted” way)?
- How would you know if a person understood the meaning of an utterance?

What Does It All Mean?

- Paraphrase, “state in your own words” (English to English translation)
- Translation into another language
- Reading comprehension questions
- Drawing appropriate inferences
- Carrying out appropriate actions
- Open-ended dialogue (Turing test)

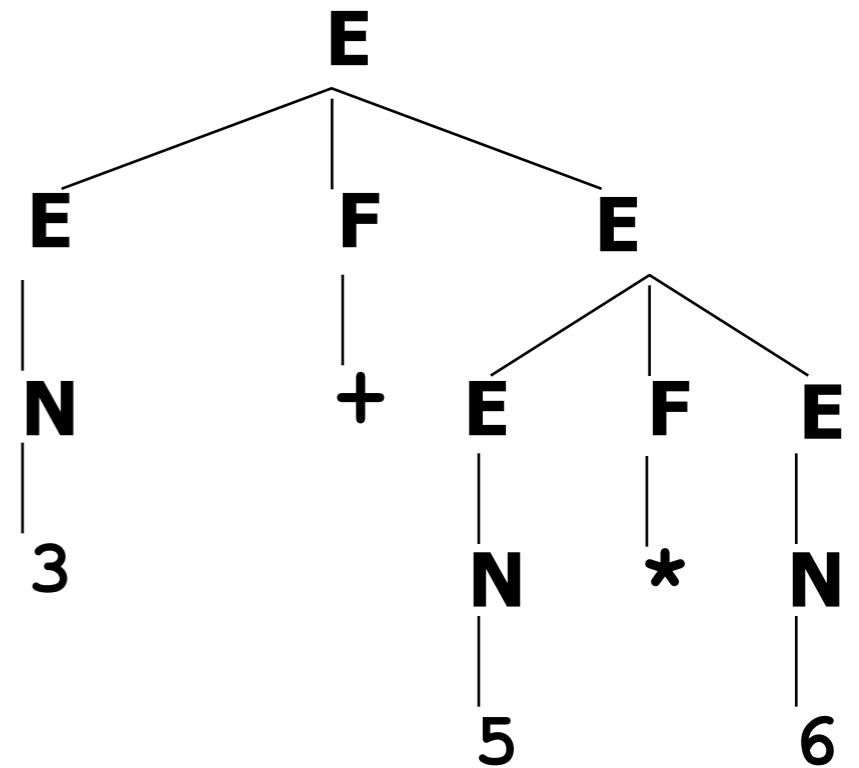
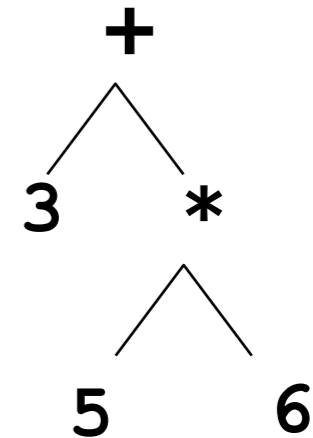
Programming Language Interpreter

- What is meaning of $3+5*6$?
- First parse it into $3+(5*6)$



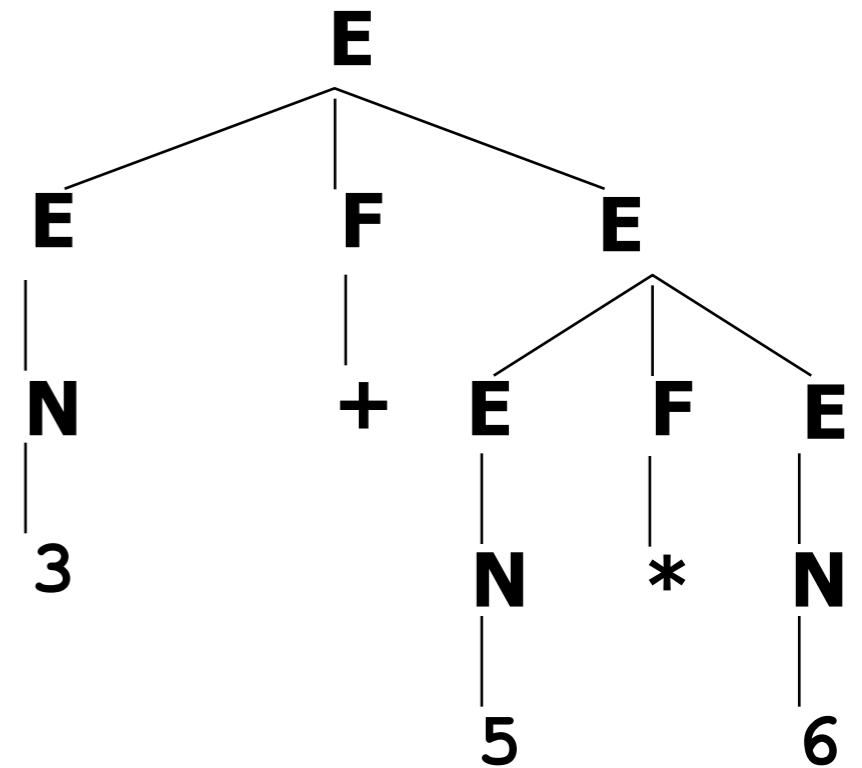
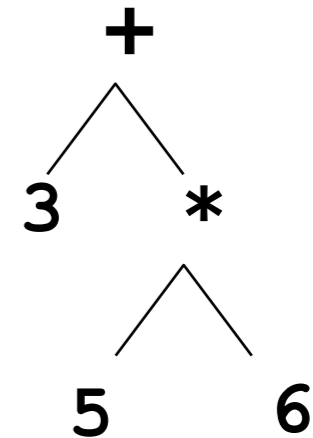
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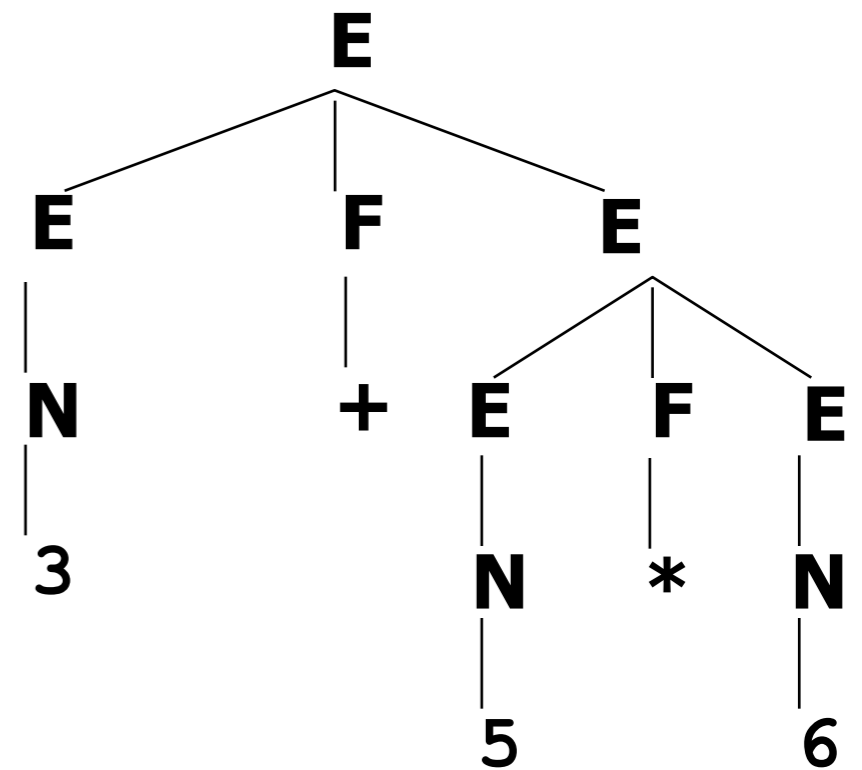
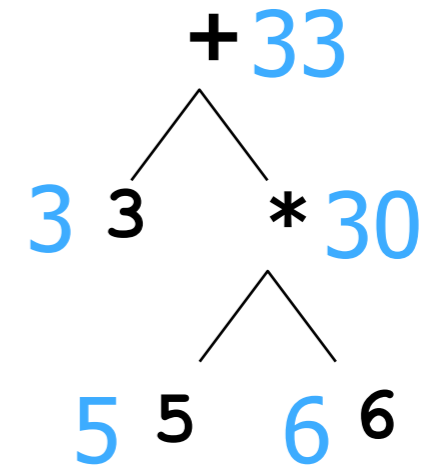
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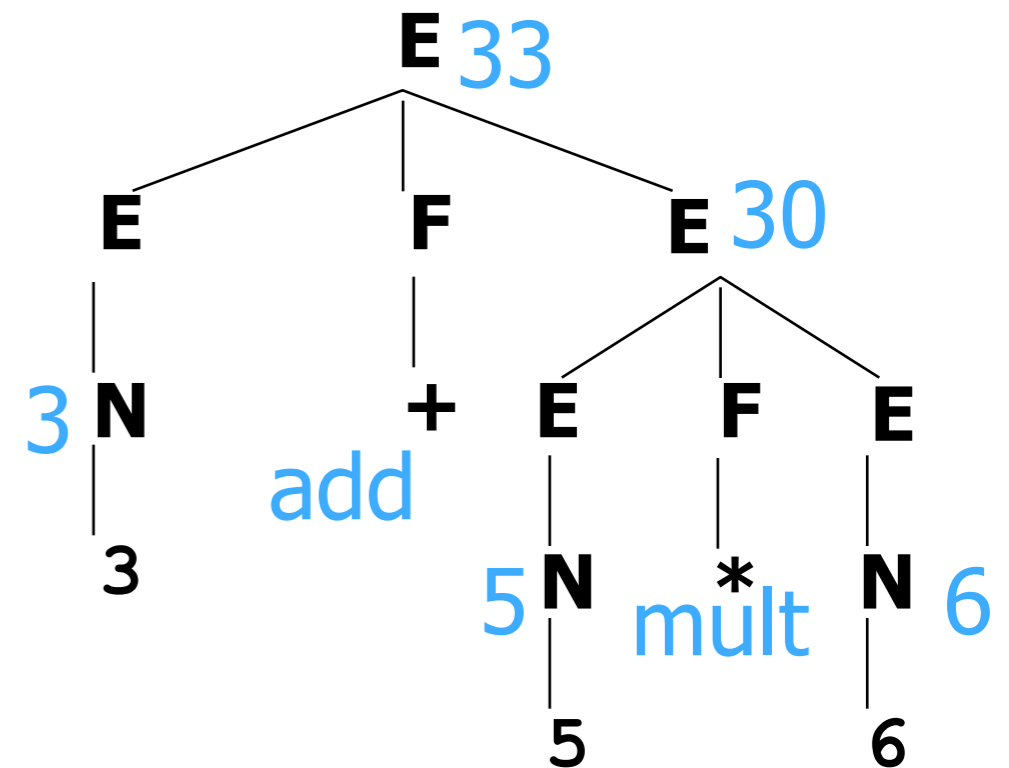
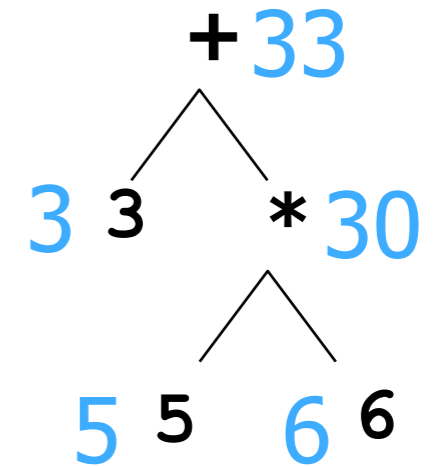
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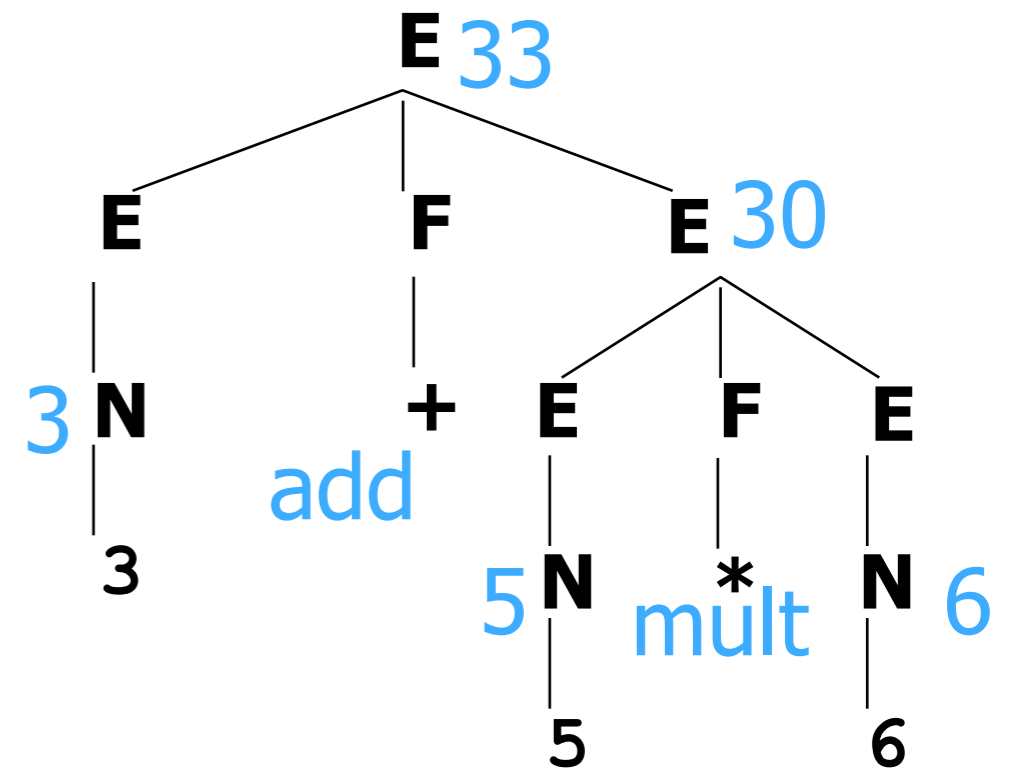
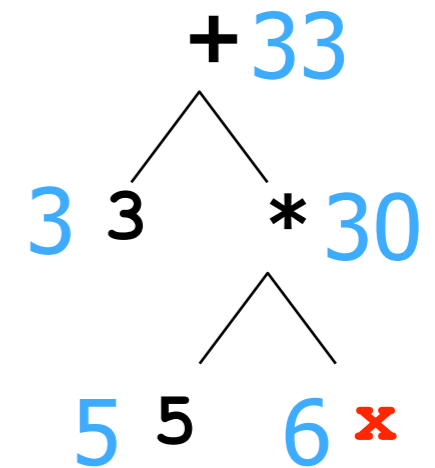


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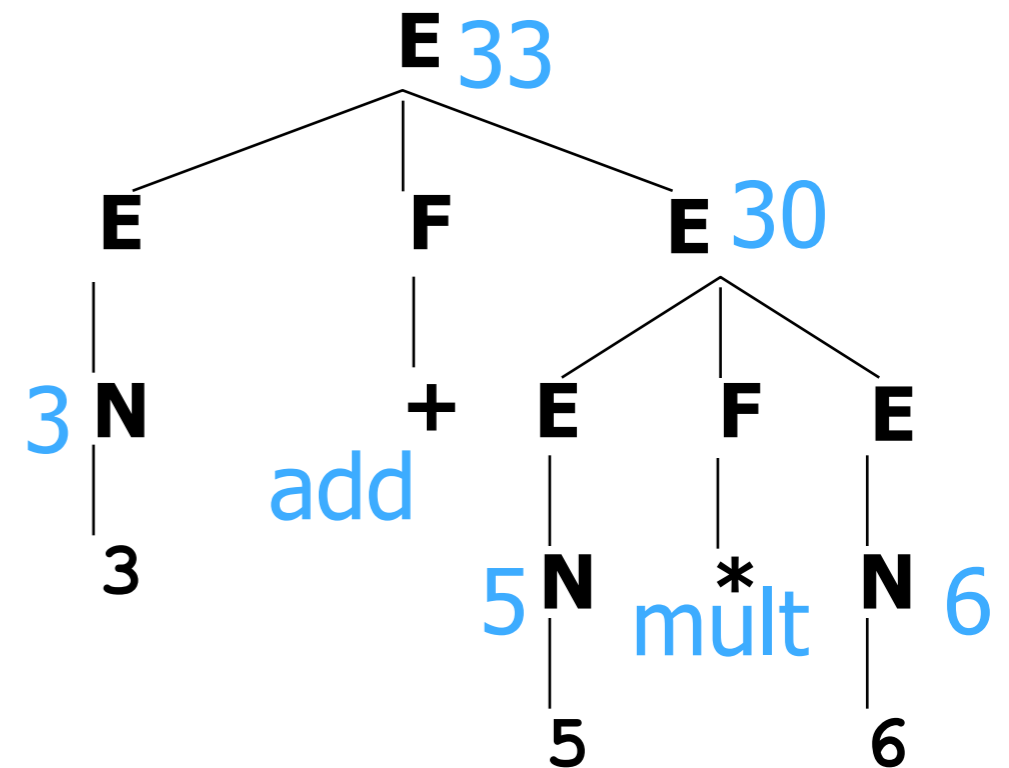
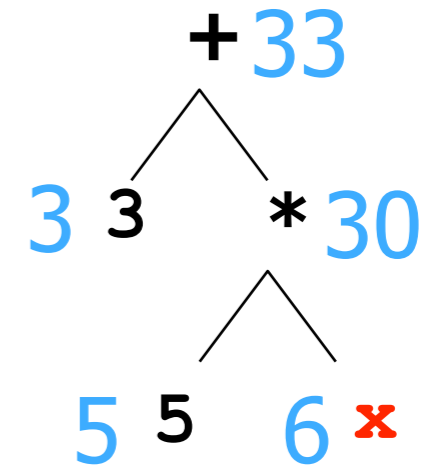


Interpreting in an Environment



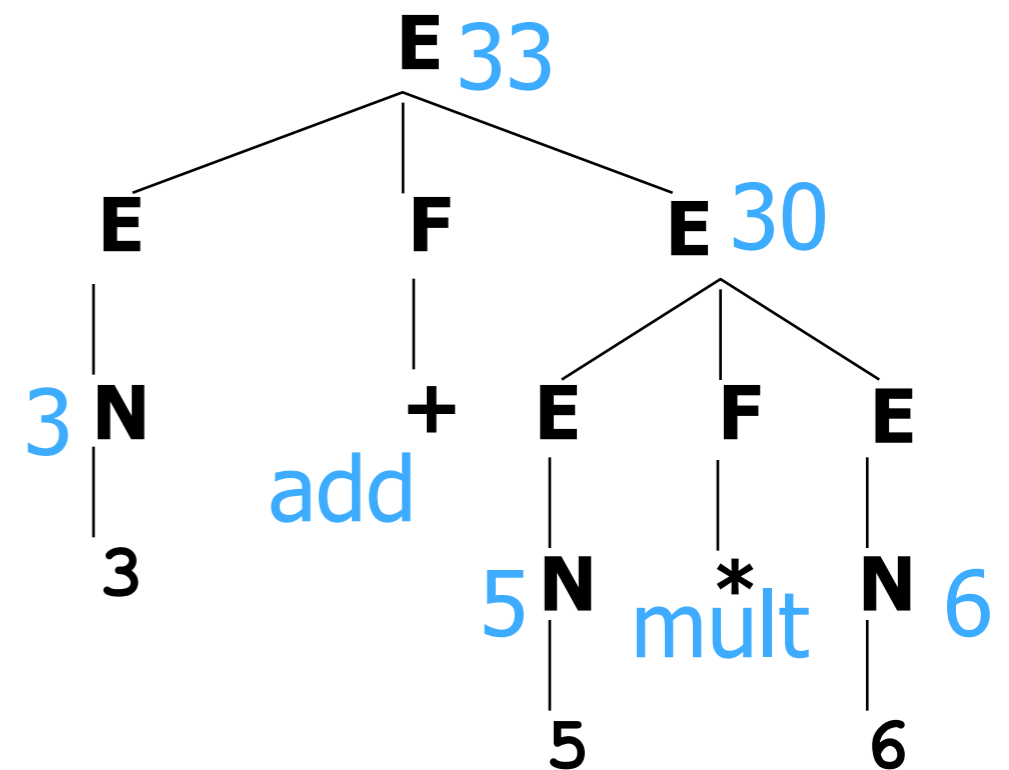
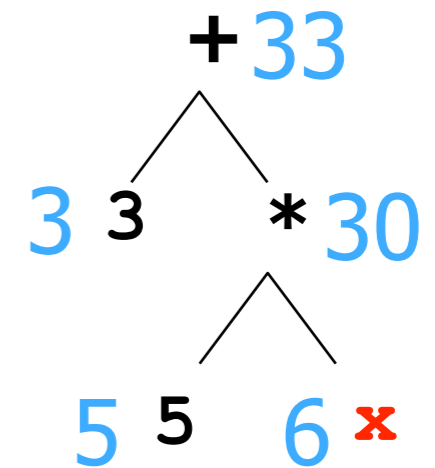
Interpreting in an Environment

- How about $3+5*x$?



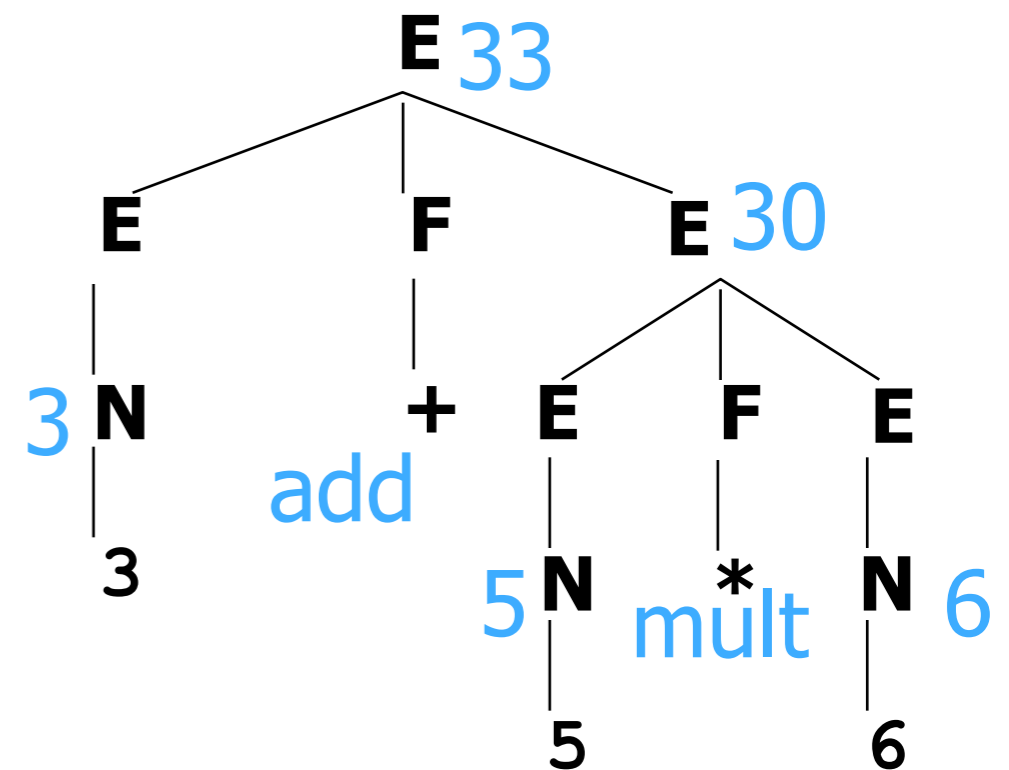
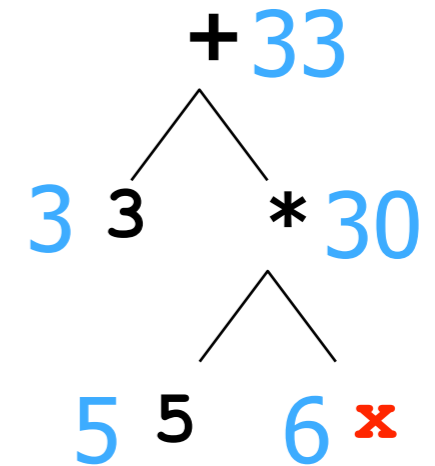
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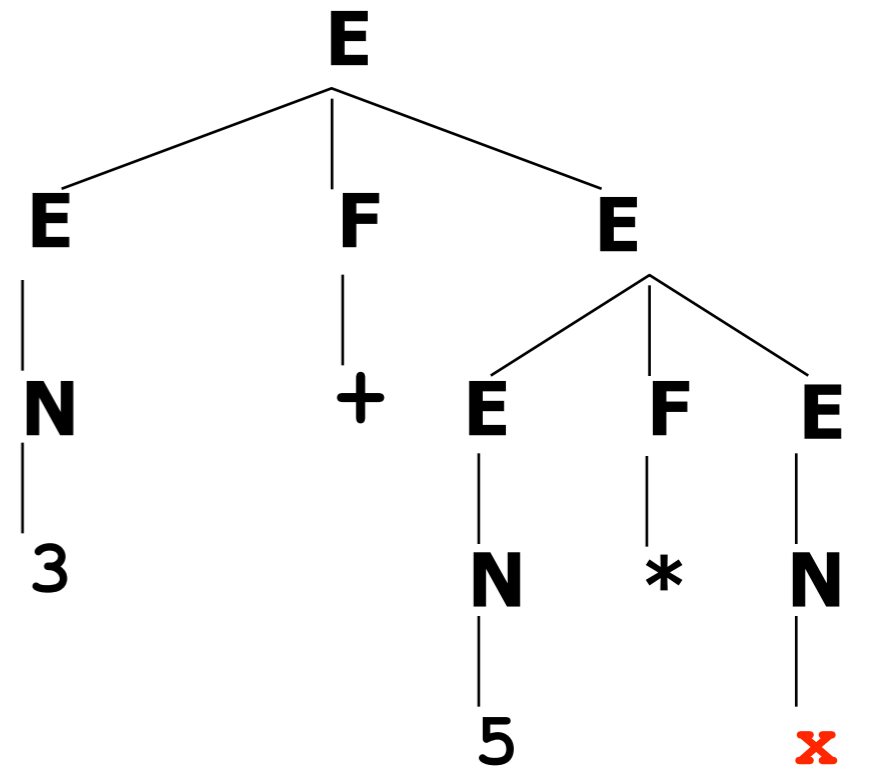


Interpreting in an Environment

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- Analogies in language?

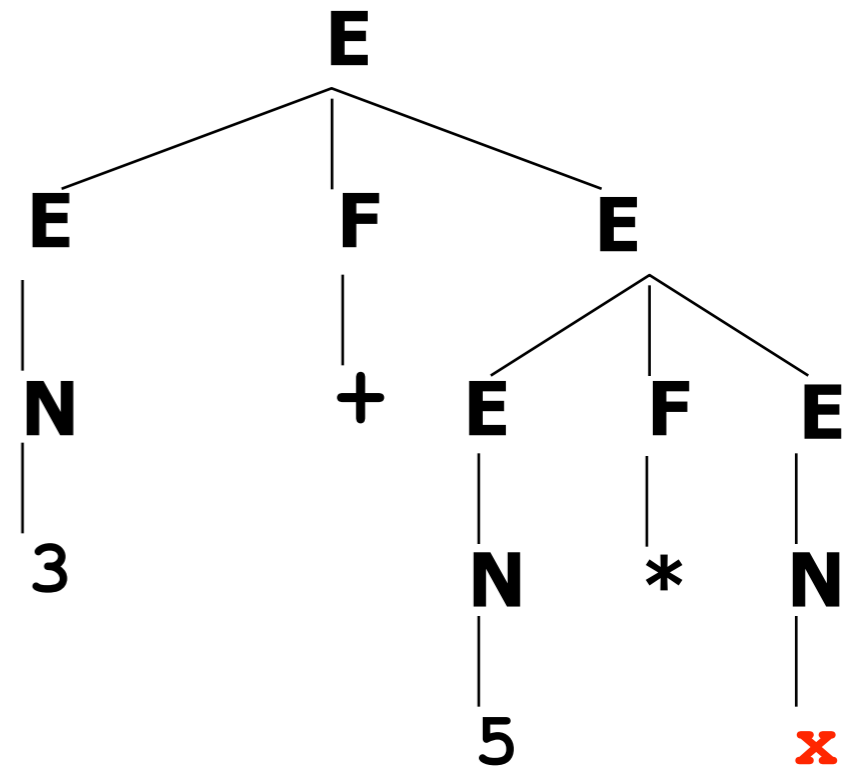


Compiling



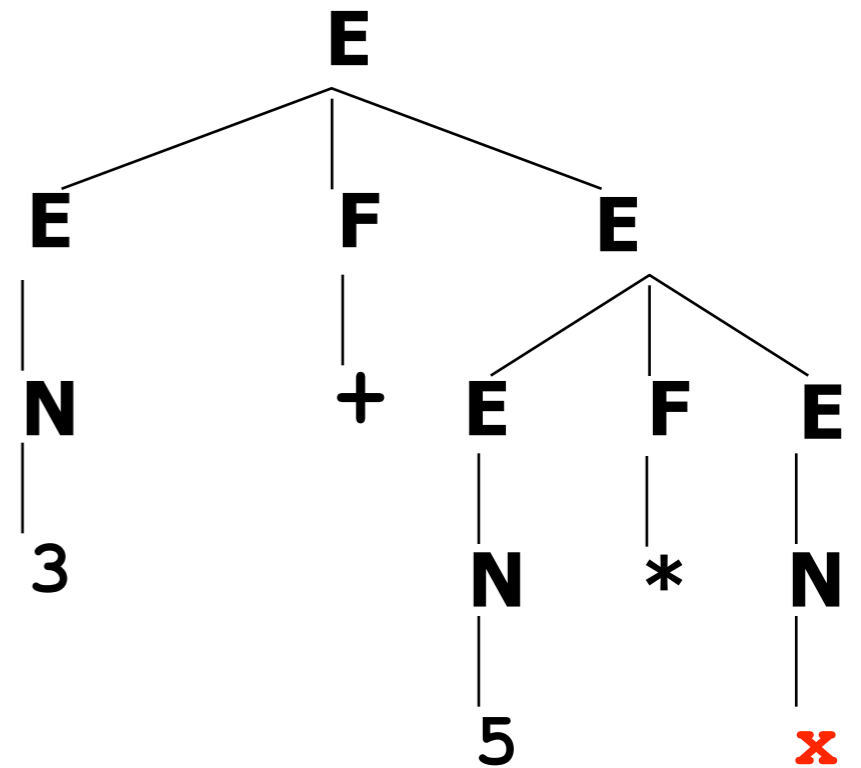
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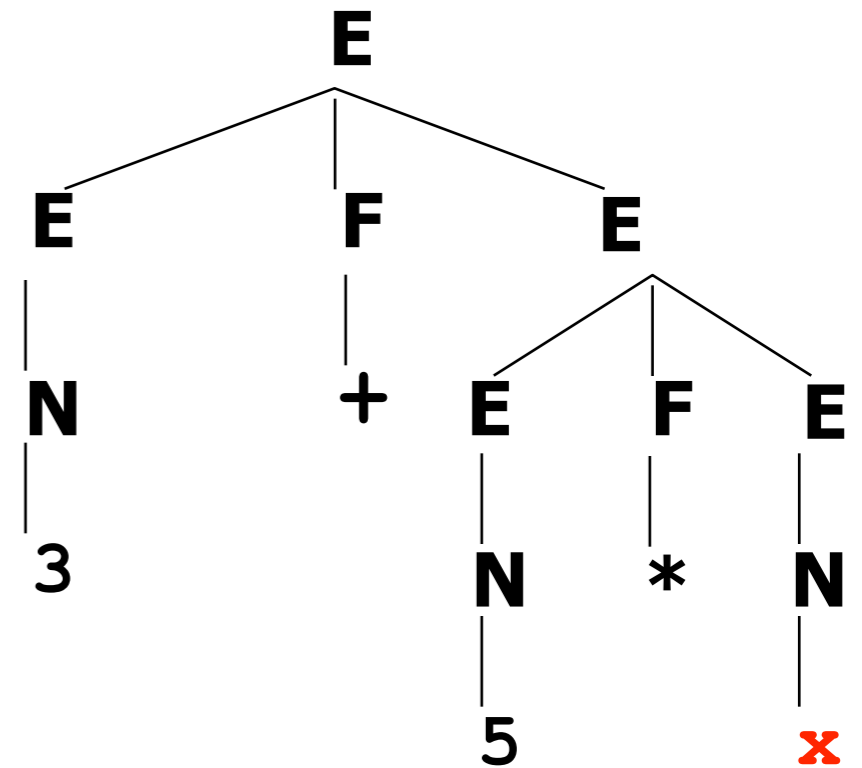
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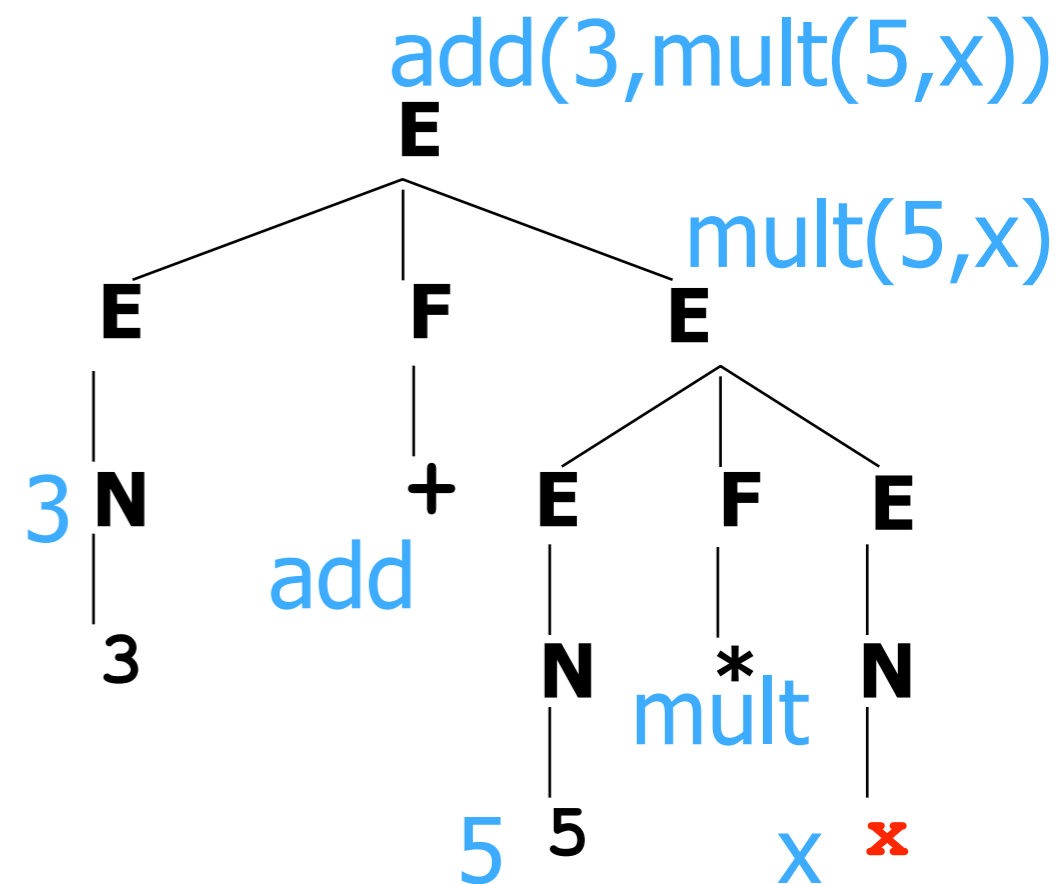
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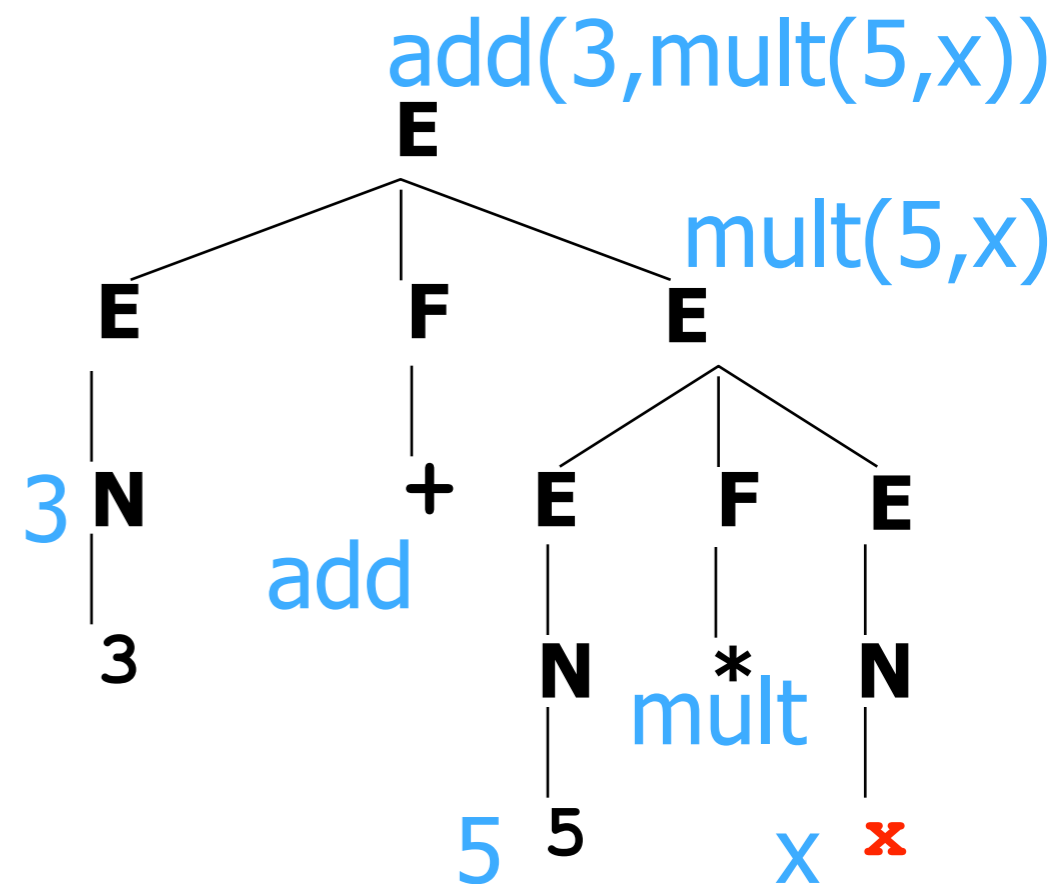
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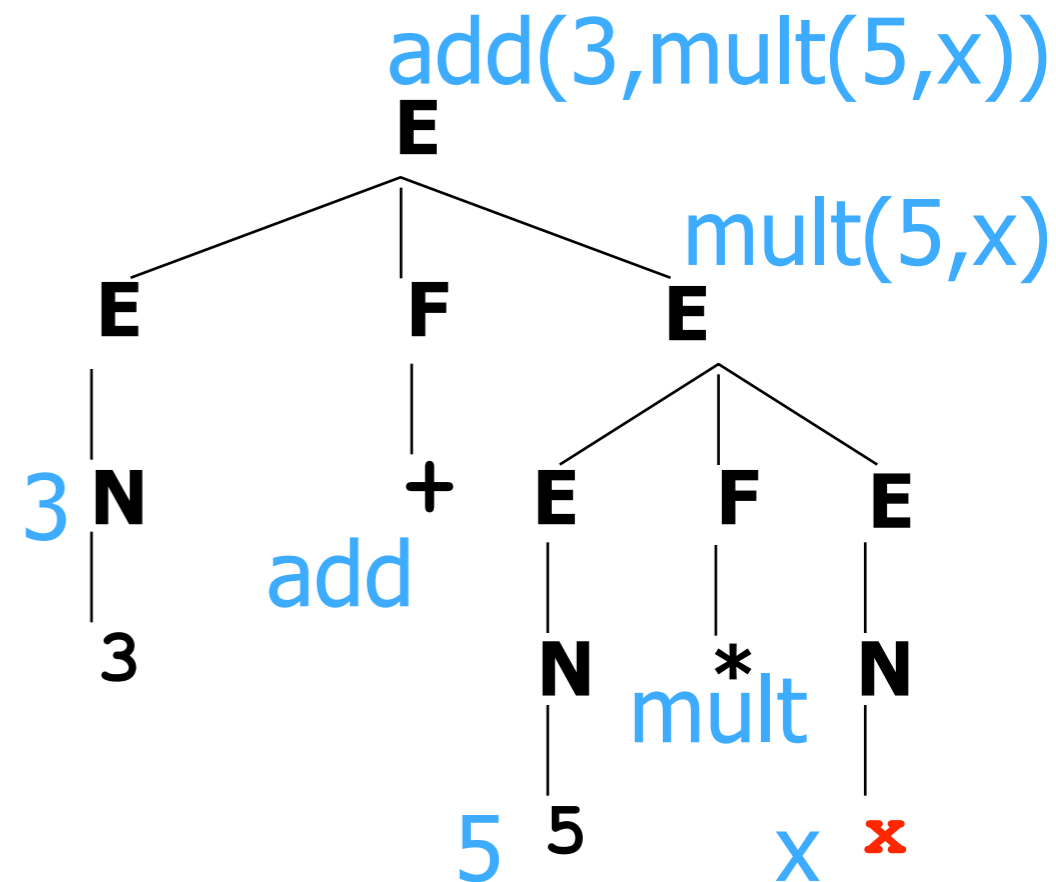
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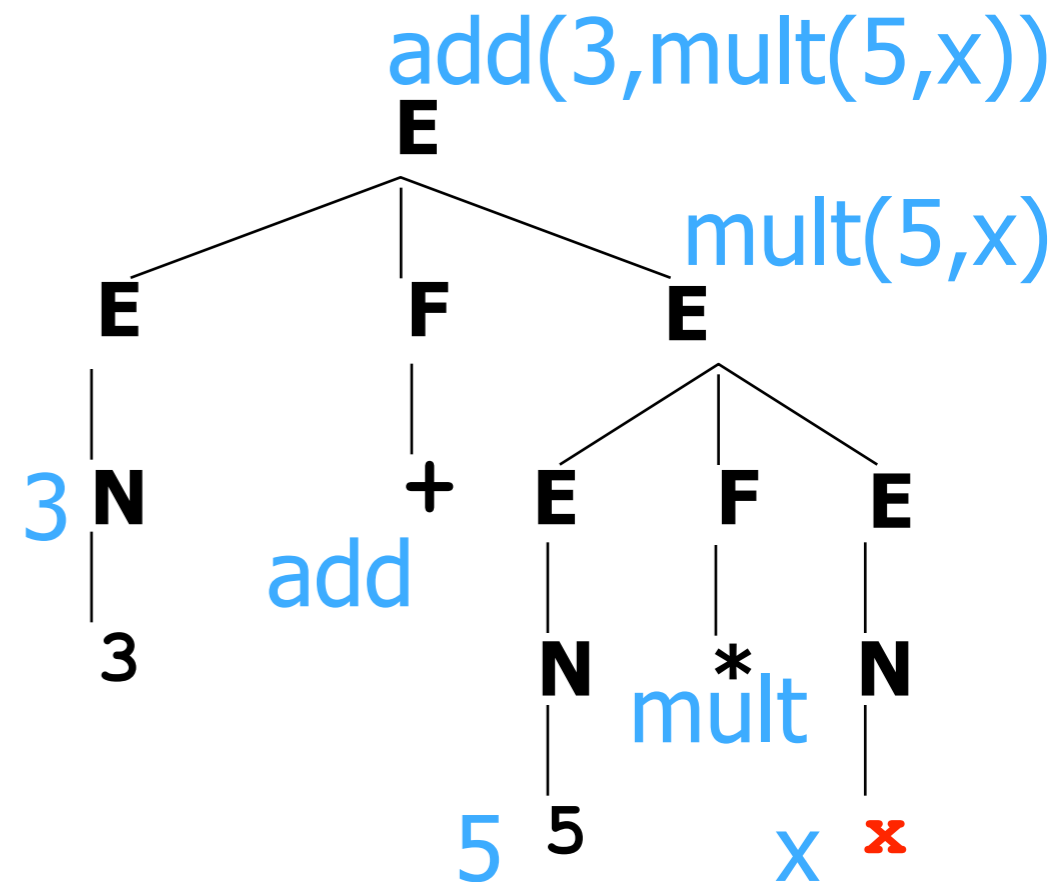
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Analogies in language?



What Counts as Understanding?

some notions

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- We understand if we can respond appropriately
 - ok for commands, questions (these demand response)
 - “Computer, warp speed 5”
 - “throw axe at dwarf”
 - “put all of my blocks in the red box”
 - imperative programming languages
 - SQL database queries and other questions
- We understand statement if we can determine its truth
 - ok, but if you knew whether it was true, why did anyone bother telling it to you?
 - comparable notion for understanding NP is to compute what the NP refers to, which might be useful

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- We understand statement if we know how one could (in principle) determine its truth
 - What are exact conditions under which it would be true?
 - necessary + sufficient
 - Equivalently, derive all its consequences
 - what else must be true if we accept the statement?
 - Match statements with a “domain theory”
 - Philosophers tend to use this definition

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 - Match statements with a “domain theory”
 - Philosophers tend to use this definition
- We understand statement if we can use it to answer questions [very similar to above – requires reasoning]
 - **Easy:** John ate pizza. What was eaten by John?
 - **Hard:** White’s first move is P-Q4. Can Black checkmate?
 - Constructing a procedure to get the answer is enough

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- Open-ended dialogue (Turing test)
- Translation to logical form that we can reason about

(First Order) Logic

Some Preliminaries

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Three major kinds of objects

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- Roughly, the semantic values of sentences

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- Values of NPs, e.g., objects like this slide
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- Functions from booleans to booleans (and, or, not)
- A function from entity to boolean is called a “predicate” – e.g., `frog(x)`, `green(x)`
- Functions might return other functions!

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- Functions might return other functions!
- Function might take other functions as arguments!

Logic: Lambda Terms

- Lambda terms:
 - A way of writing “anonymous functions”
 - No function header or function name
 - But defines the key thing: **behavior** of the function
 - Just as we can talk about 3 without naming it “x”
 - Let `square = $\lambda p p * p$`
 - Equivalent to `int square(p) { return p * p; }`
 - But we can talk about `$\lambda p p * p$` without naming it
 - Format of a lambda term: `λ variable expression`

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(proving that these functions are equal – and indeed they are,
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- How about $\text{even}(\text{square}(x))$?
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- Suppose `times` is defined as $\lambda x \lambda y (x*y)$
- Claim that `times(5)(6)` is 30
 - $\text{times}(5) = (\lambda x \lambda y x*y) (5) = \lambda y 5*y$

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 - And now times can be written as $\lambda x \lambda y \text{times}(x,y)$

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- What is executed by `loves(john, mary)` ?

Logic: Interesting Constants

- Thus, have “constants” that name some of the entities and functions (e.g., *):
 - `GeorgeWBush` - an entity
 - `red` – a predicate on entities
 - holds of just the red entities: `red(x)` is true if `x` is red!
 - `loves` – a predicate on 2 entities
 - `loves(GeorgeWBush, LauraBush)`
 - Question: What does `loves(LauraBush)` denote?
- Constants used to define meanings of words
- Meanings of phrases will be built from the constants

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- **most** – a predicate on 2 predicates on entities
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 - Equivalently, **most(λx pig(x), λx big(x))**
 - returns true if most of the things satisfying the first predicate also satisfy the second predicate
- similarly for other quantifiers
 - **all(pig, big)** (equivalent to **$\forall x$ pig(x) \Rightarrow big(x)**)
 - **exists(pig, big)** (equivalent to **$\exists x$ pig(x) AND big(x)**)
 - can even build complex quantifiers from English phrases:
 - “between 12 and 75”; “a majority of”; “all but the smallest 2”

A reasonable representation?

- `Gilly` swallowed a goldfish
- First attempt: `swallowed(Gilly, goldfish)`

- Returns true or false. Analogous to
 - `prime(17)`
 - `equal(4,2+2)`
 - `loves(GeorgeWBush, LauraBush)`
 - `swallowed(Gilly, Jilly)`
- ... or is it analogous?

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- In particular, don't want
`Gilly swallowed a goldfish and Milly
swallowed a goldfish`
to translate as
`swallowed(Gilly, goldfish) AND swallowed(Milly, goldfish)`
since probably not the same goldfish ...

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- Or using one of our quantifier predicates:
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- Here `goldfish` is a predicate on entities
 - This is the same semantic type as `red`
 - But `goldfish` is noun and `red` is adjective .. #@!?

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(Simplify Notation)

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 - Specifies who what why when ...

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 - Specifies who what why when ...
- Replace time variable t with an event variable e
 - $\exists e \text{ past}(e), \text{act}(e, \text{swallowing}), \text{swallower}(e, \text{Gilly}), \text{exists}(\text{goldfish}, \text{swallowee}(e)), \text{exists}(\text{booth}, \text{location}(e)), \dots$
 - As with probability notation, a comma represents AND
 - Could define past as $\lambda e \exists t \text{ before}(t, \text{now}), \text{ended-at}(e, t)$

Quantifier Order

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- Does this mean what we'd expect??
 - says that there's only one event
 - with a single goldfish getting swallowed
 - that took place in a lot of booths ...

Quantifier Order

- Groucho Marx celebrates quantifier order ambiguity:
 - In this country a woman gives birth every 15 min. Our job is to find that woman and stop her.
 - $\exists \text{woman} (\forall 15\text{min gives-birth-during}(\text{woman}, 15\text{min}))$
 - $\forall 15\text{min} (\exists \text{woman gives-birth-during}(15\text{min}, \text{woman}))$
 - Surprisingly, both are possible in natural language!
 - Which is the joke meaning (where it's always the same woman) and why?

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 - Probably false unless Gilly can be in every booth during her swallowing of a single goldfish

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 - “for all booths b, there was such an event in b”

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- **Intensional verbs besides** want: hope, doubt, believe,...

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 - Then `wants a unicorn = wants a dodo`. Oops!

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 - $\exists e \text{ act}(e, \text{wanting}), \text{wanter}(e, \text{Willy}), \text{wantee}(e, \lambda u \text{ unicorn}(u))$
 - “Willy wants anything that satisfies the unicorn predicate”
 - here the wantee is a type of entity
- Problem (a fine point I’ll gloss over):
 - $\lambda g \text{ unicorn}(g)$ is defined by the actual set of unicorns (“extension”)
 - But this set is empty: $\lambda g \text{ unicorn}(g) = \lambda g \text{ FALSE} = \lambda g \text{ dodo}(g)$
 - Then `wants a unicorn = wants a dodo`. Oops!
 - So really the wantee should be criteria for unicornness (“intension”)

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- Traditional solution involves “possible-world semantics”
 - Can imagine **other worlds** where set of unicorn \neq set of dodos
 - Other worlds also useful for: You must pay the rent
You can pay the rent
If you hadn’t, you’d be homeless

Control

Control

- Willy wants Lilly to get married

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 - $\exists e$ present(e), act(e,wanting), wanter(e,Willy), wantee(e, λf [act(f,marriage), marrier(f,Lilly)])

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 - **Just as easy to represent as** Willy wants Lilly ...

Control

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- Willy wants to get married
 - **Same as** Willy wants Willy to get married
 - **Just as easy to represent as** Willy wants Lilly ...
 - The only trick is to construct the representation from the syntax. The empty subject position of “to get married” is said to be controlled by the subject of “wants.”

Nouns and Their Modifiers

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- expert
 - λg expert(g)

Nouns and Their Modifiers

- expert
 - $\lambda g \text{ expert}(g)$
- big fat expert
 - $\lambda g \text{ big}(g), \text{ fat}(g), \text{ expert}(g)$
 - **But:** bogus expert
 - Wrong: $\lambda g \text{ bogus}(g), \text{ expert}(g)$
 - Right: $\lambda g (\text{bogus}(\text{expert}))(g)$... bogus maps to new concept

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- Baltimore expert (white-collar expert, TV expert ...)
 - $\lambda g \text{ Related}(\text{Baltimore}, g), \text{ expert}(g)$ – expert from Baltimore
 - Or with different intonation:
 - $\lambda g (\text{Modified-by}(\text{Baltimore}, \text{expert}))(g)$ – expert on Baltimore
 - Can't use **Related** for this case: law expert and dog catcher
= $\lambda g \text{ Related}(\text{law}, g), \text{ expert}(g), \text{ Related}(\text{dog}, g), \text{ catcher}(g)$
= dog expert and law catcher

Nouns and Their Modifiers

- the goldfish that Gilly swallowed
- every goldfish that Gilly swallowed
- three goldfish that Gilly swallowed

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- three $\overbrace{\text{swallowed-by-Gilly}}^{\text{like an adjective!}}$ goldfish

Nouns and Their Modifiers

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λg [goldfish(g), swallowed(Gilly, g)]

like an adjective!

- three swallowed-by-Gilly goldfish

Or for real: λg [goldfish(g), $\exists e$ [past(e), act(e,swallowing),
swallower(e,Gilly), swallowee(e,g)]]

Adverbs

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- Lili passionately wants Billy
 - Wrong?: `passionately(want(Lili,Billy)) = passionately(true)`
 - Better: `(passionately(want))(Lili,Billy)`
 - Best: `∃e present(e), act(e,wanting), wanter(e,Lili), wantee(e, Billy), manner(e, passionate)`

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- Lili often stalks Billy
 - $(\text{often}(\text{stalk}))(\text{Lili}, \text{Billy})$
 - $\text{many}(\text{day}, \lambda d \exists e \text{ present}(e), \text{act}(e, \text{stalking}), \text{stalker}(e, \text{Lili}), \text{stalkee}(e, \text{Billy}), \text{during}(e, d))$

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- Lili obviously likes Billy
 - $(\text{obviously}(\text{like}))(\text{Lili}, \text{Billy})$ – one reading
 - $\text{obvious}(\text{like}(\text{Lili}, \text{Billy}))$ – another reading

Speech Acts

Speech Acts

- What is the meaning of a full sentence?
 - Depends on the punctuation mark at the end. 😊
 - Billy likes Lili. → **assert**(like(B,L))
 - Billy likes Lili? → **ask**(like(B,L))
 - or more formally, "Does Billy like Lili?"
 - Billy, like Lili! → **command**(like(B,L))
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 - or more accurately, "Let Billy like Lili!"
- Let's try to do this a little more precisely, using event variables etc.

Speech Acts

Speech Acts

- What did Gilly swallow?
 - **ask**($\lambda x \exists e \text{ past}(e), \text{act}(e, \text{swallowing}),$
 $\text{swallower}(e, \text{Gilly}), \text{swallowee}(e, x)$)
 - Argument is identical to the modifier “that Gilly swallowed”
 - Is there any common syntax?

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- Eat your fish!
 - **command**($\lambda f \text{ act}(f, \text{eating}), \text{eater}(f, \text{Hearer}), \text{eatee}(\dots))$)

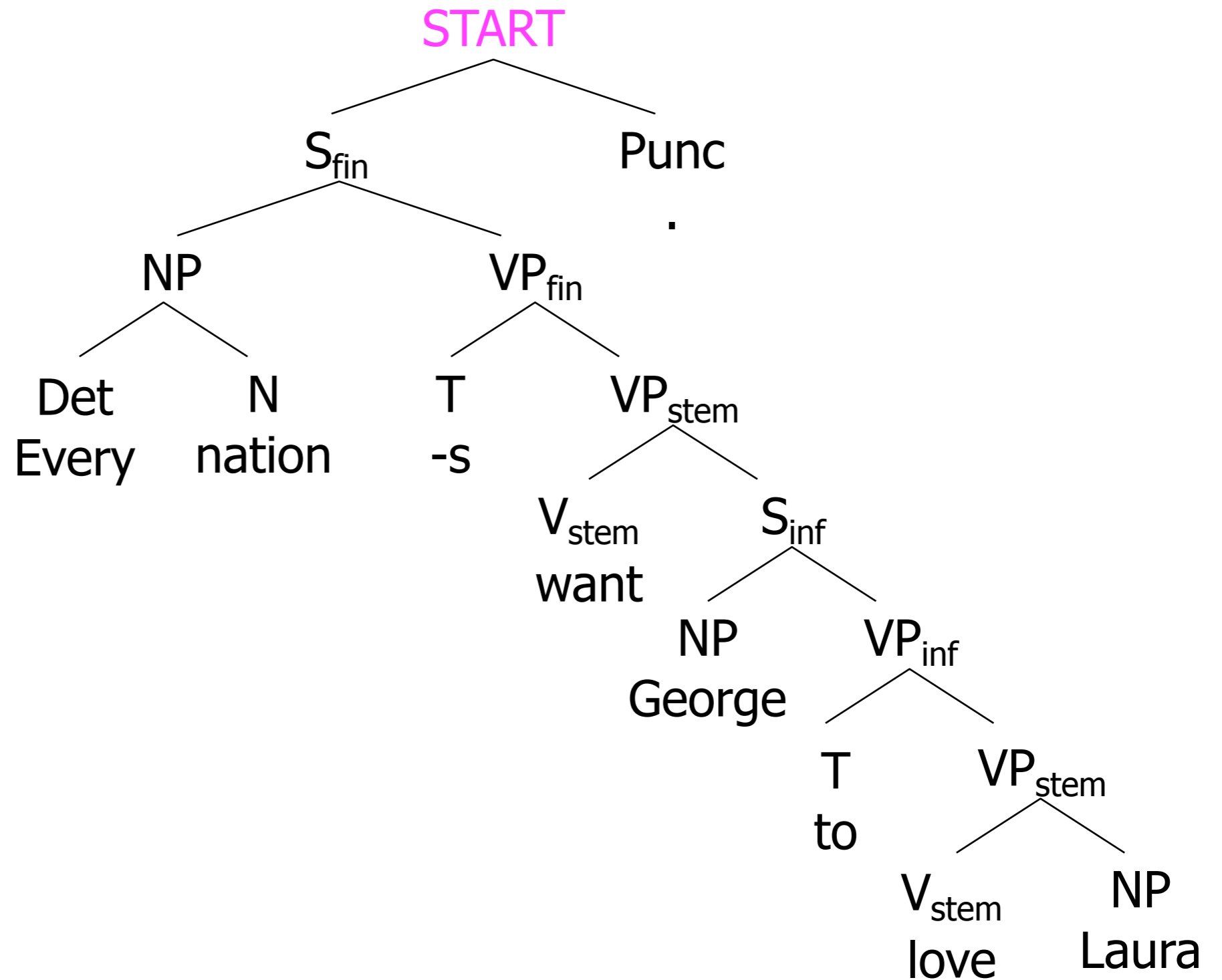
Speech Acts

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 - Is there any common syntax?
- Eat your fish!
 - **command**($\lambda f \text{ act}(f, \text{eating}), \text{eater}(f, \text{Hearer}), \text{eatee}(\dots)$)
- I ate my fish.
 - **assert**($\exists e \text{ past}(e), \text{act}(e, \text{eating}), \text{eater}(f, \text{Speaker}),$
 $\text{eatee}(\dots)$)

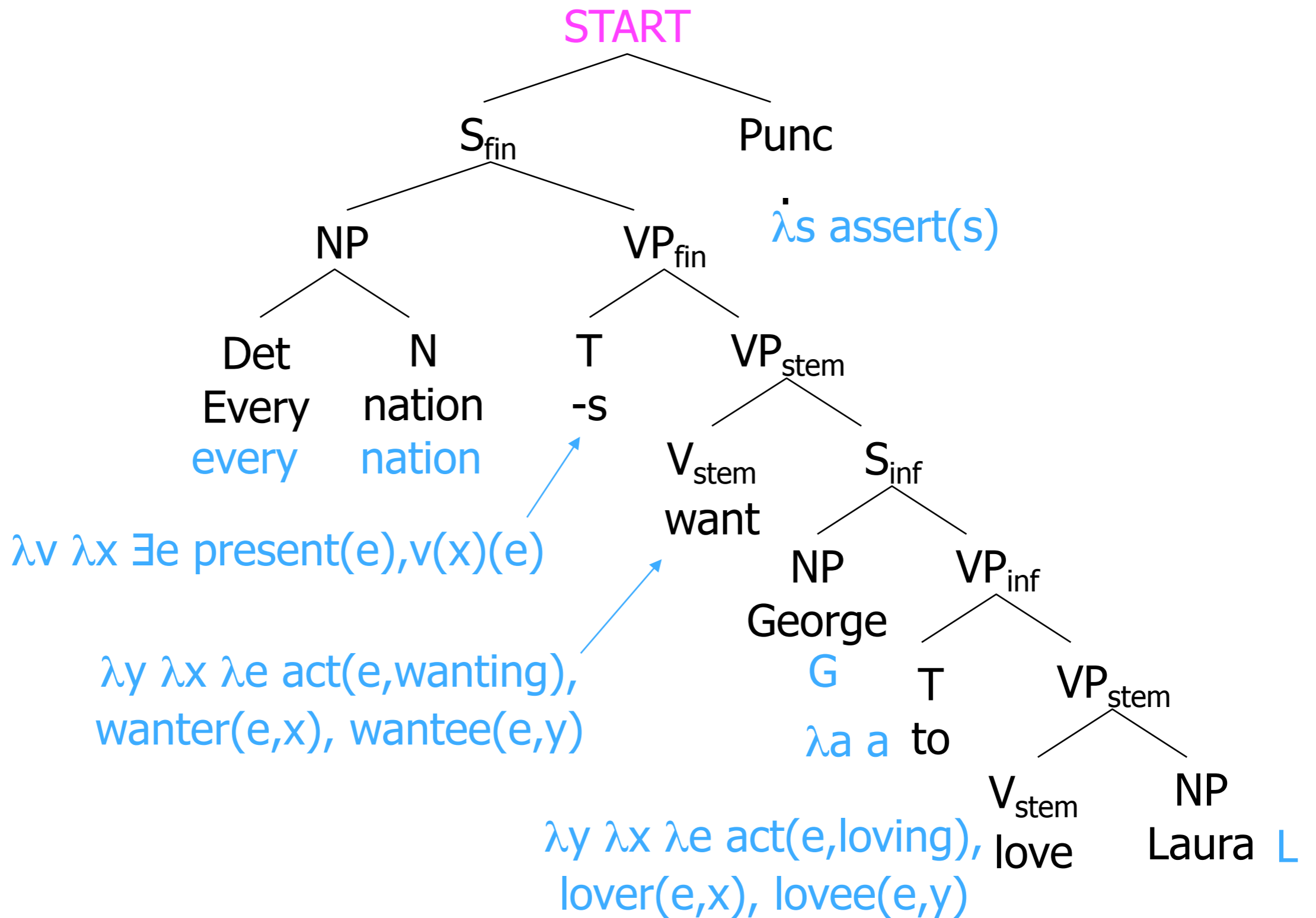
Compositional Semantics

- We've discussed what semantic representations should look like.
- **But how do we get them from sentences???**
- **First** - parse to get a syntax tree.
- **Second** - look up the semantics for each word.
- **Third** - build the semantics for each constituent
 - Work from the bottom up
 - The syntax tree is a "recipe" for how to do it

Compositional Semantics

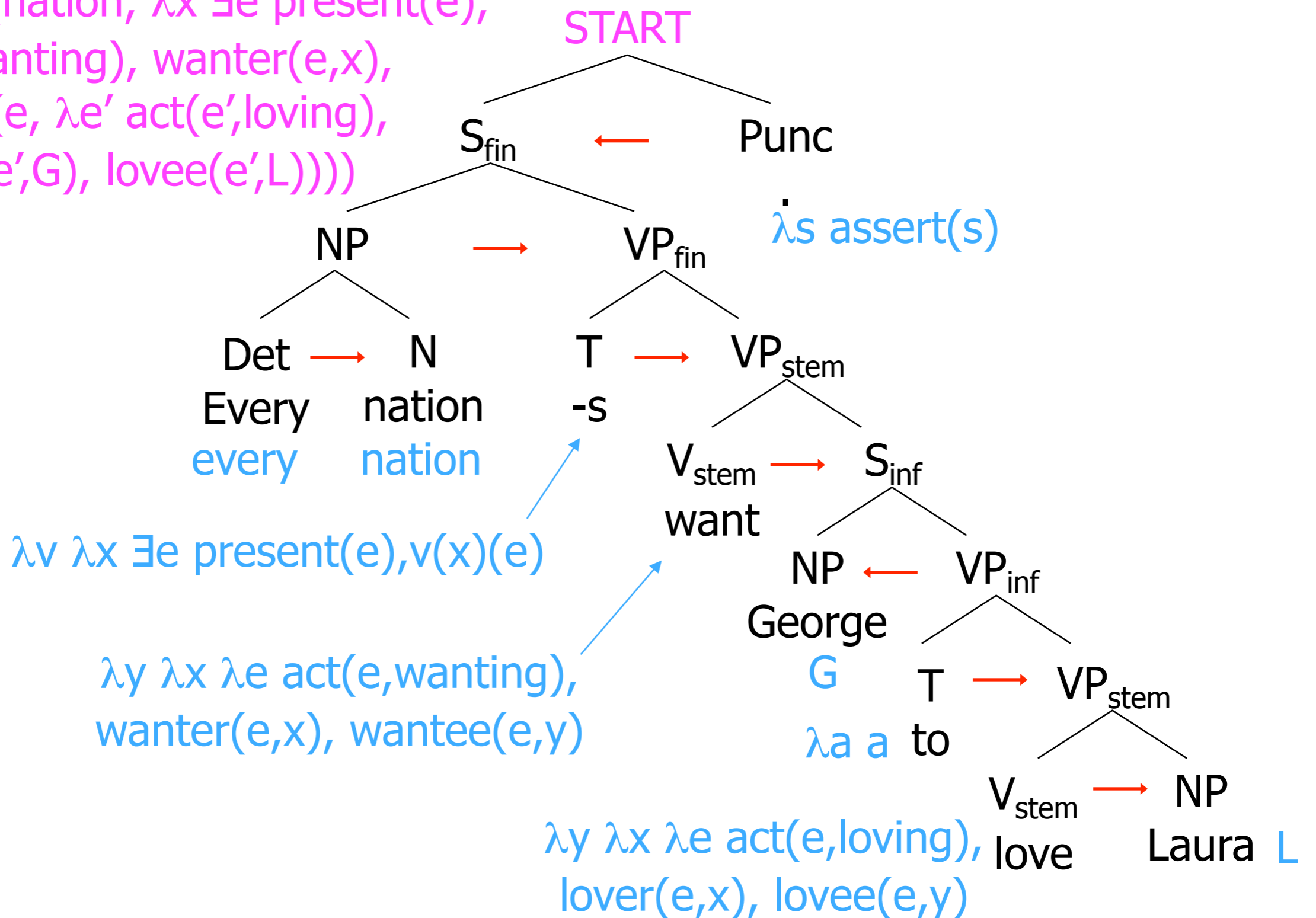


Compositional Semantics



Compositional Semantics

assert(every(nation, $\lambda x \exists e$ present(e),
 act(e,wanting), wanter(e,x),
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 lover(e',G), lovee(e',L))))



Compositional Semantics

Compositional Semantics

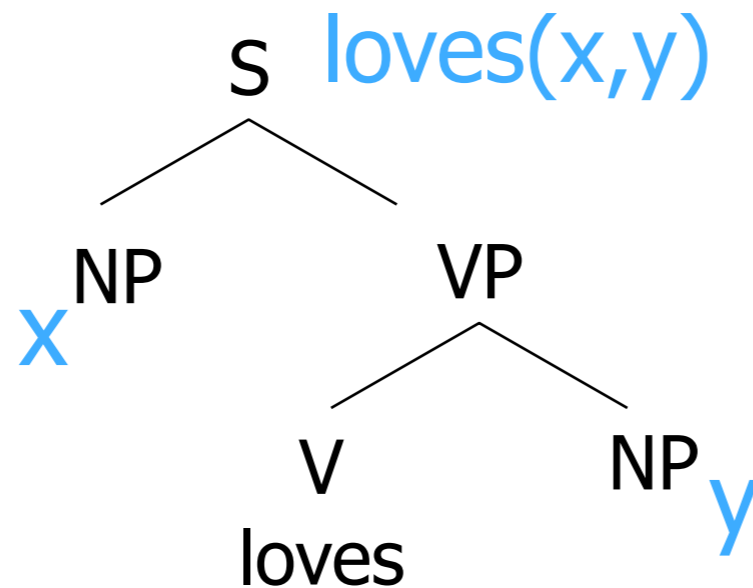
- Add a “sem” feature to each context-free rule
 - $S \rightarrow NP \text{ loves } NP$
 - $S[\text{sem}=\text{loves}(x,y)] \rightarrow NP[\text{sem}=x] \text{ loves } NP[\text{sem}=y]$
 - Meaning of S depends on meaning of NPs

Compositional Semantics

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- TAG version:

Compositional Semantics

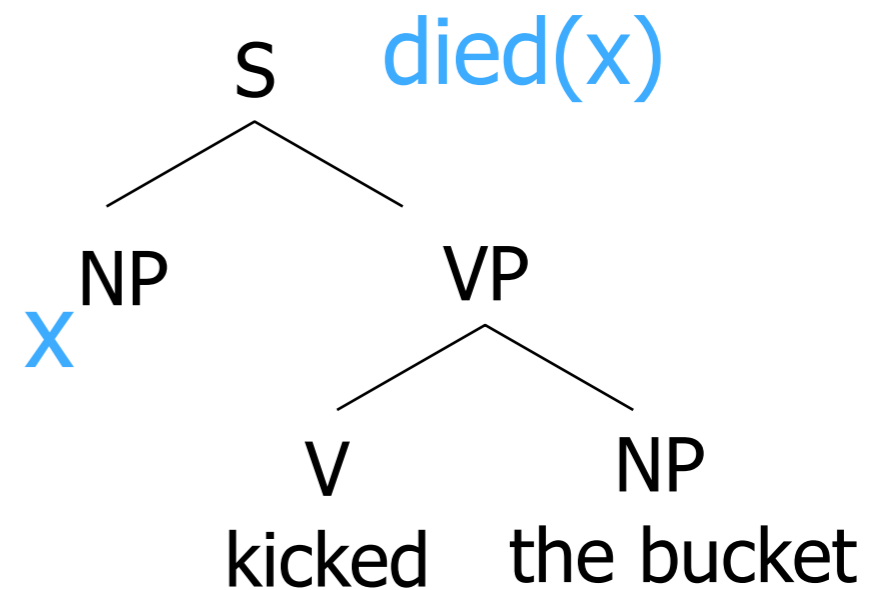
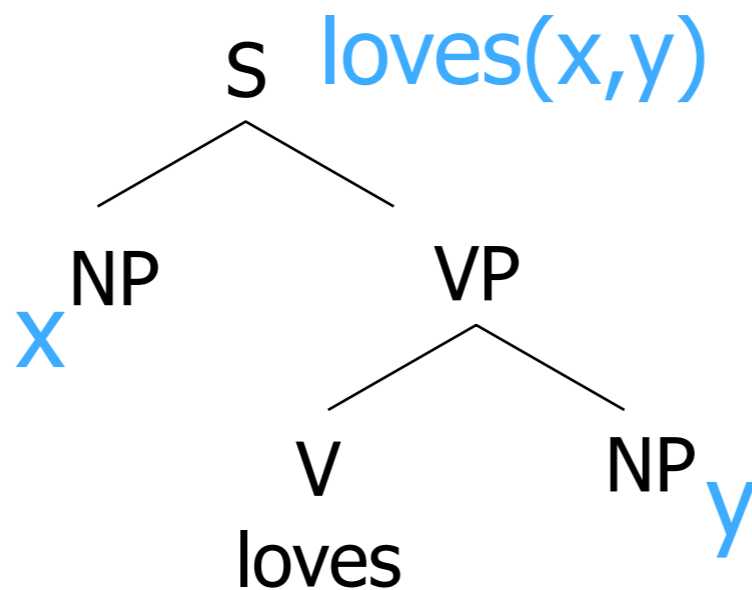
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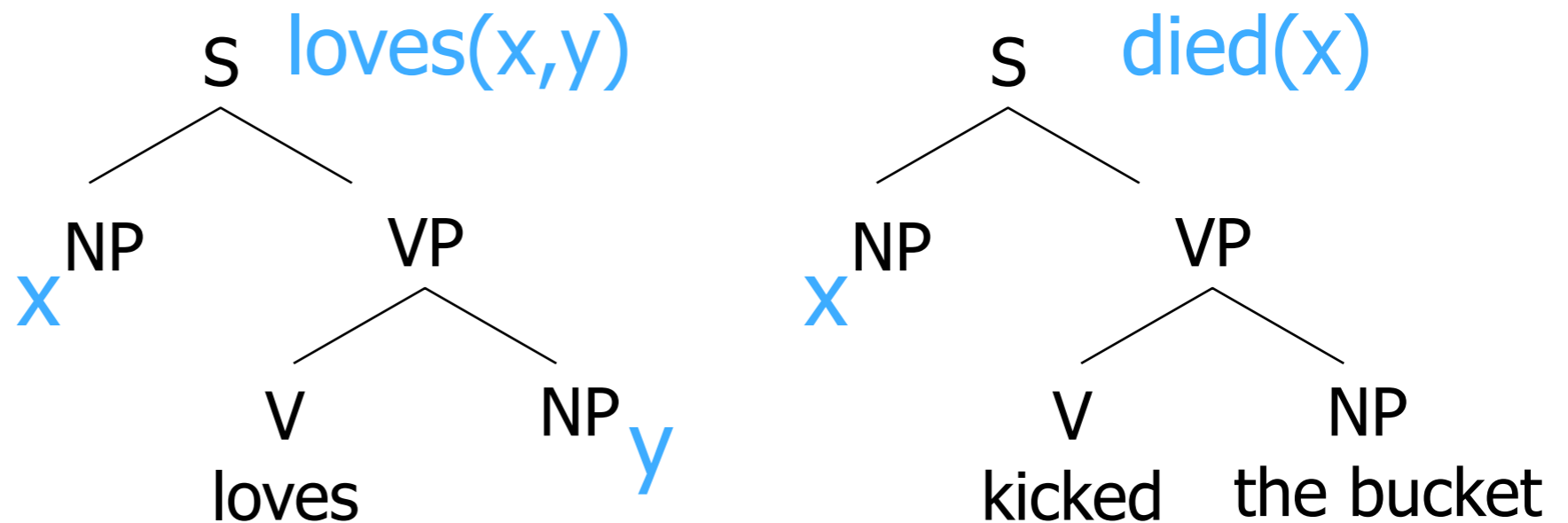
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- Template filling: $S[\text{sem}=\text{showflights}(x,y)] \rightarrow$
I want a flight from $NP[\text{sem}=x]$ to $NP[\text{sem}=y]$

Compositional Semantics

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 - $VP[\text{sem}=\text{v}(\text{obj})] \rightarrow V[\text{sem}=\text{v}] NP[\text{sem}=\text{obj}]$
 - $S[\text{sem}=\text{vp}(\text{subj})] \rightarrow NP[\text{sem}=\text{subj}] VP[\text{sem}=\text{vp}]$

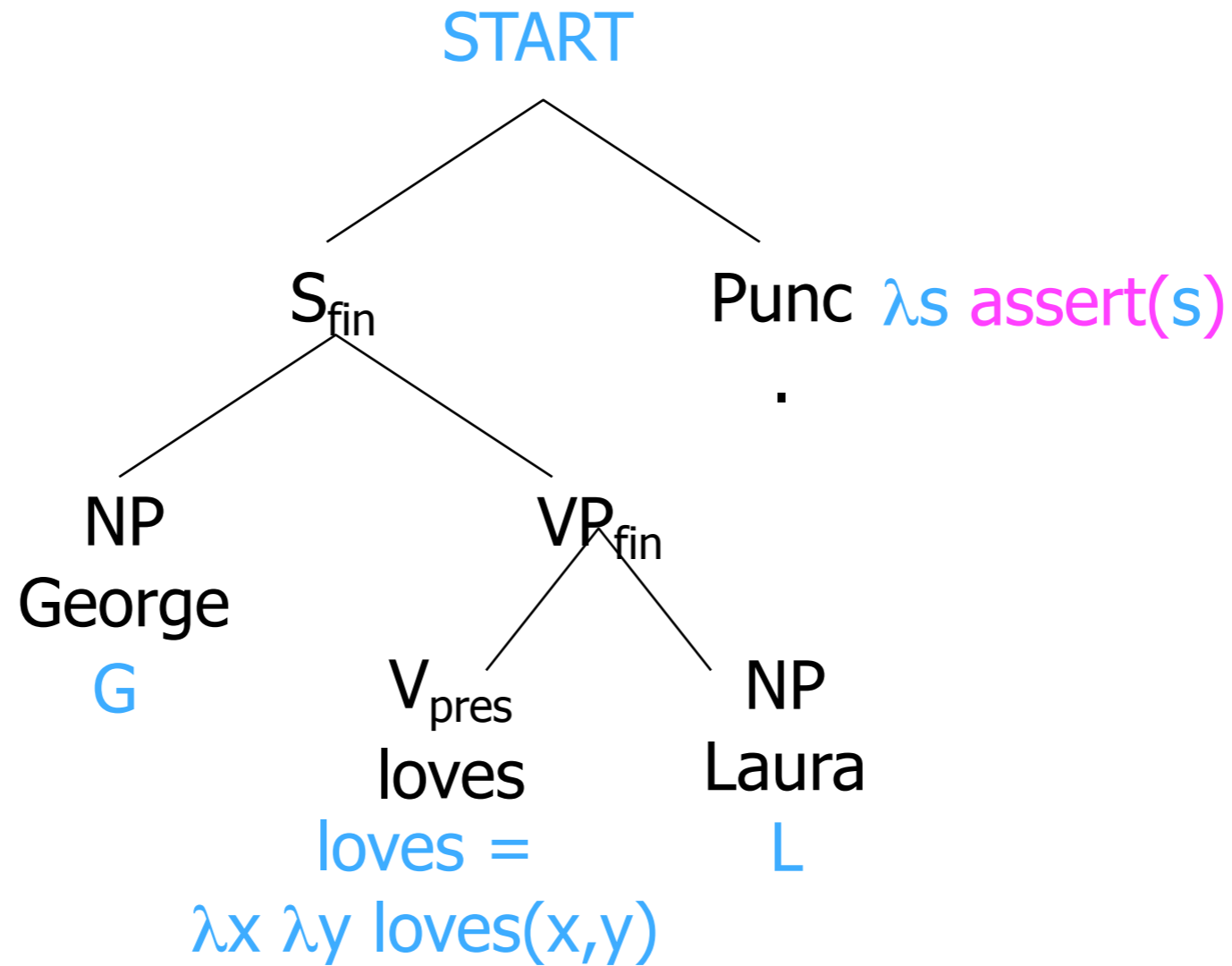
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- **Now** George loves Laura **has** $\text{sem}=\text{loves}(\text{Laura})(\text{George})$

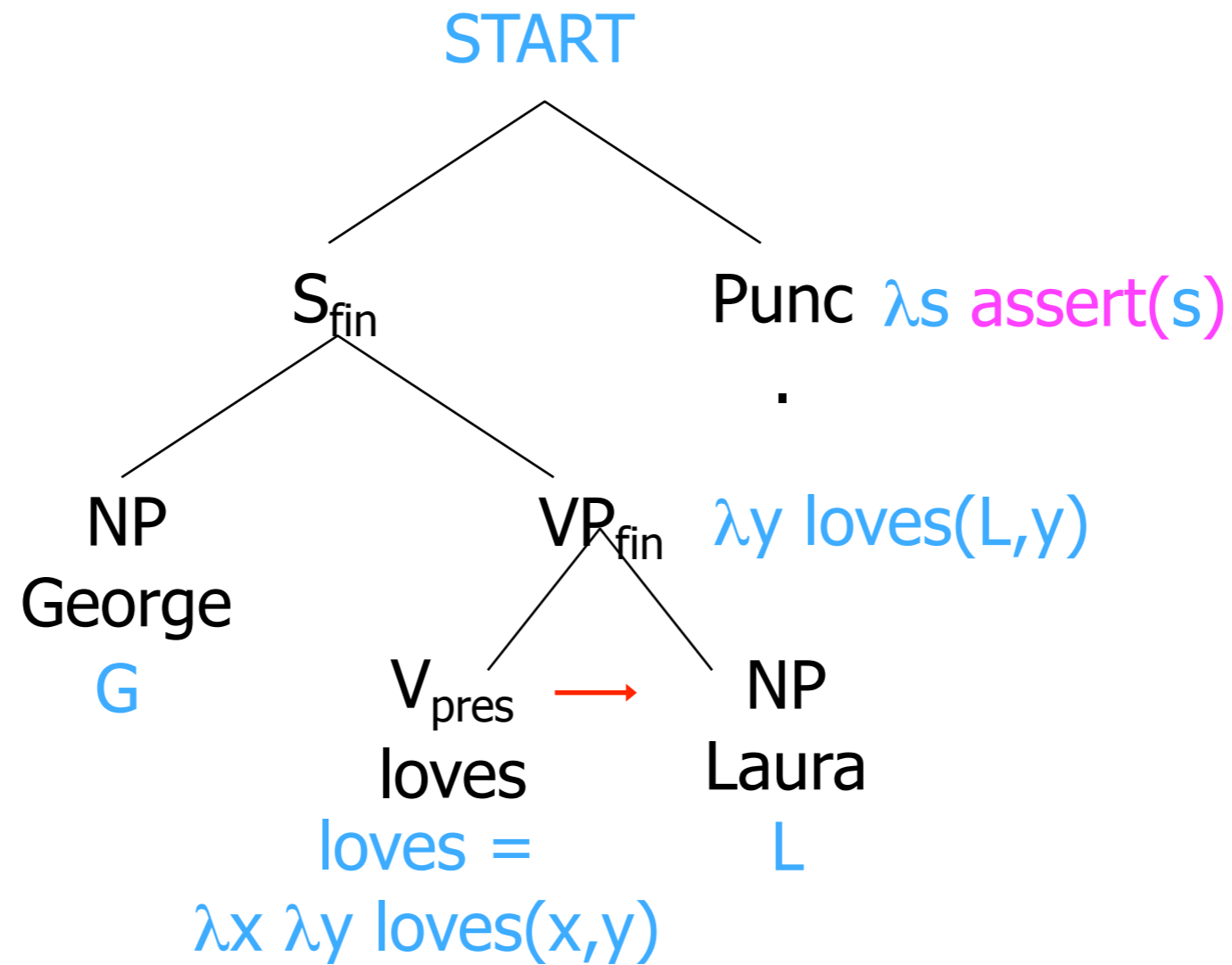
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- **Now** `George loves Laura` has $\text{sem}=\text{loves}(\text{Laura})(\text{George})$
- In this manner we'll sketch a version where
 - Still compute semantics bottom-up
 - Grammar is in Chomsky Normal Form
 - So each node has 2 children: 1 function & 1 argument
 - **To get its semantics, apply function to argument!**

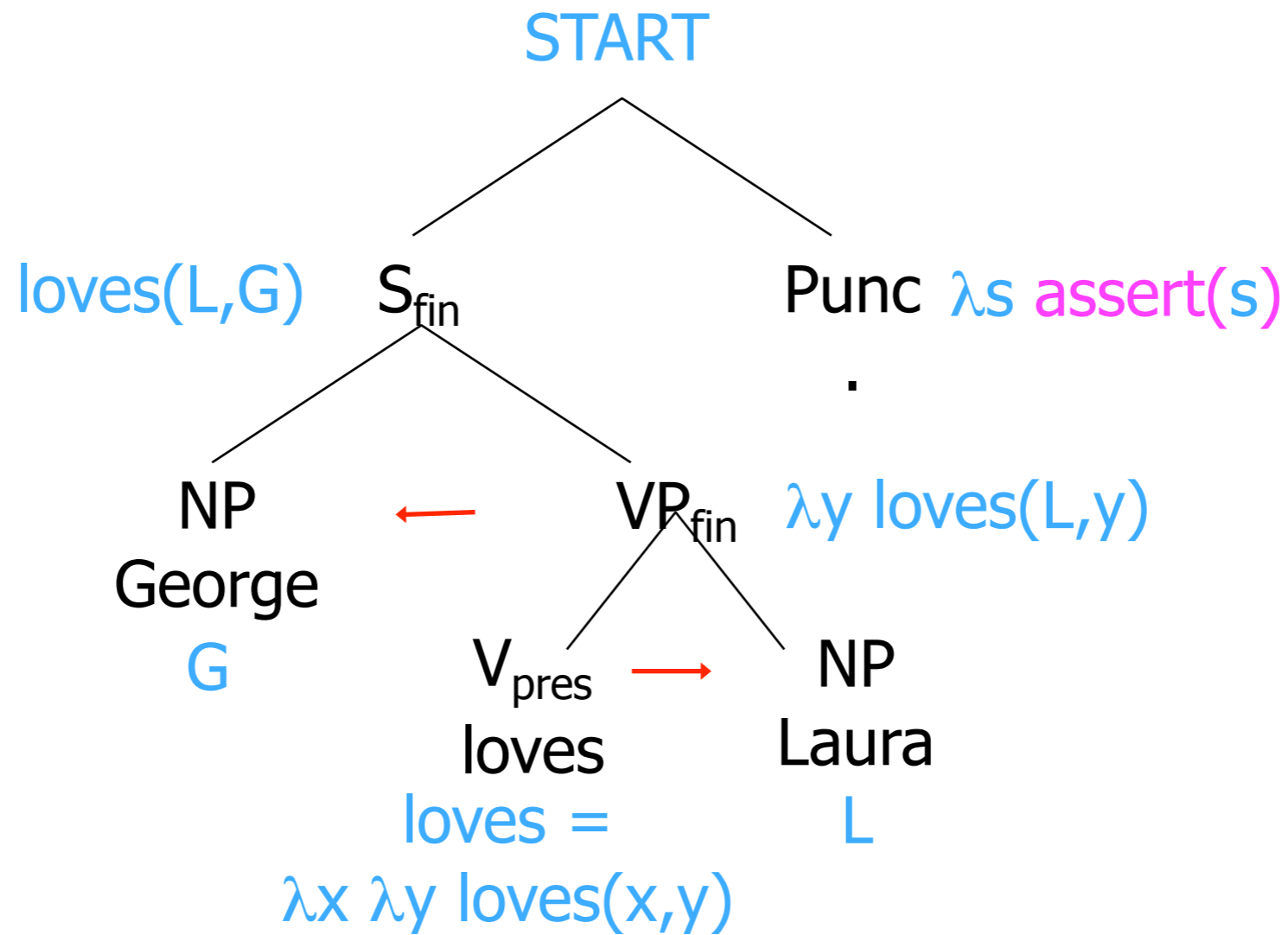
Compositional Semantics



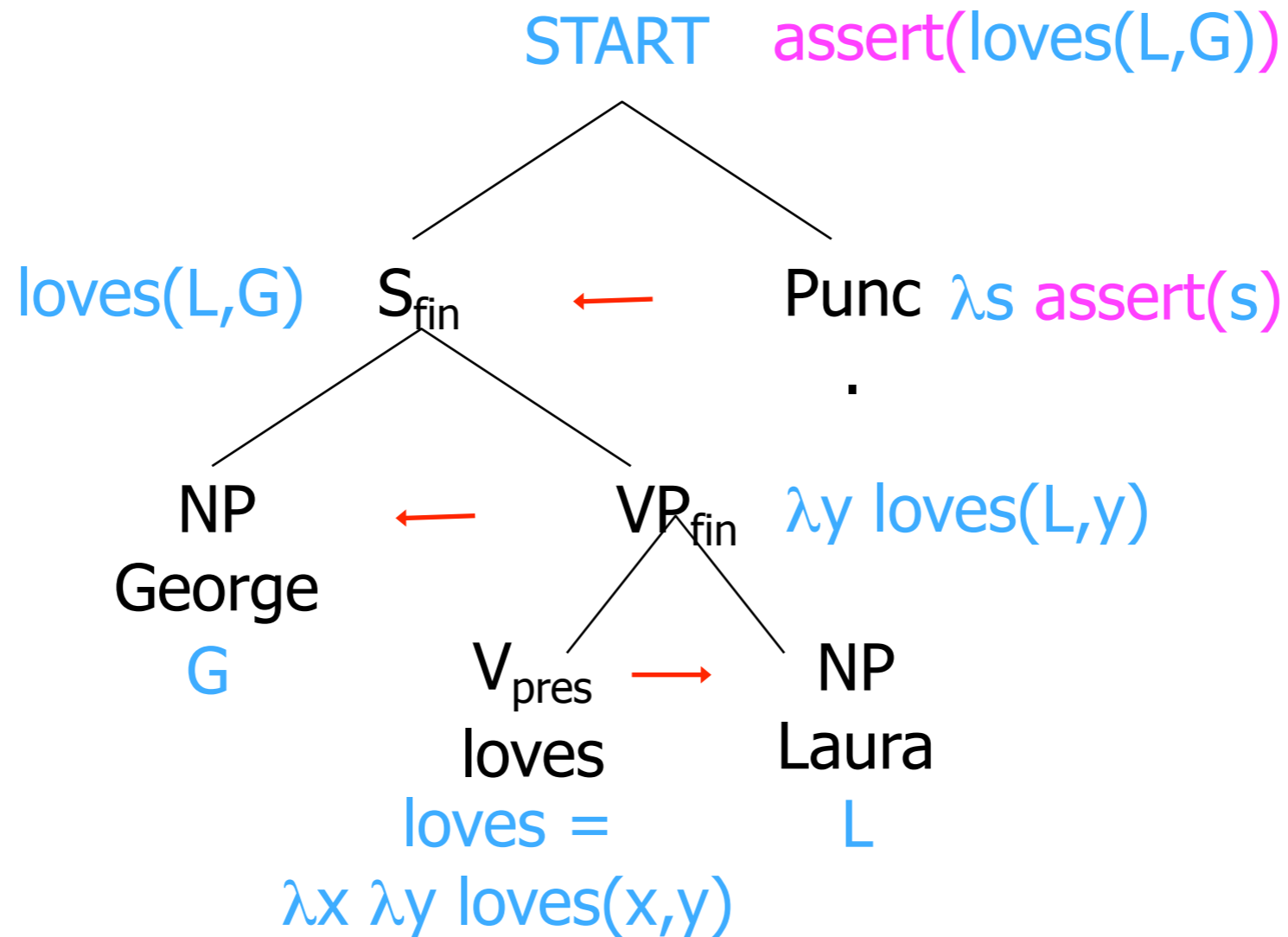
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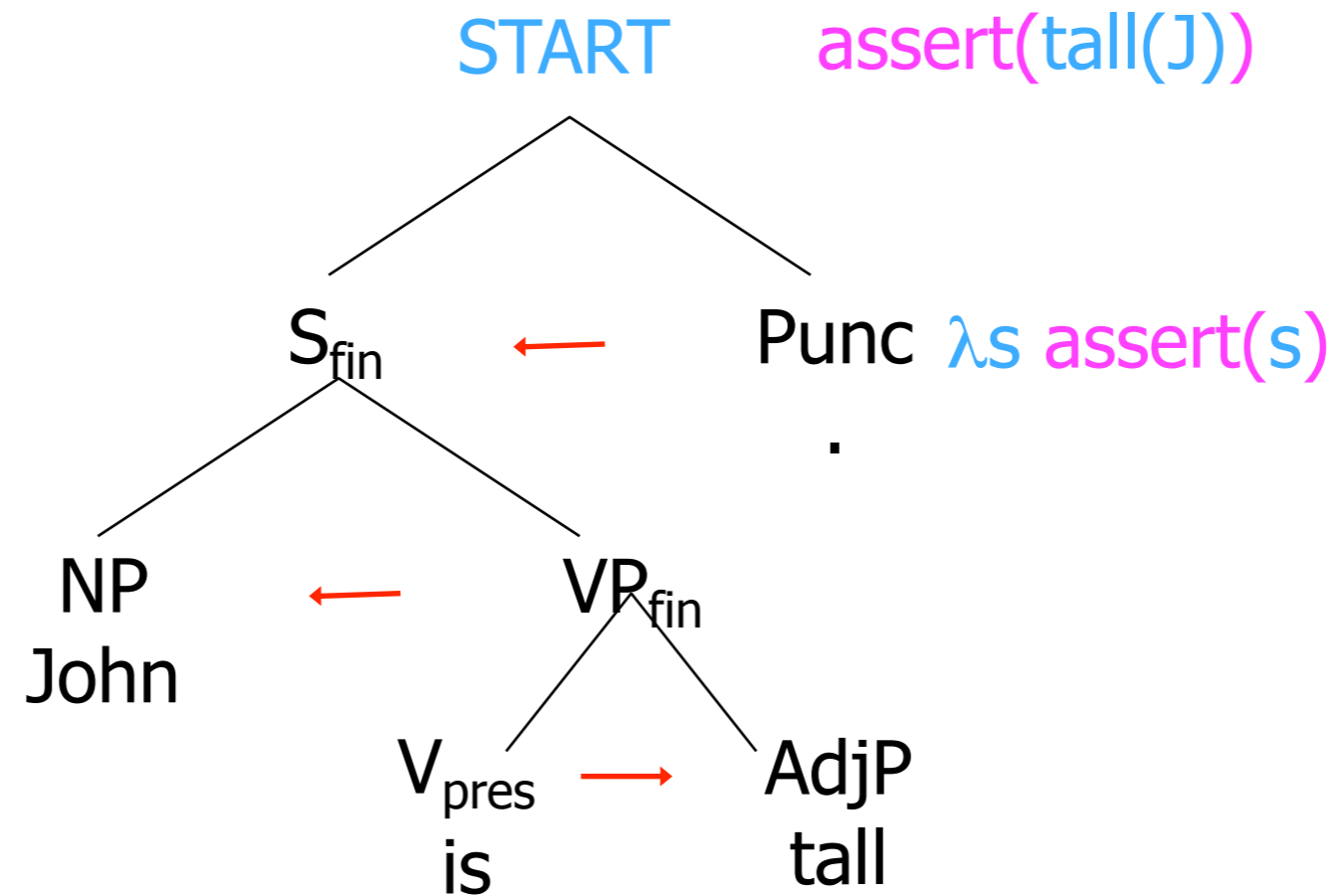
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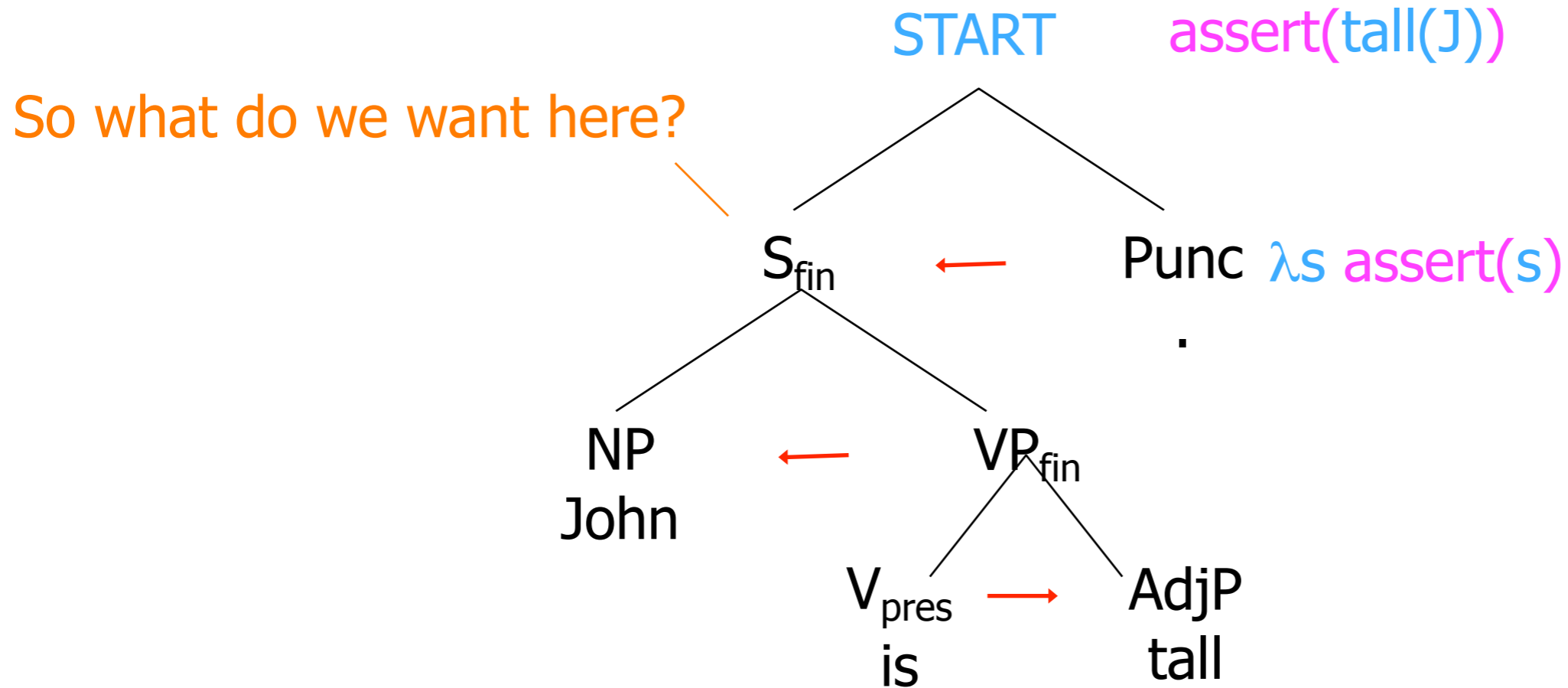
Compositional Semantics



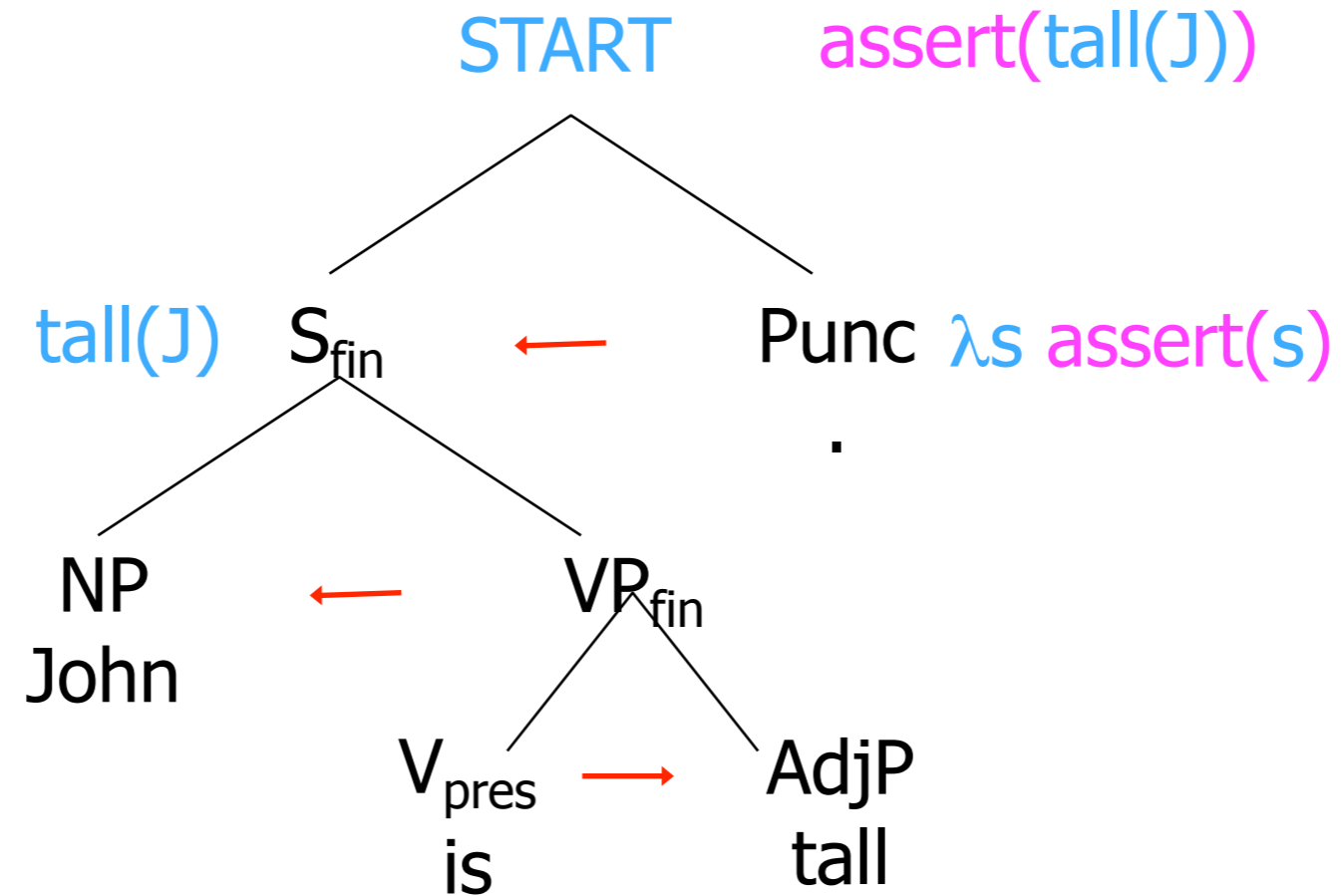
Compositional Semantics



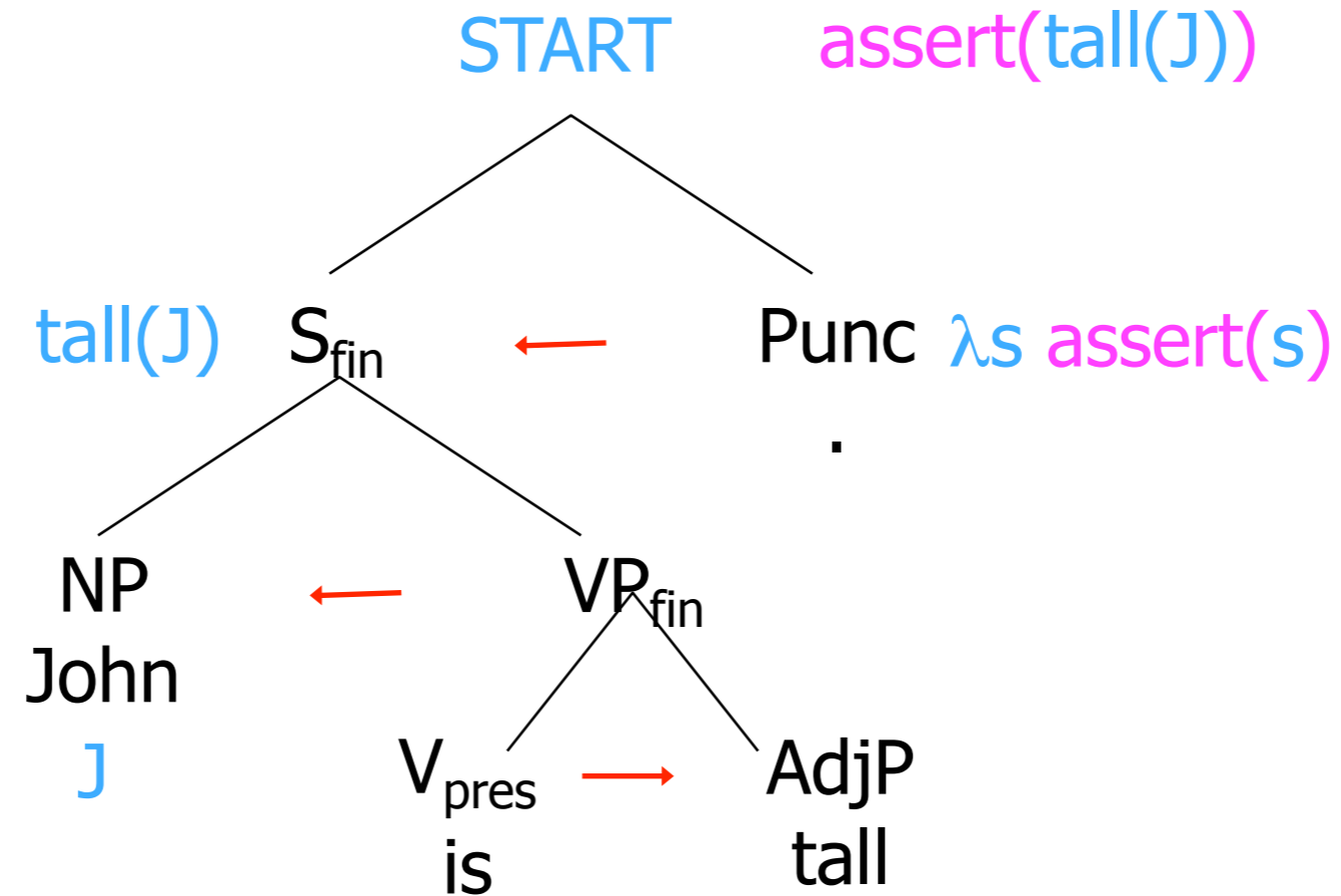
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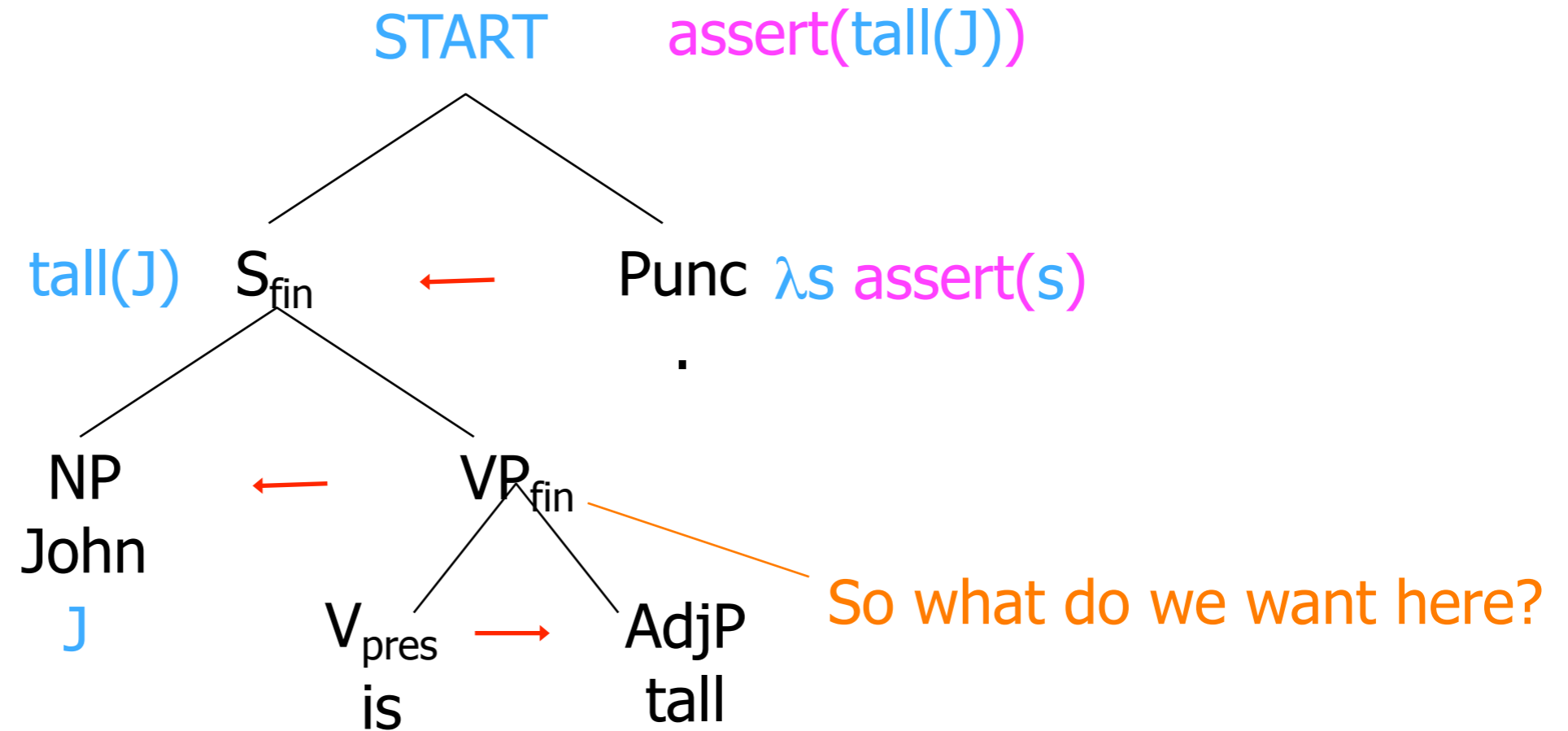
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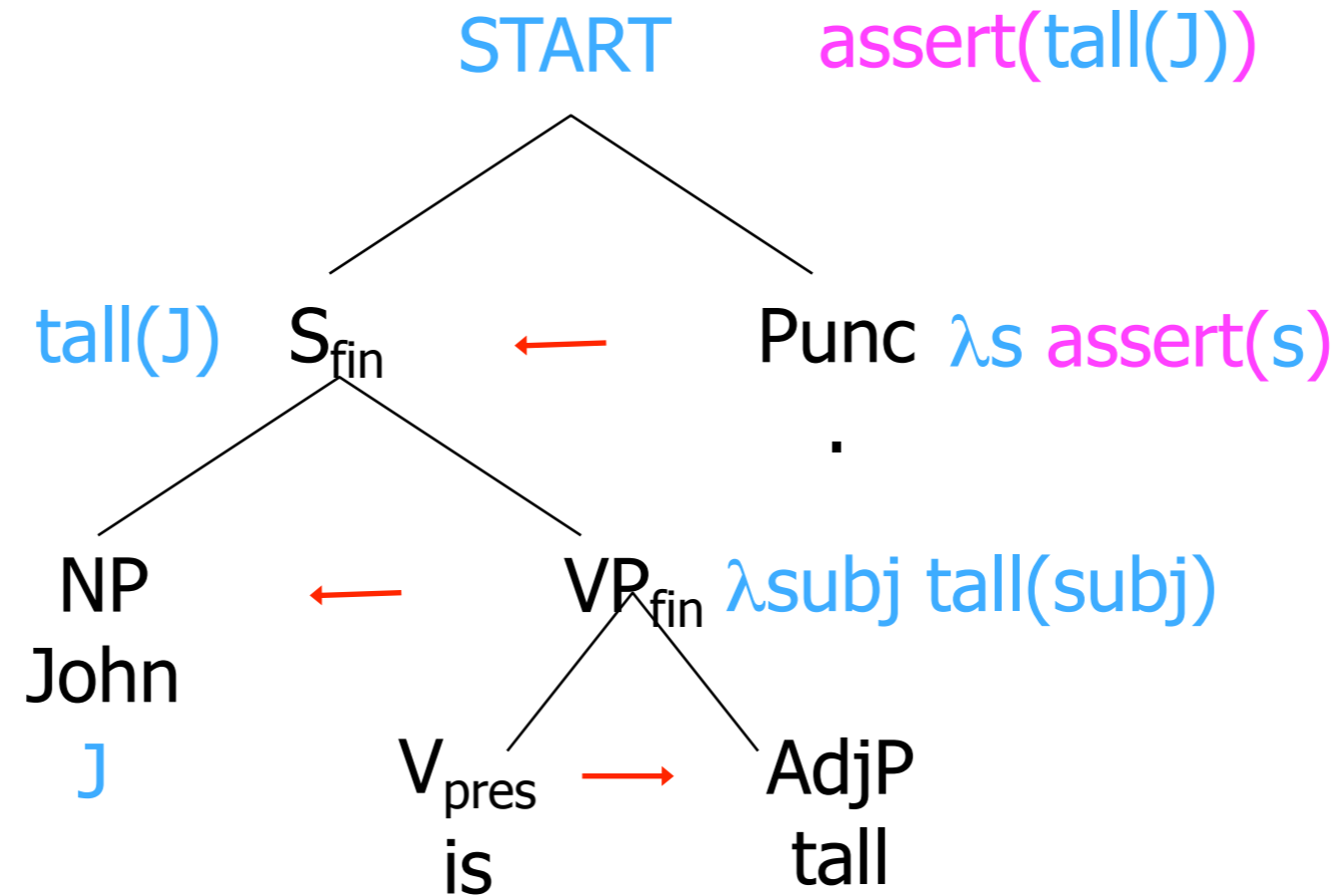
Compositional Semantics



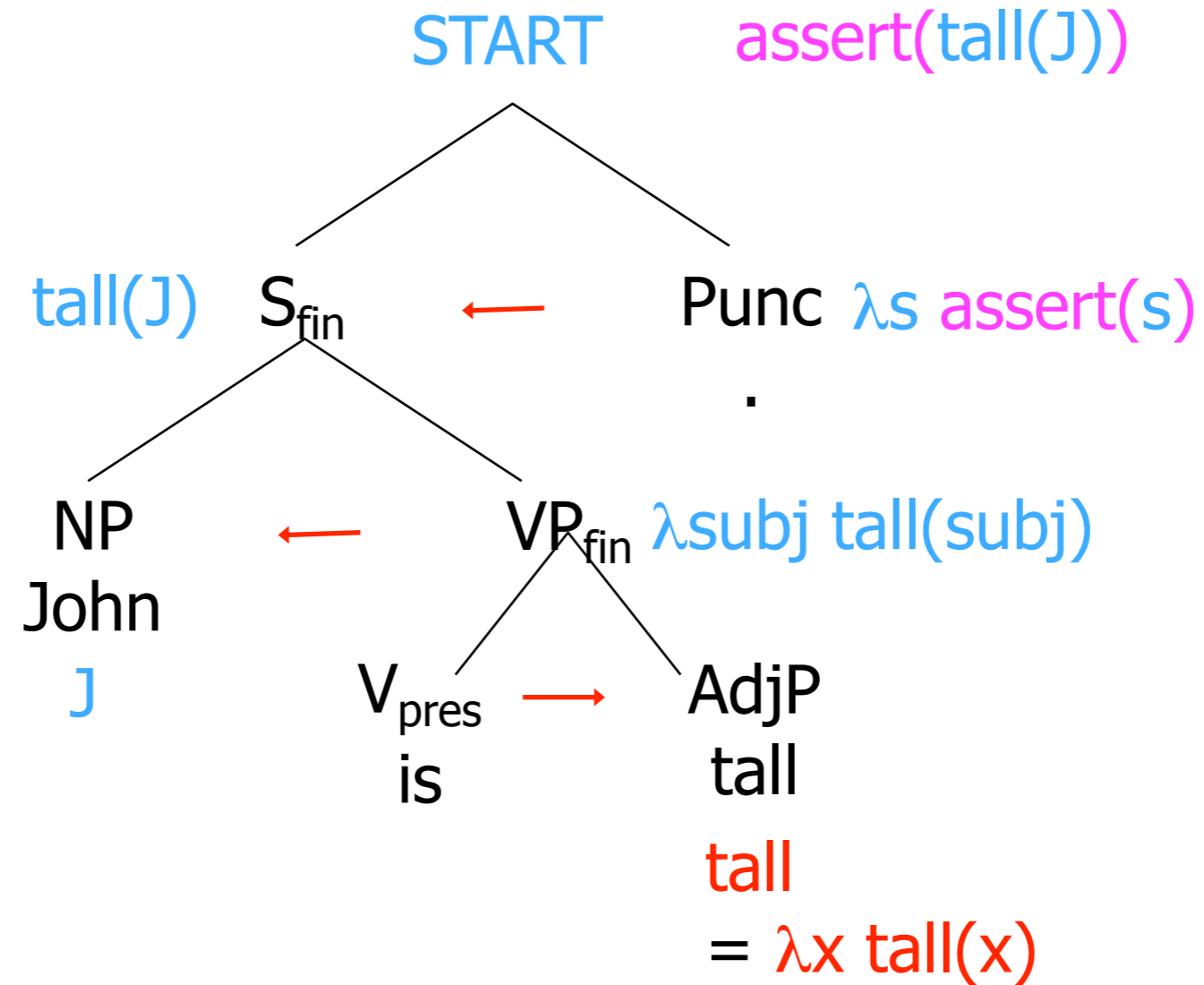
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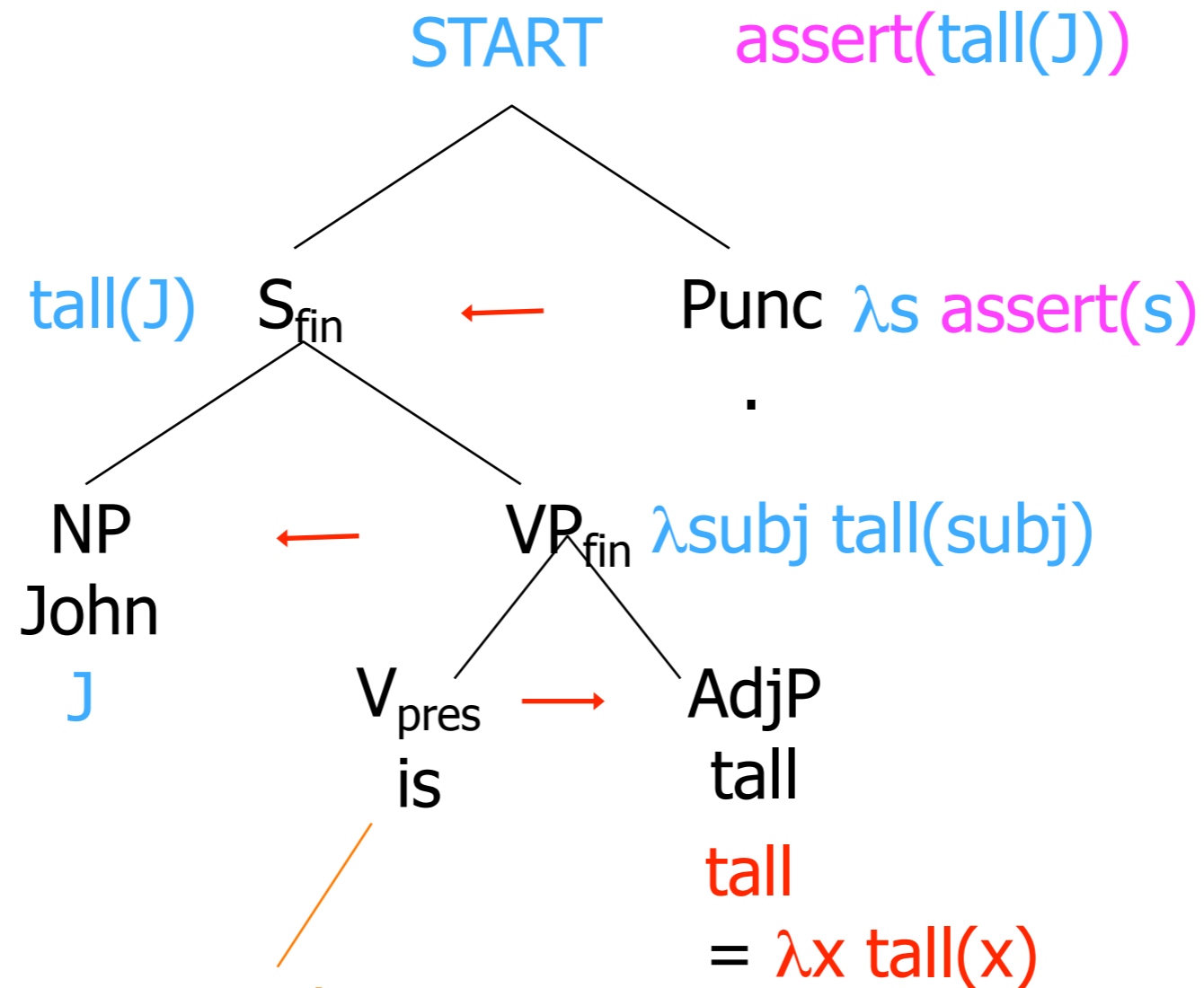
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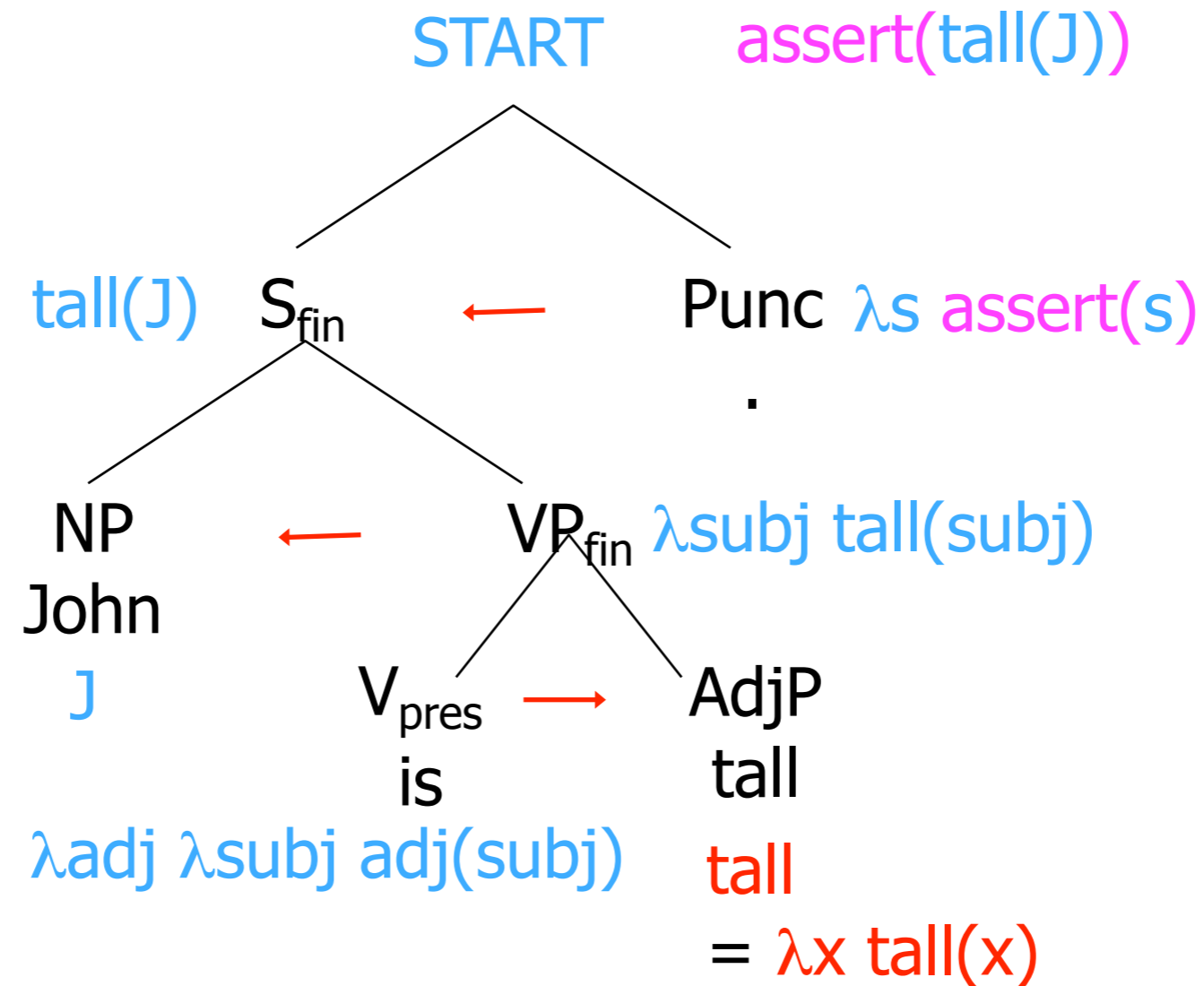


Compositional Semantics

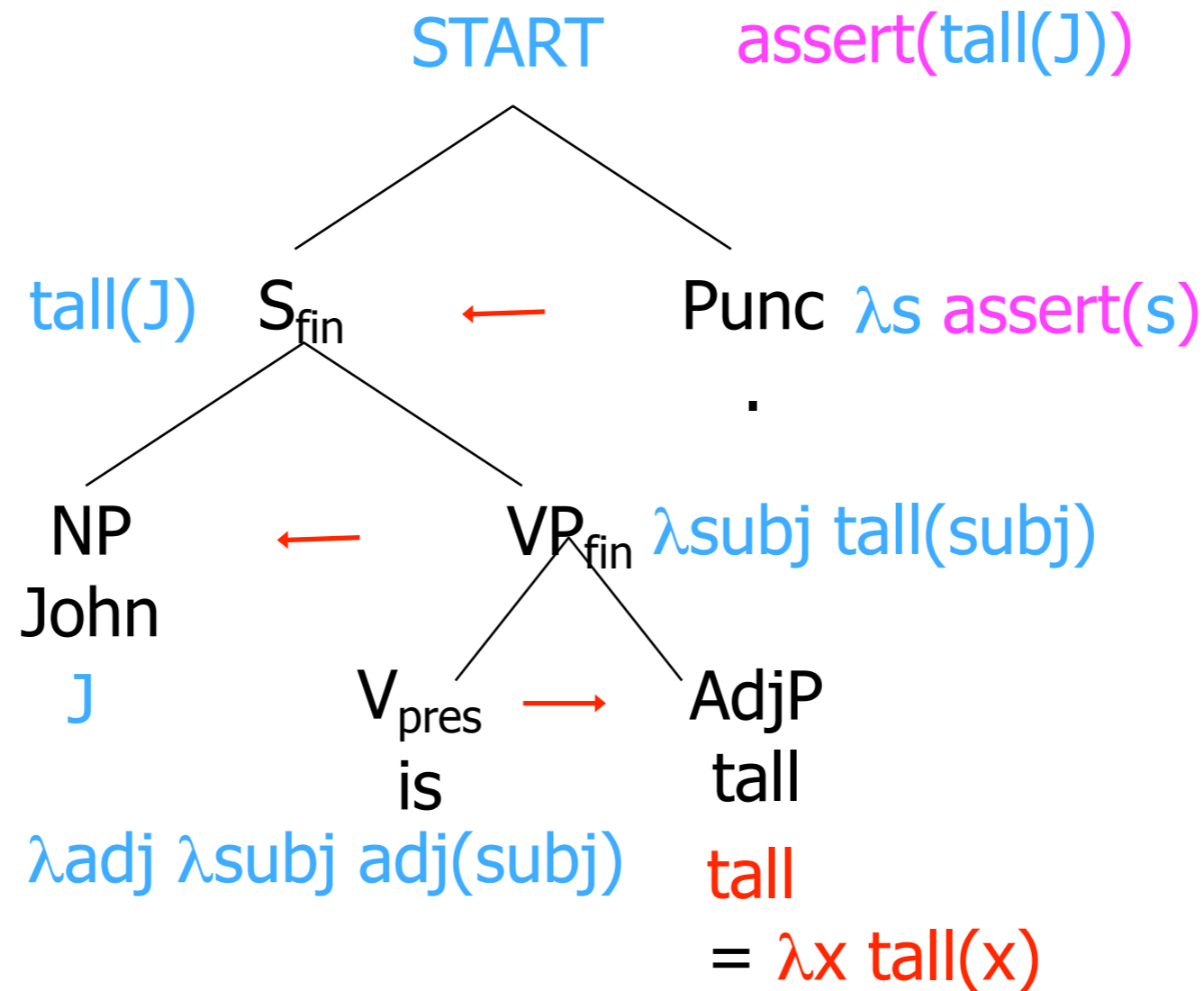


So what do we want here?

Compositional Semantics



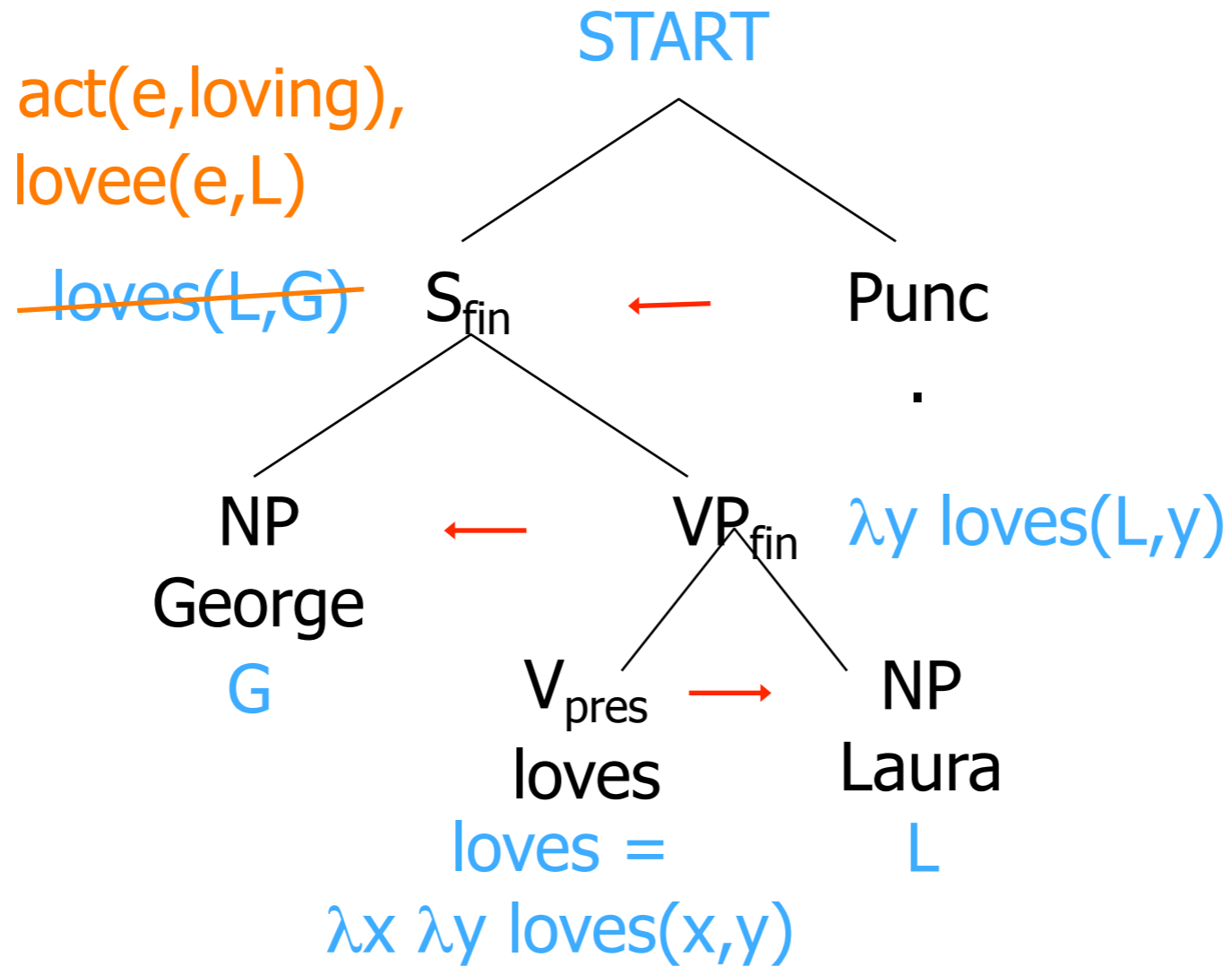
Compositional Semantics



$$\begin{aligned}
 & (\lambda \text{adj } \lambda \text{subj adj}(\text{subj}))(\lambda x \text{ tall}(x)) \\
 = & \lambda \text{subj } (\lambda x \text{ tall}(x))(\text{subj}) \\
 = & \lambda \text{subj } \text{tall}(\text{subj})
 \end{aligned}$$

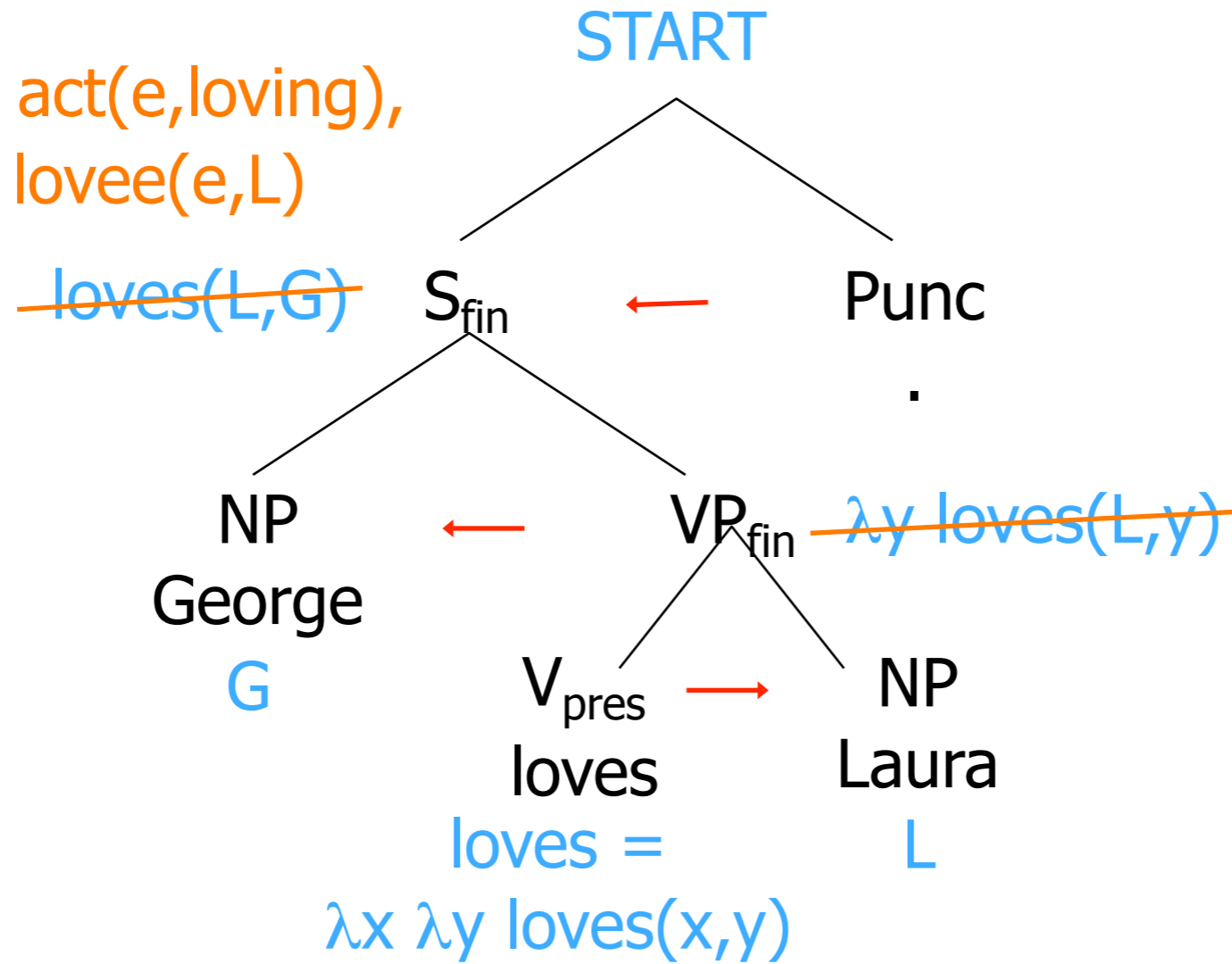
Compositional Semantics

$\exists e$ present(e), act(e,loving),
lover(e,G), lovee(e,L)



Compositional Semantics

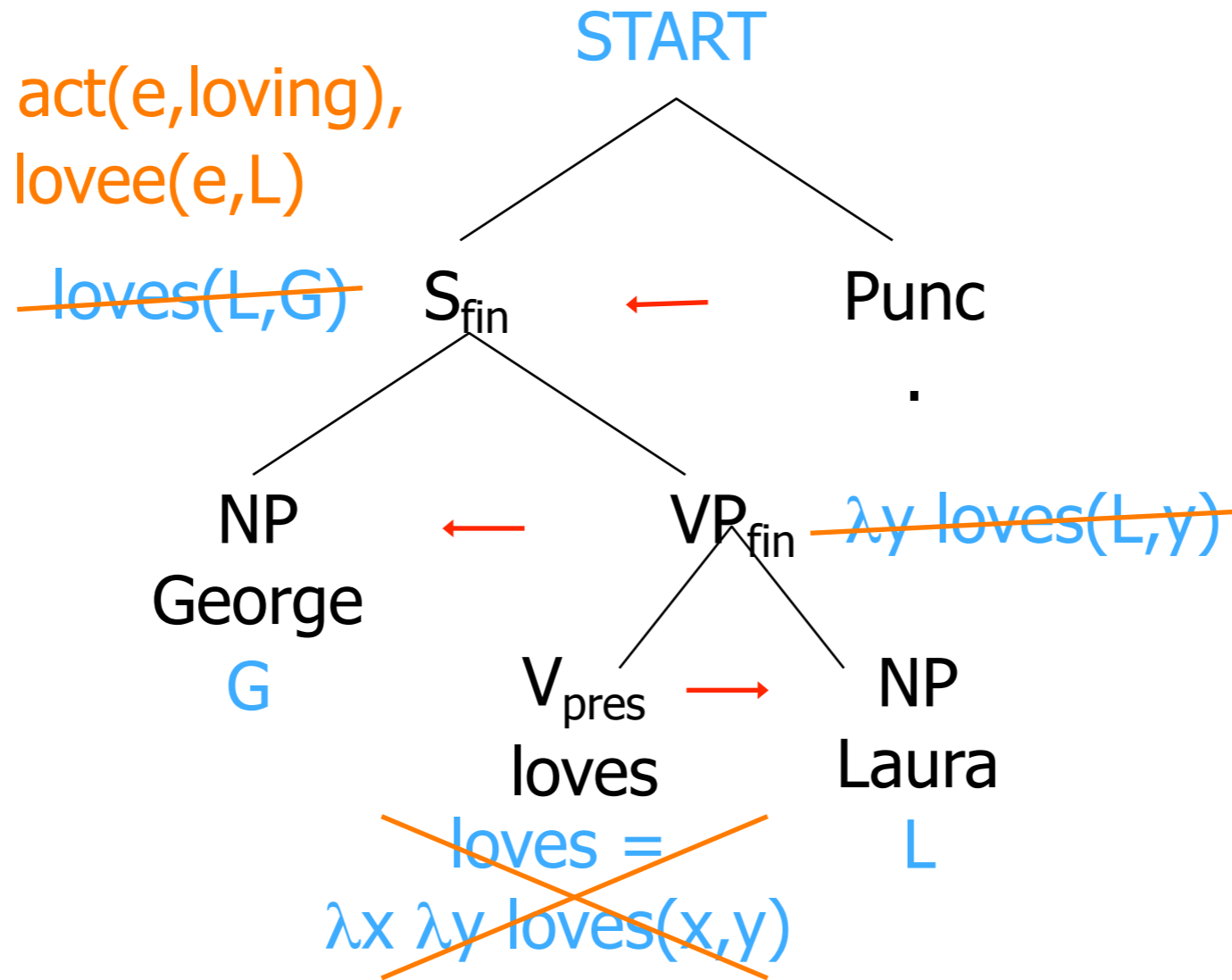
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Compositional Semantics

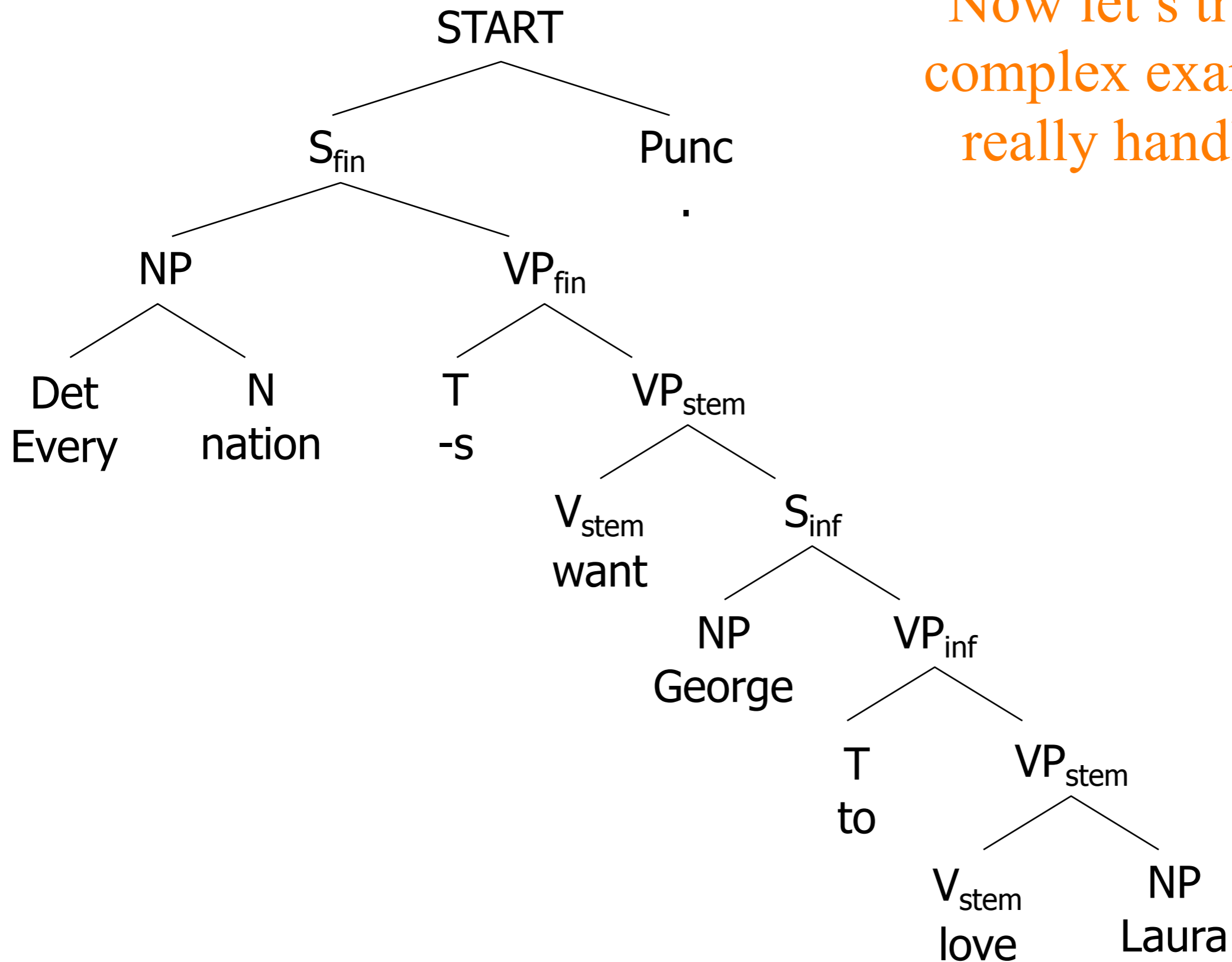
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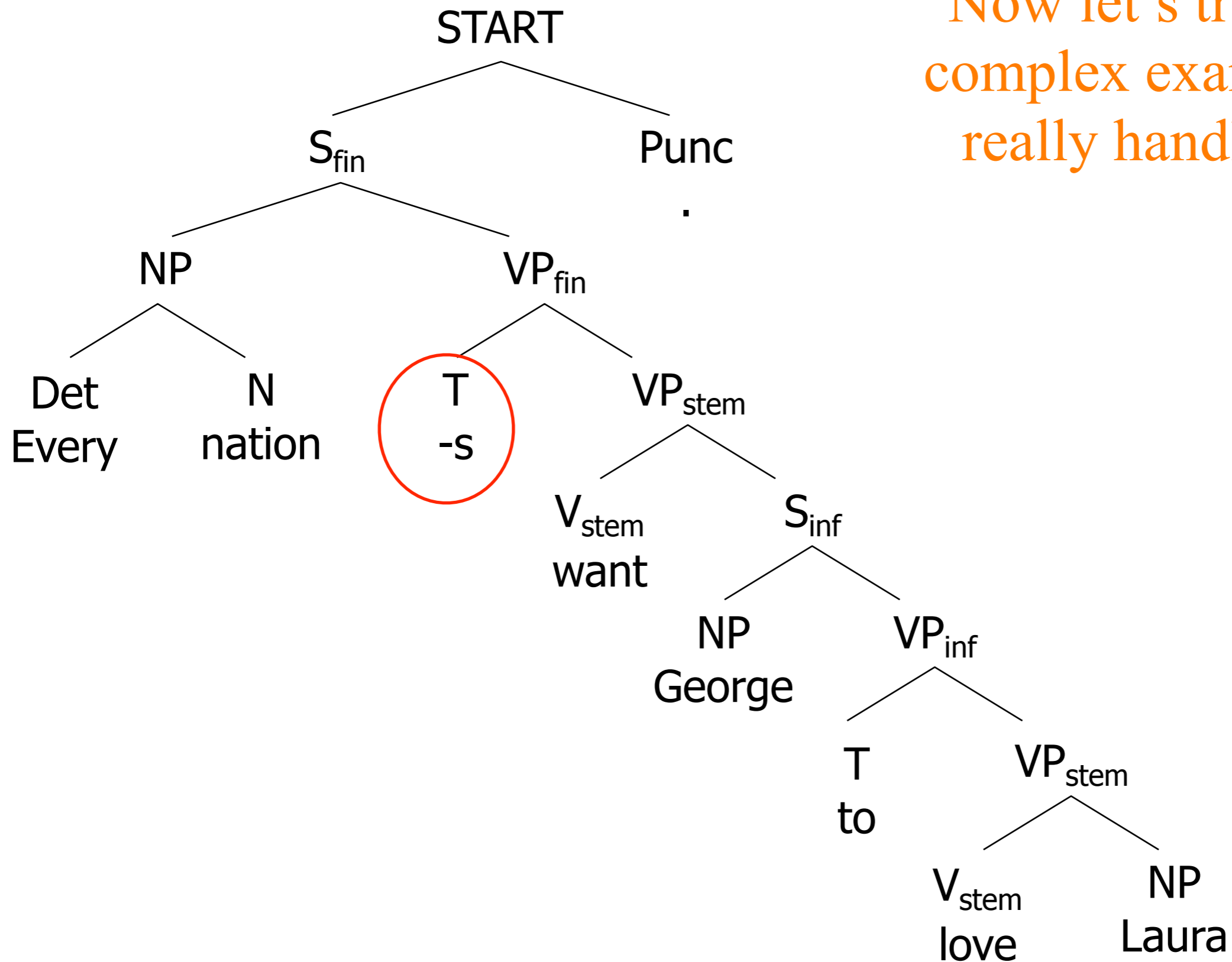
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$\lambda x \lambda y \exists e \text{ present}(e),$
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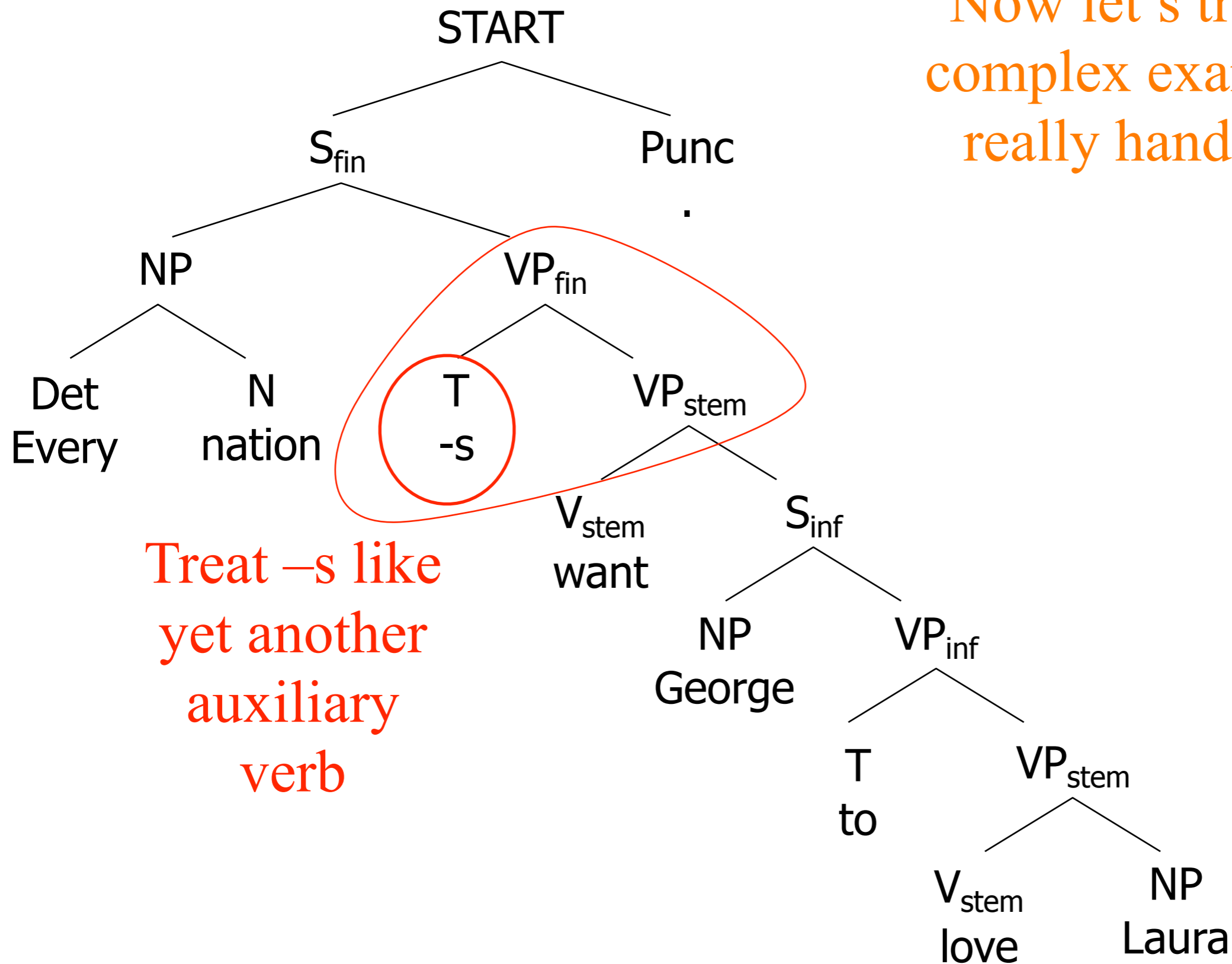
Now let's try a more complex example, and really handle tense.



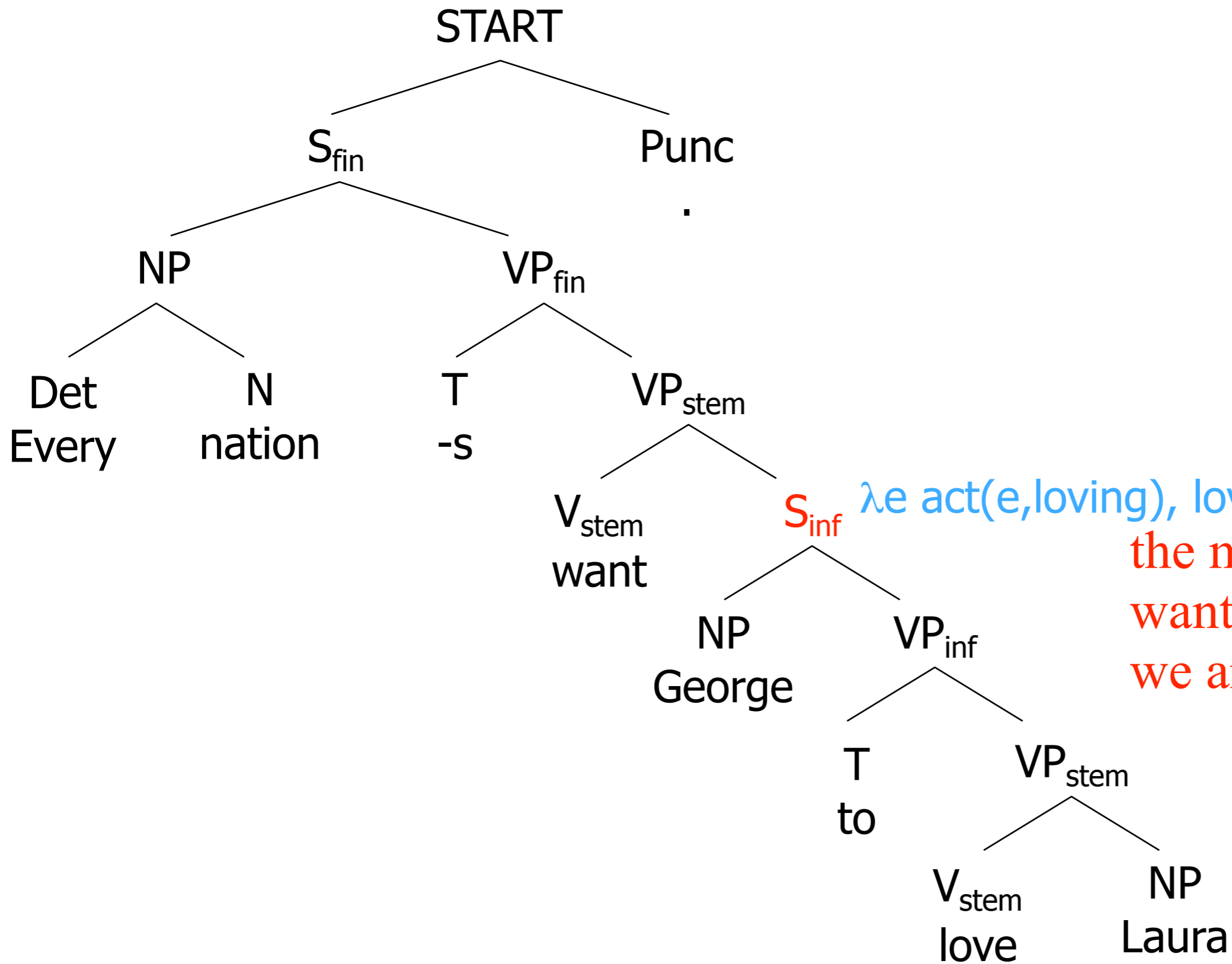
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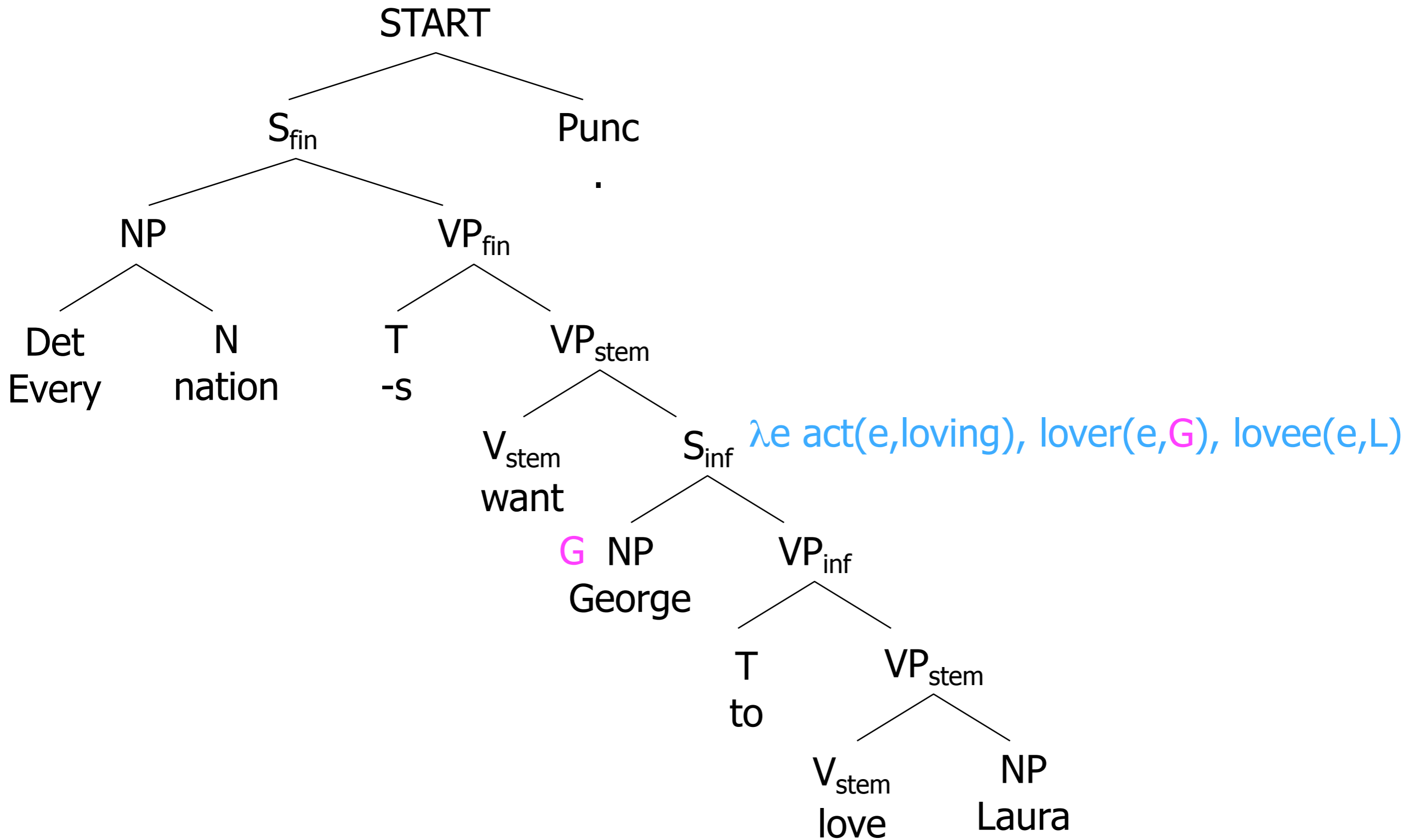
Now let's try a more complex example, and really handle tense.

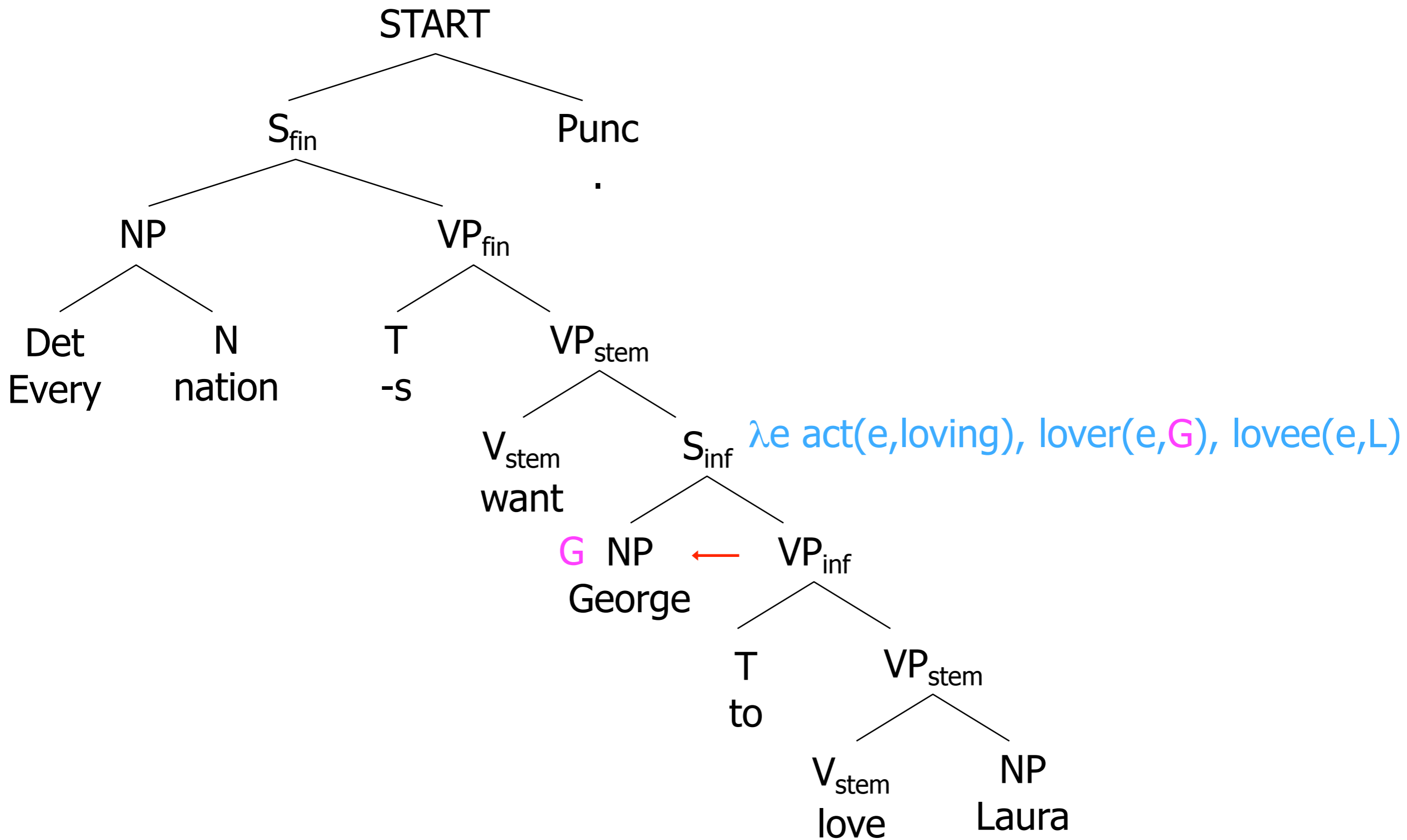


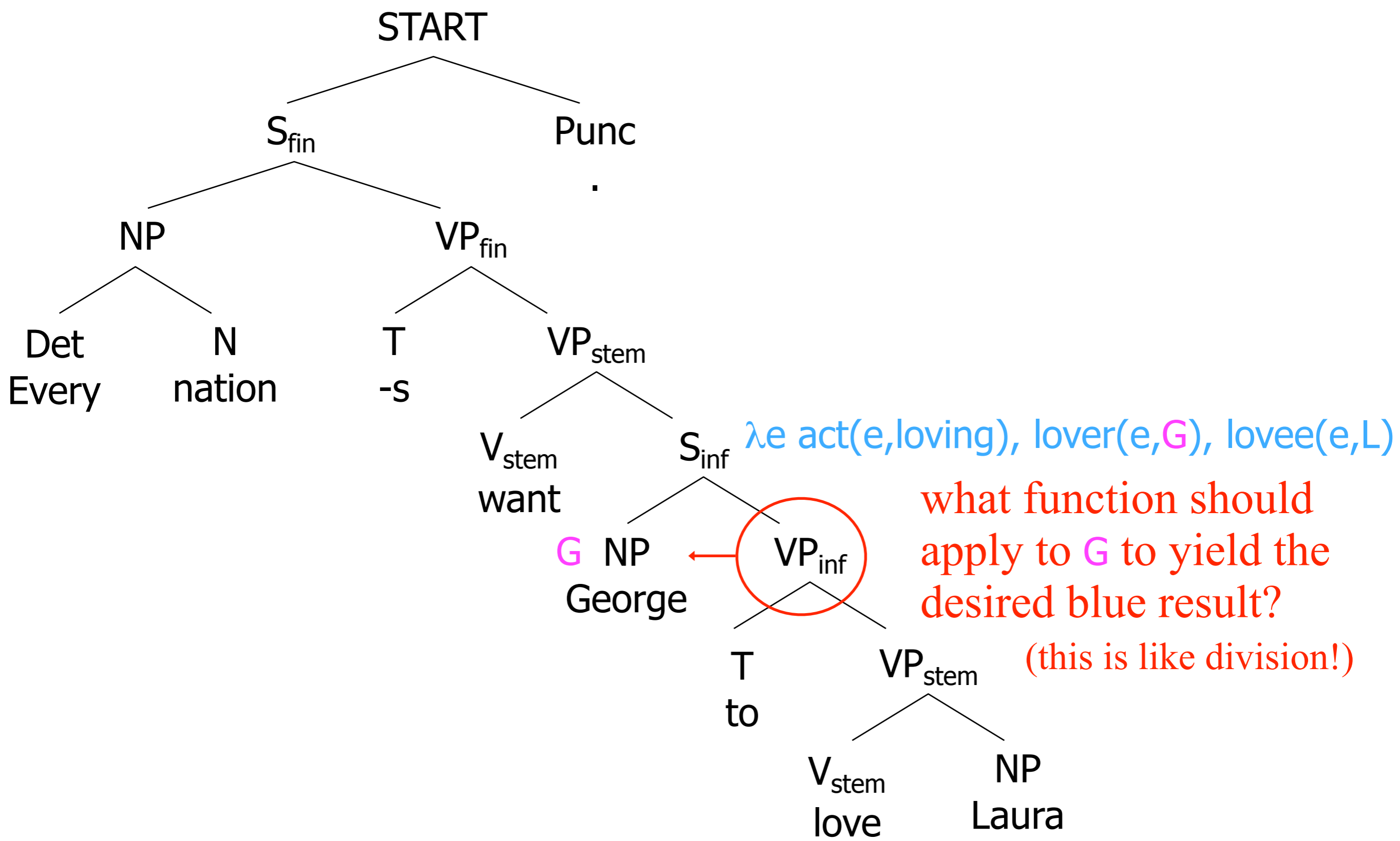
Treat -s like yet another auxiliary verb

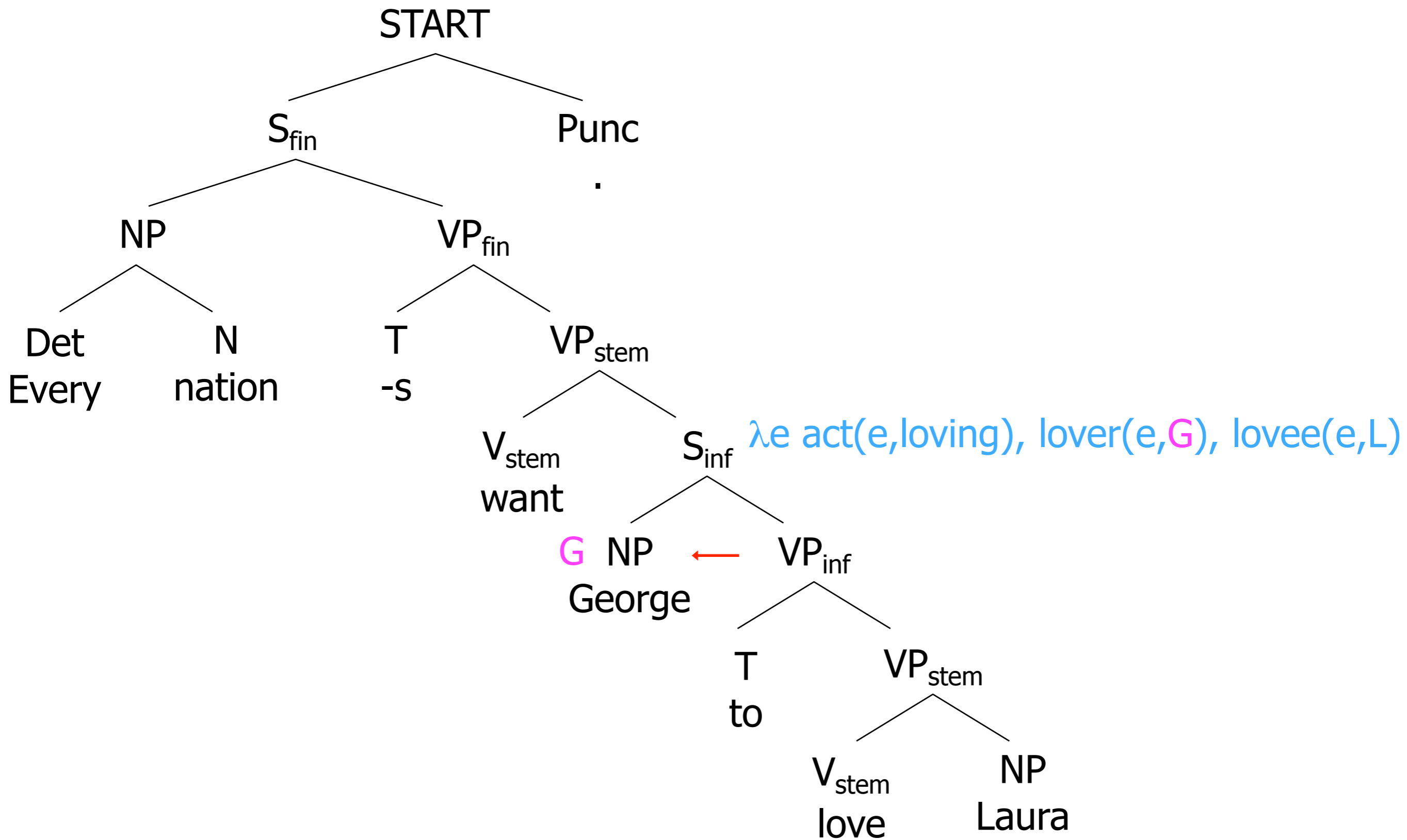


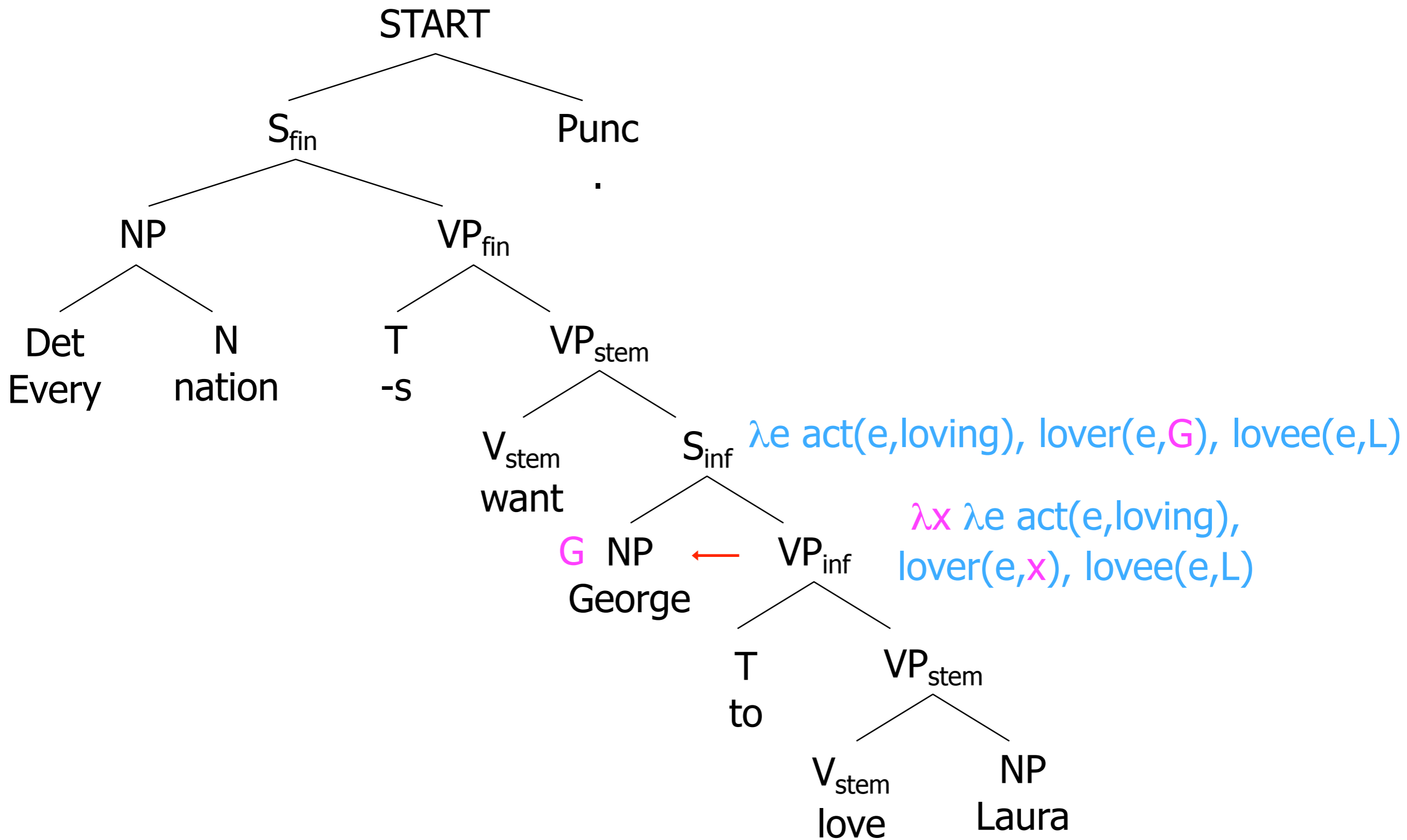
$\lambda e \text{ act}(e, \text{loving}), \text{lover}(e, G), \text{lovee}(e, L)$
 the meaning that we
 want here: how can
 we arrange to get it?

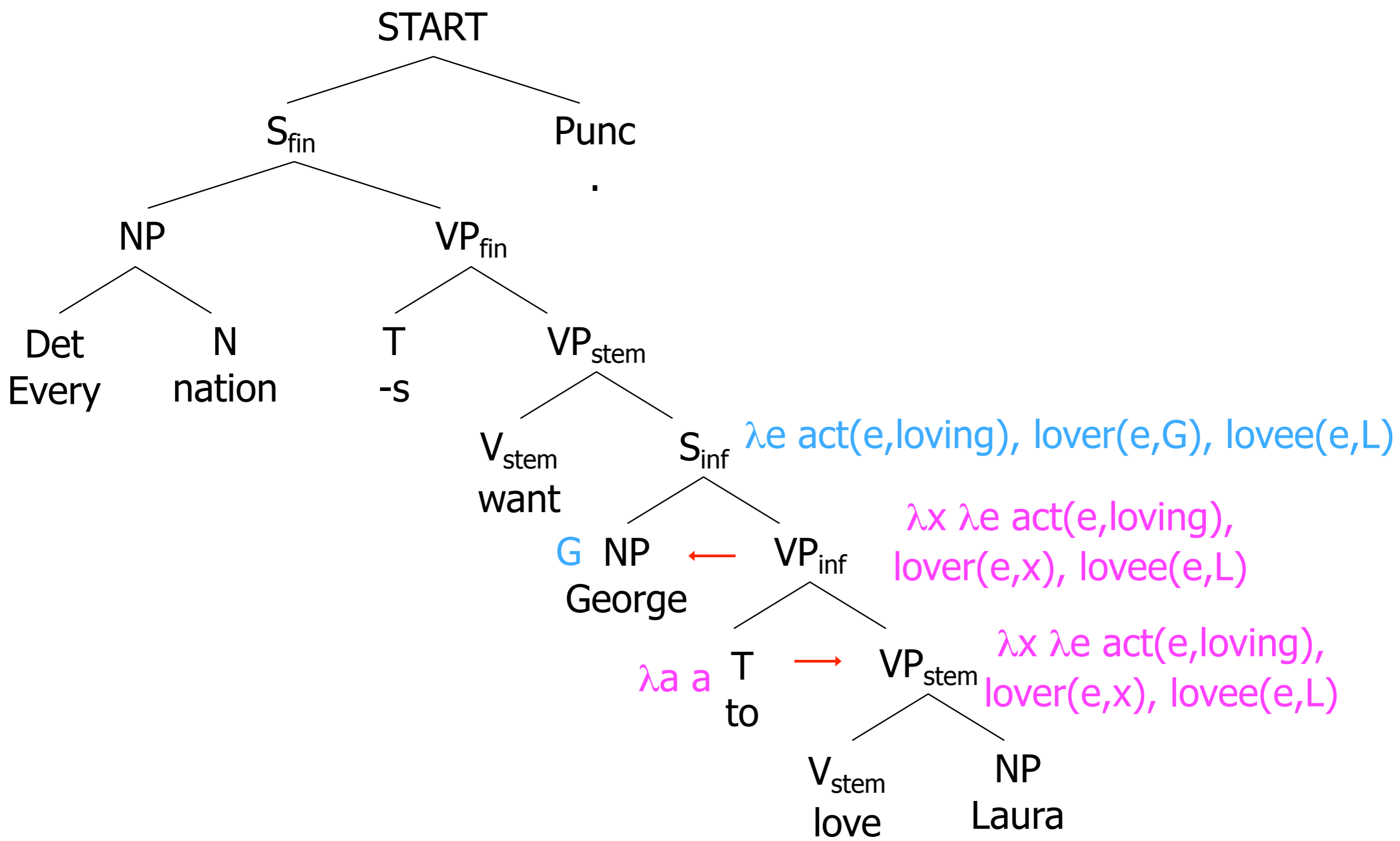


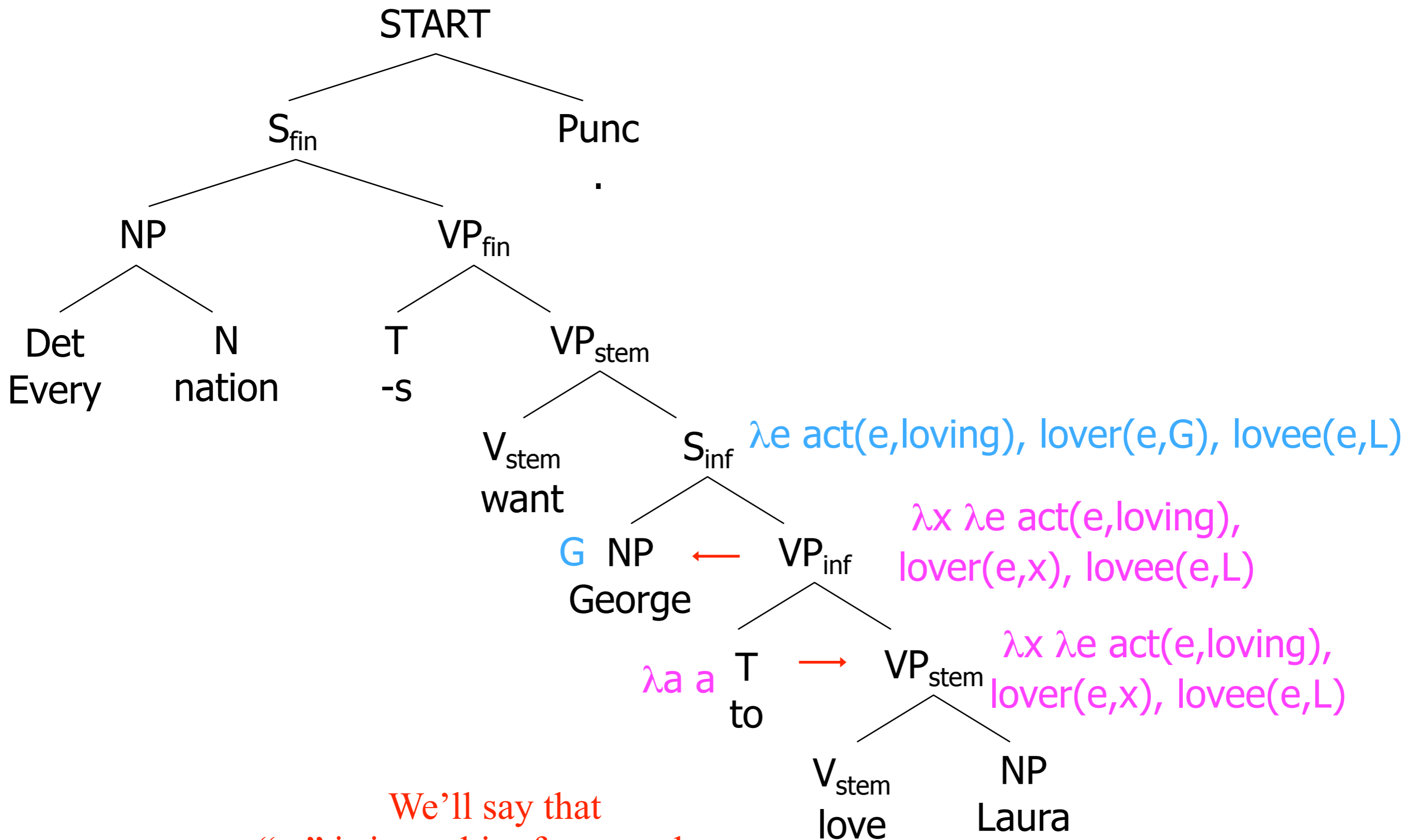




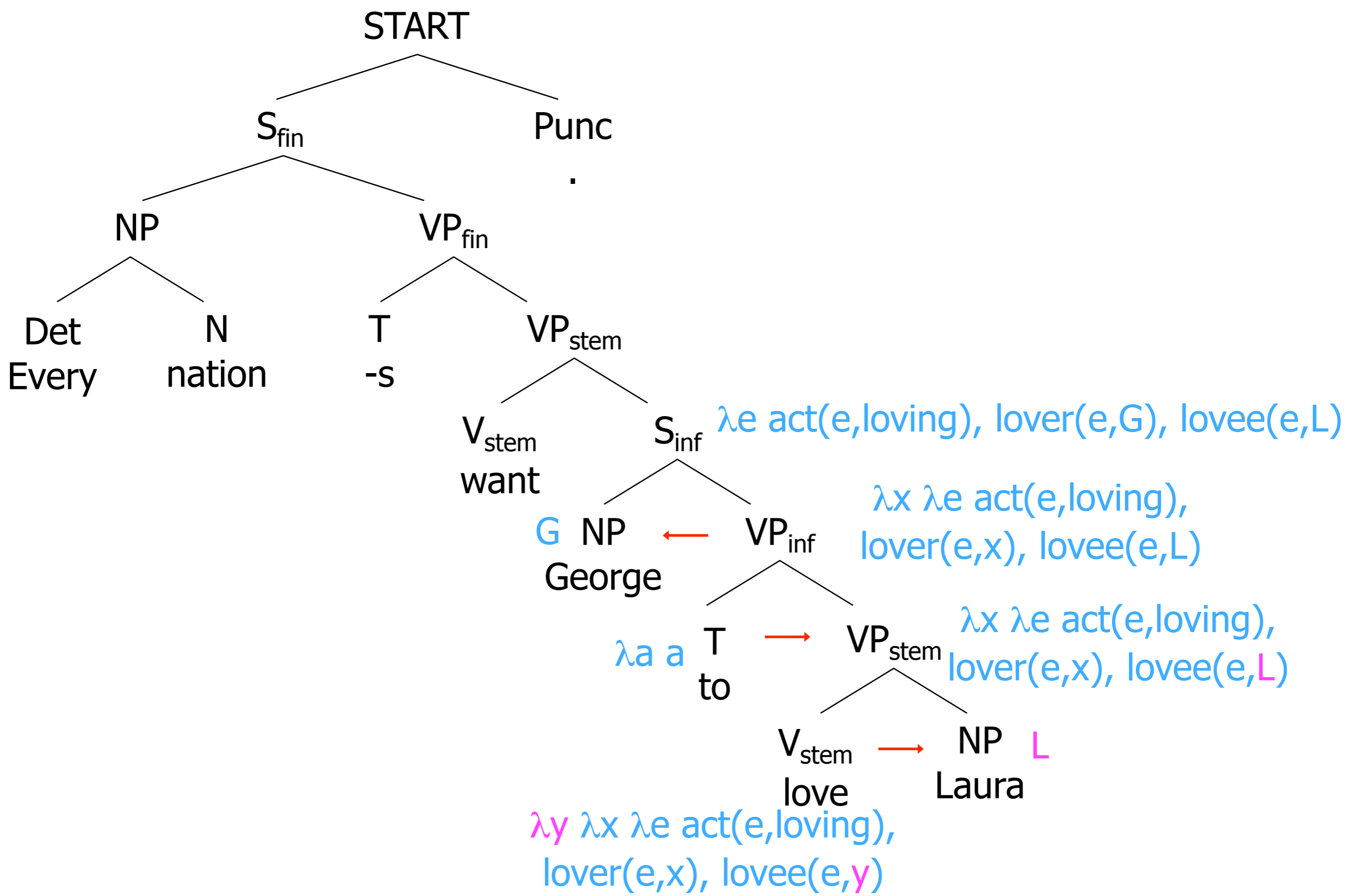


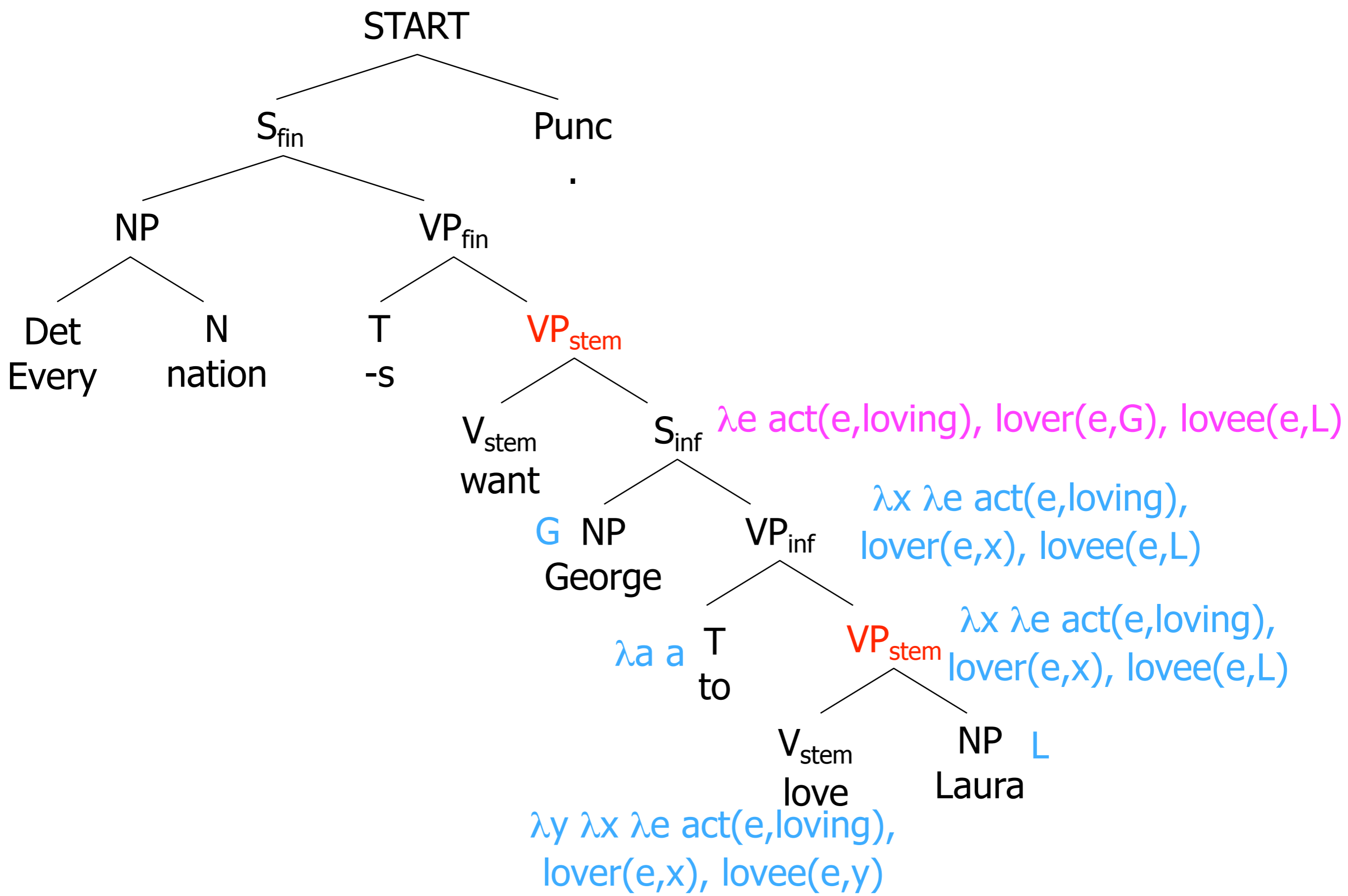


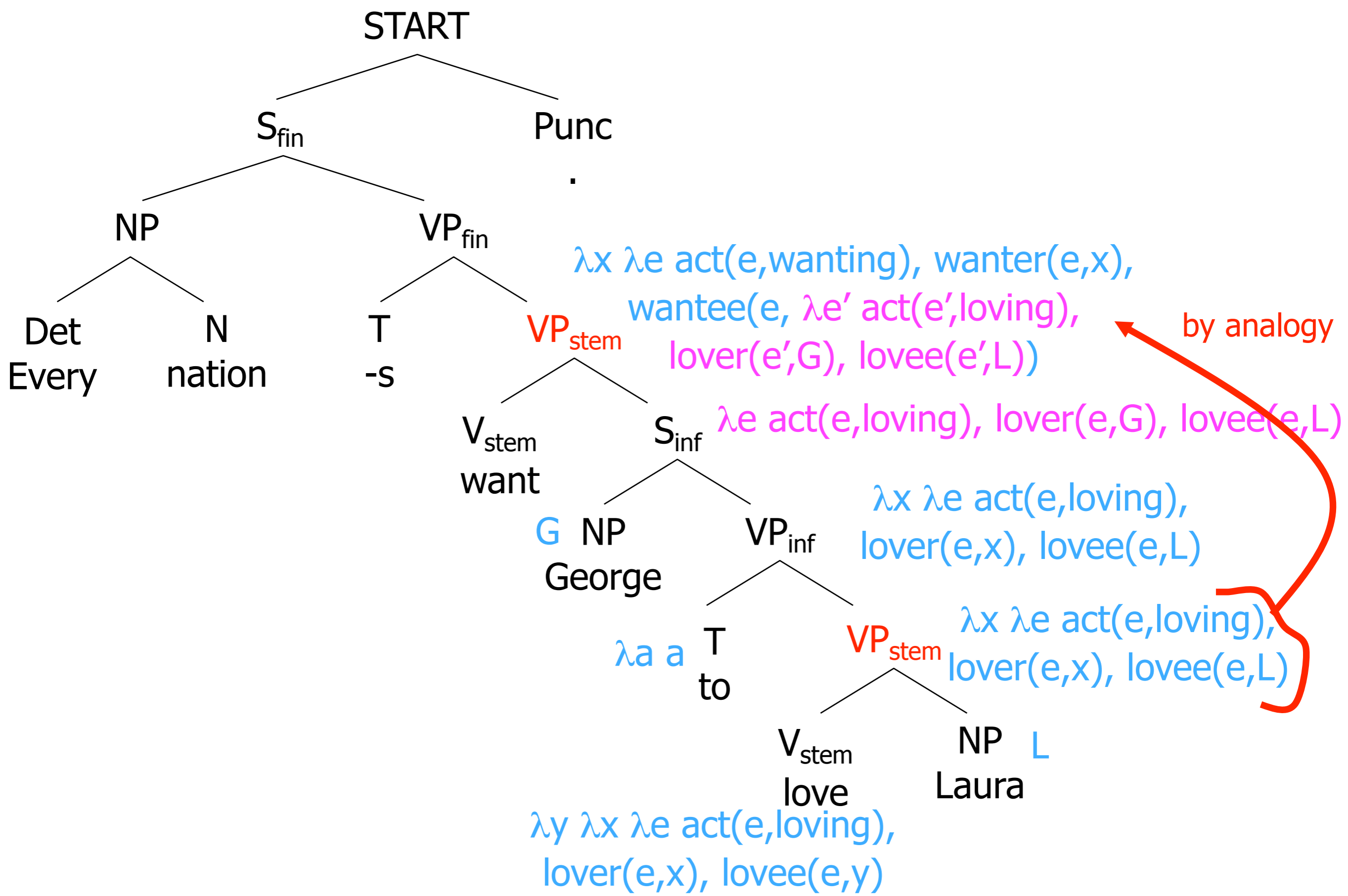


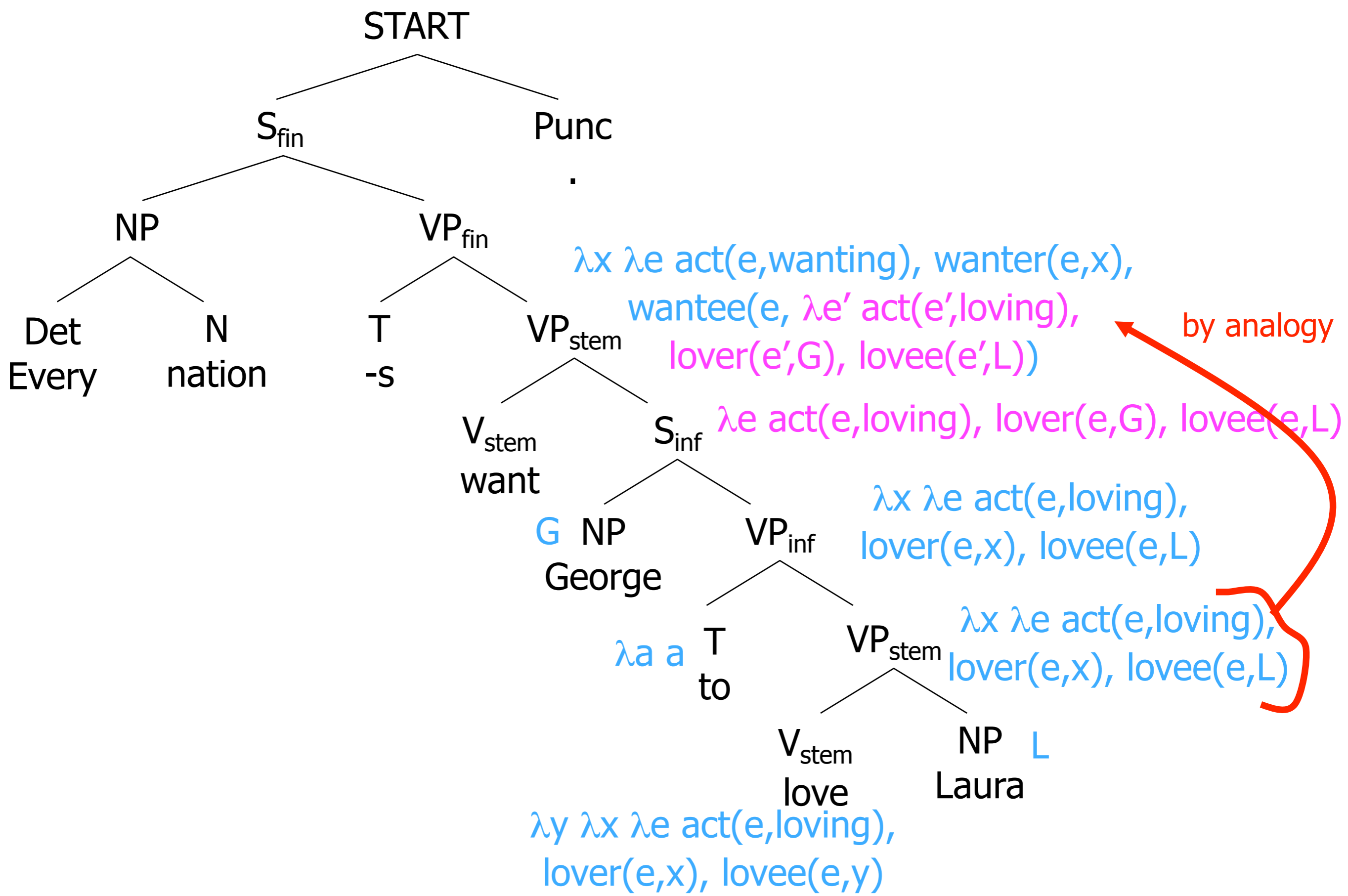


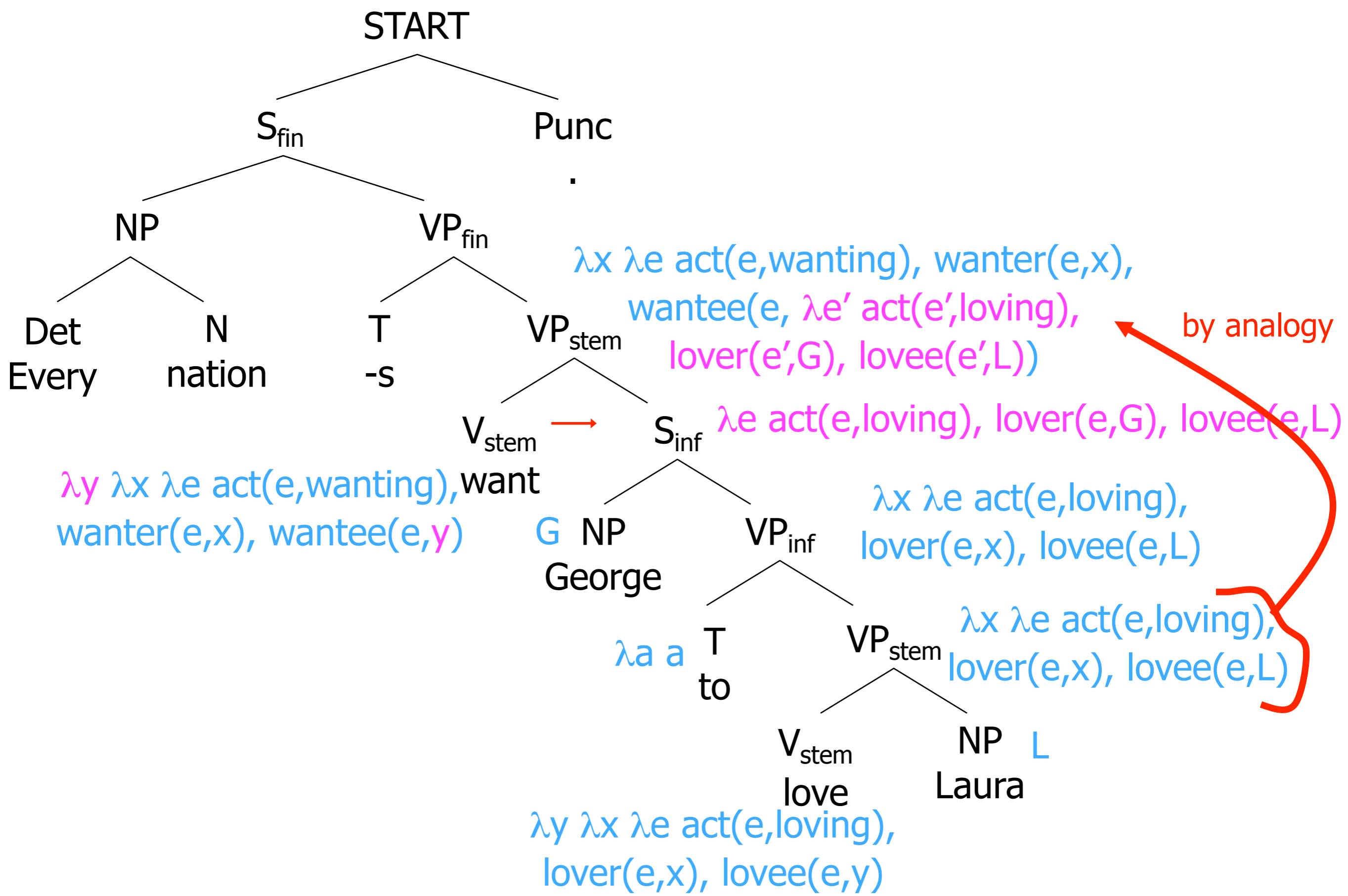
We'll say that "to" is just a bit of syntax that changes a VP_{stem} to a VP_{inf} with the same meaning.

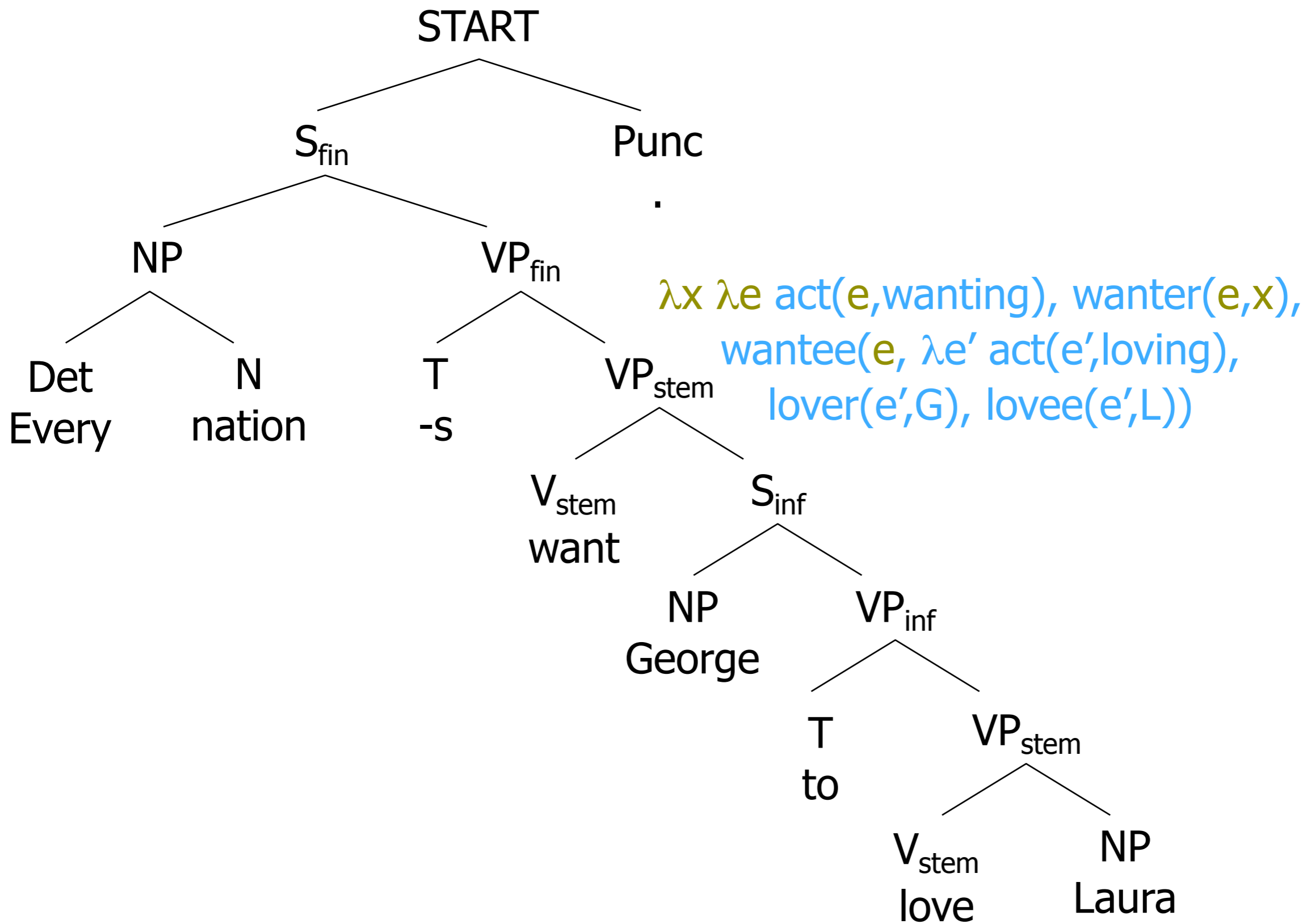


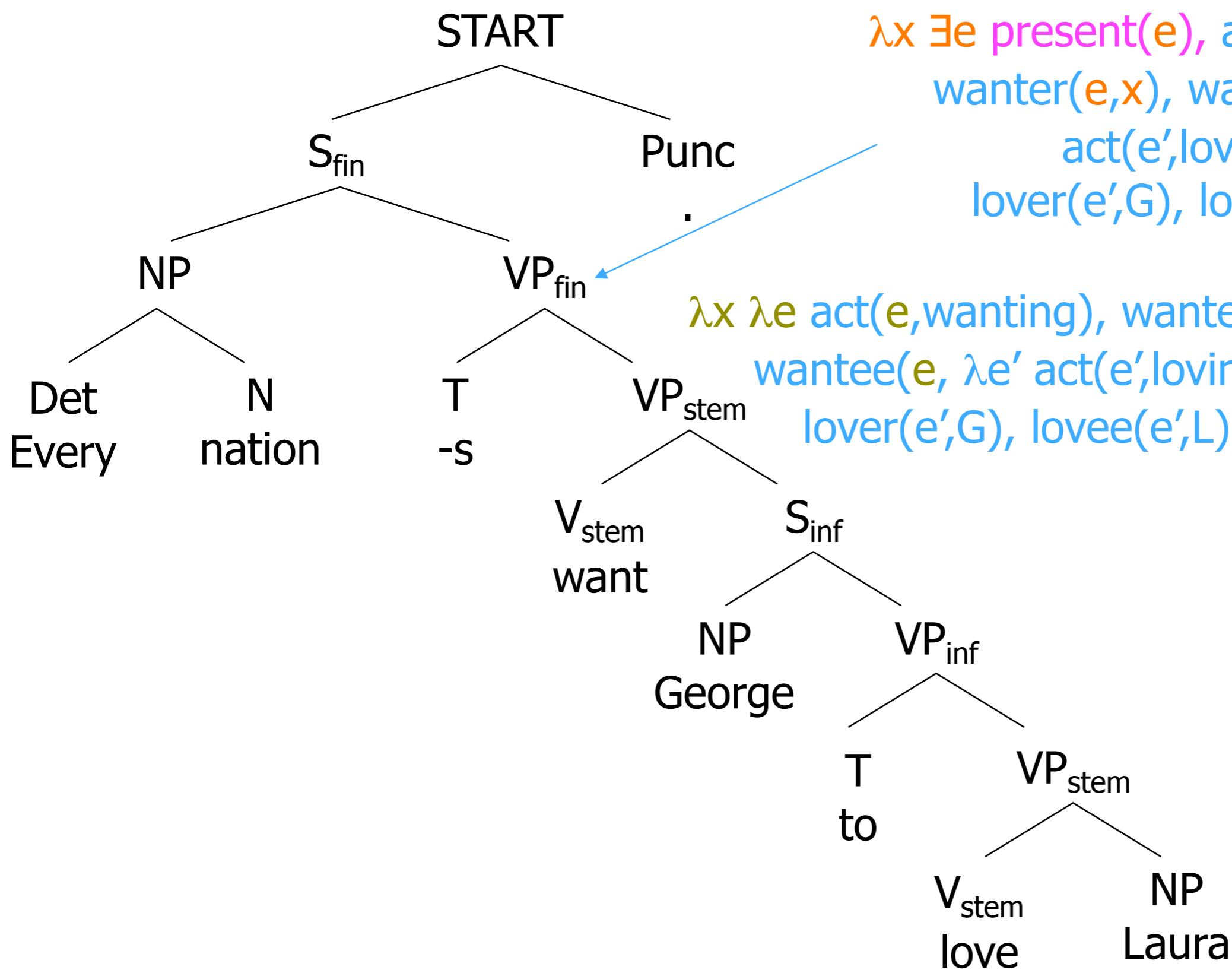






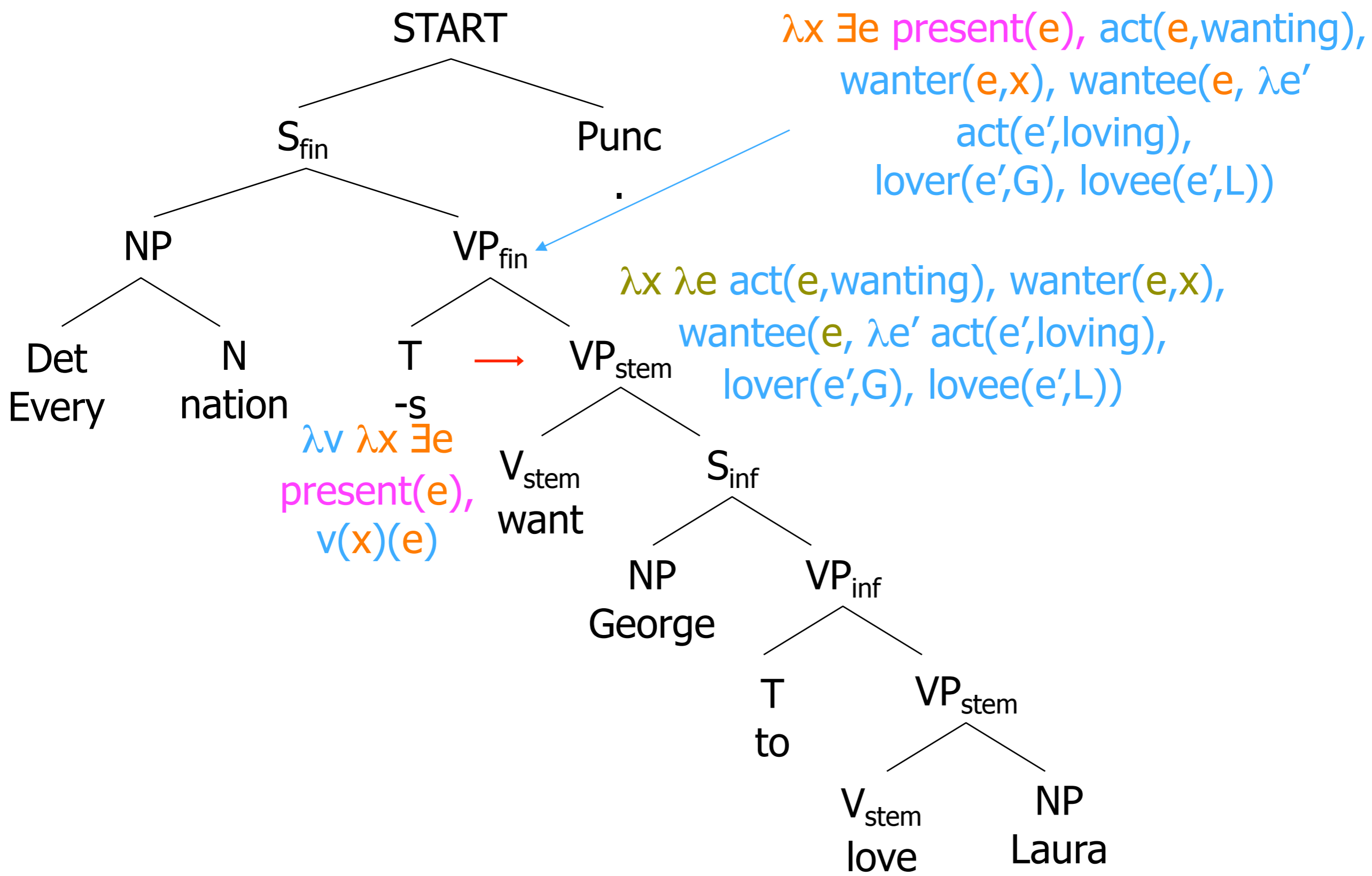


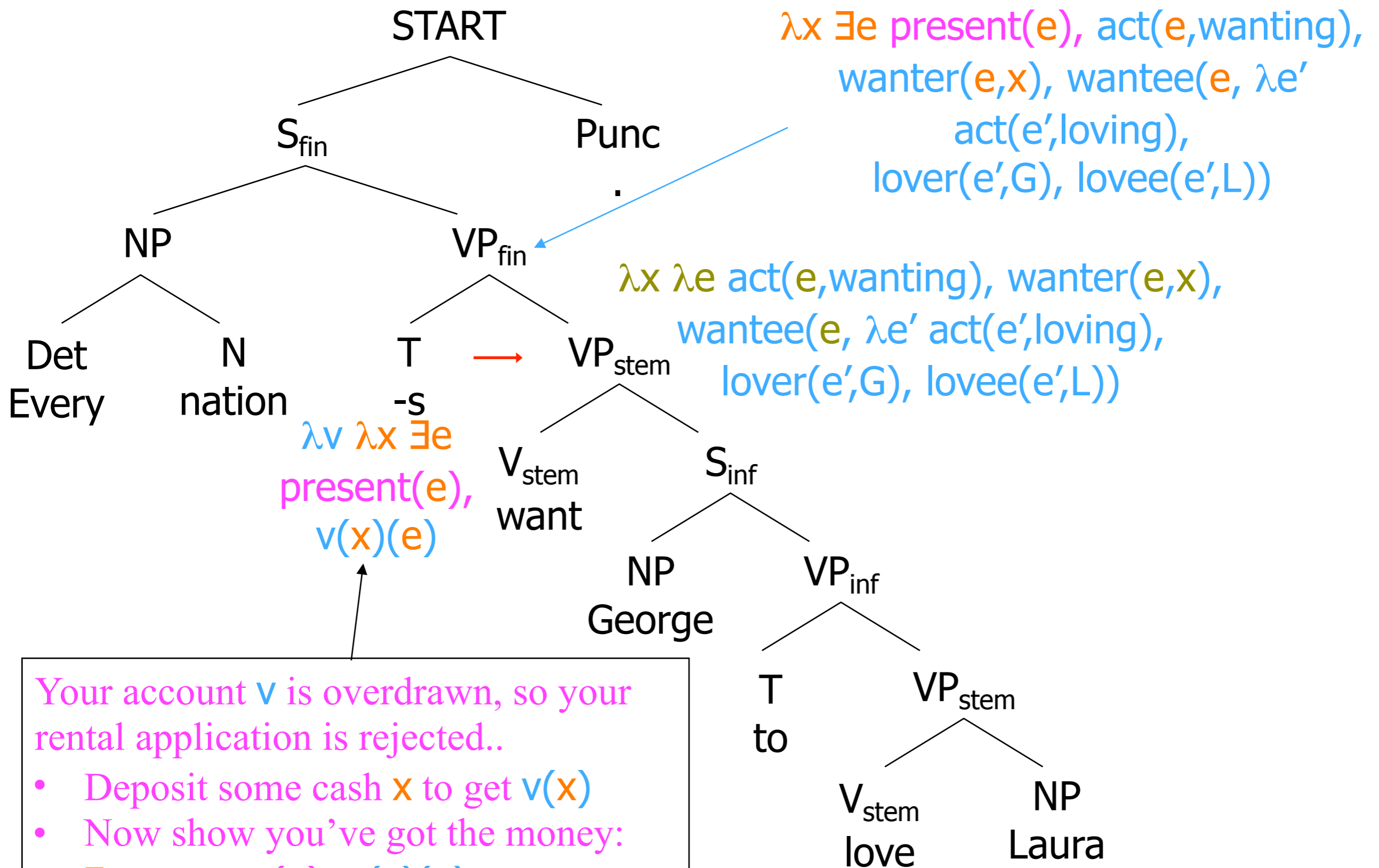




$\lambda x \exists e$ present(e), act(e,wanting),
 want(e,x), wantee(e, $\lambda e'$
 act(e',loving),
 lover(e',G), lovee(e',L))

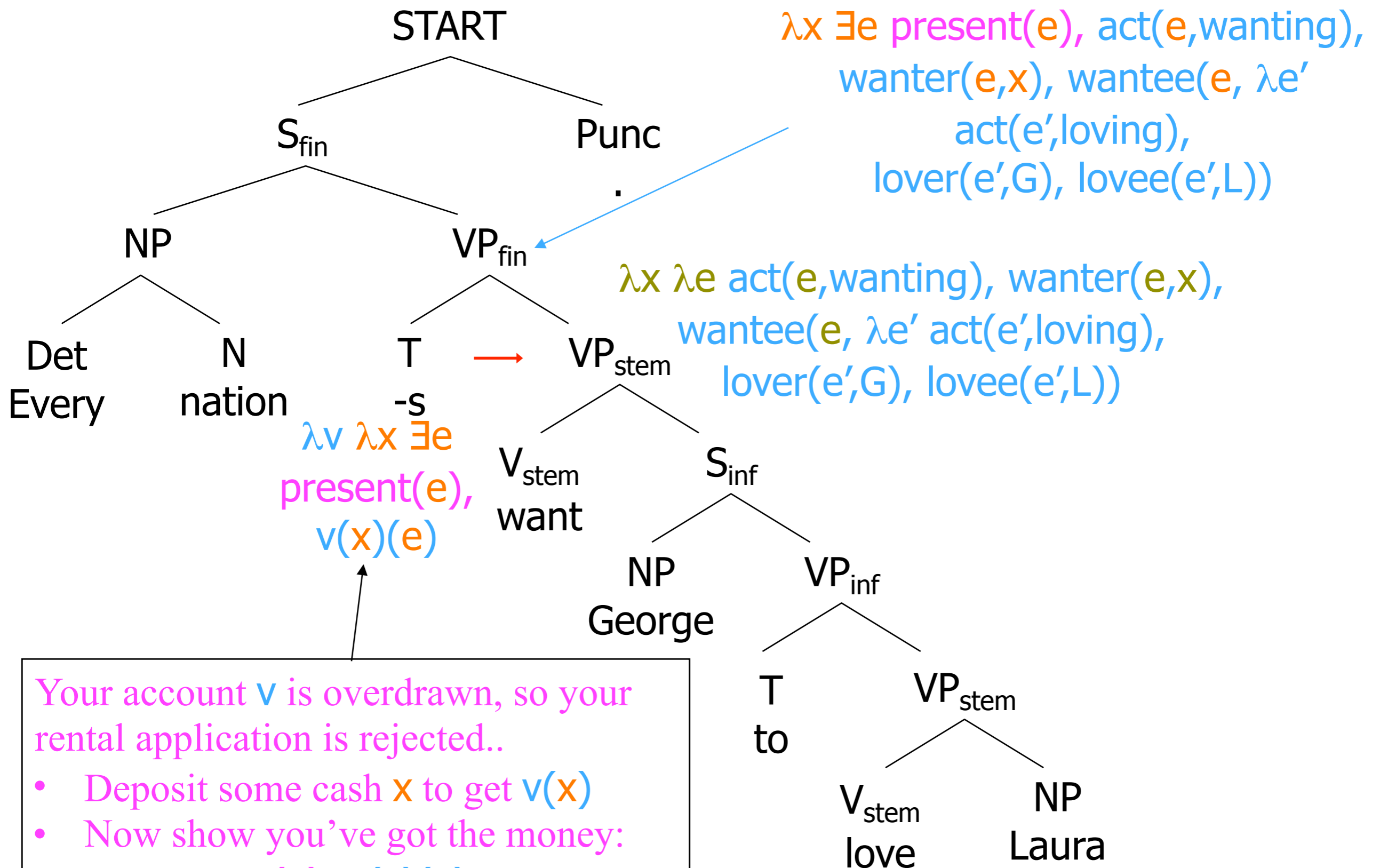
$\lambda x \lambda e$ act(e,wanting), want(e,x),
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Your account v is overdrawn, so your rental application is rejected..

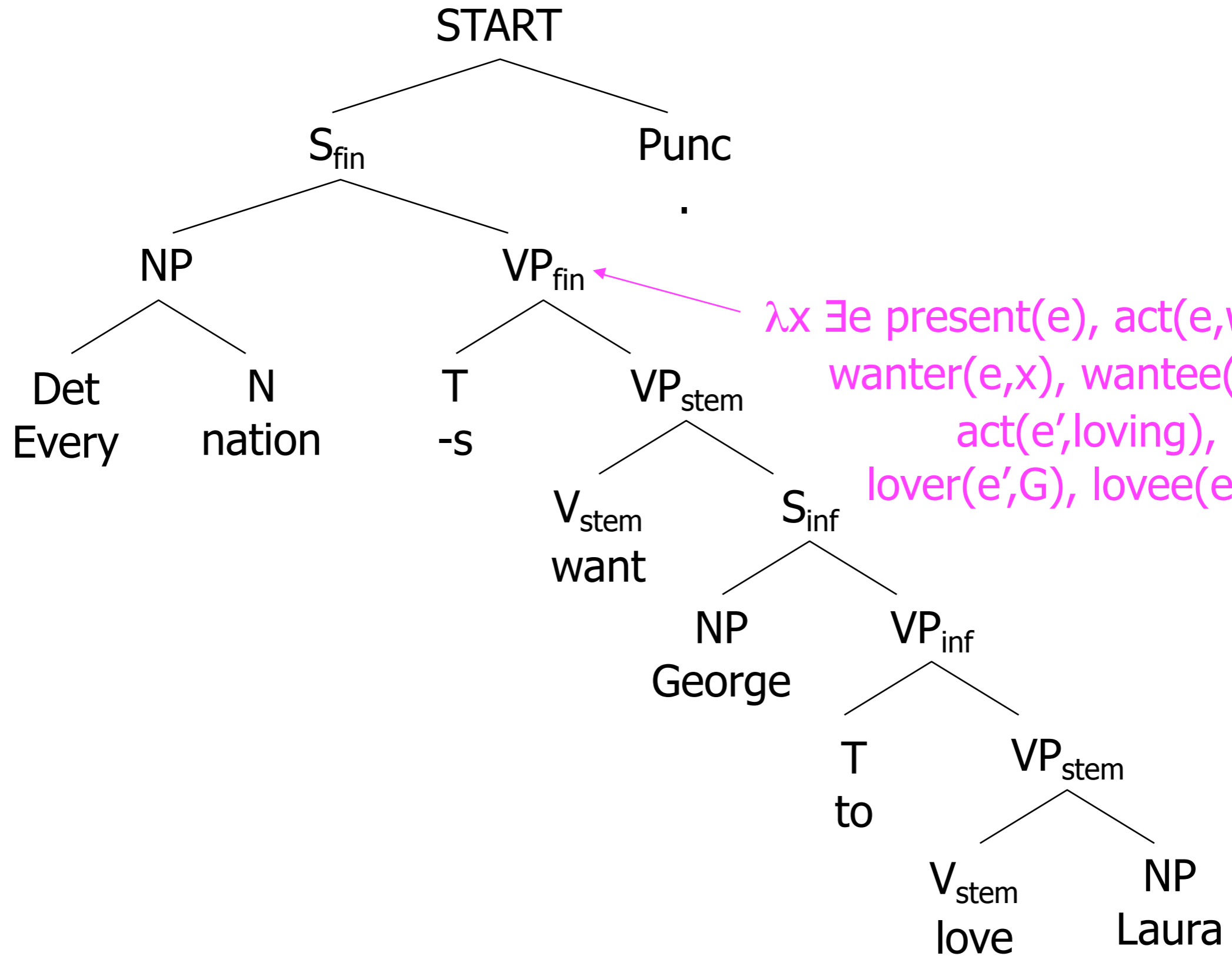
- Deposit some cash x to get $v(x)$
- Now show you've got the money:
 $\exists e$ present(e), $v(x)(e)$
- Now you can withdraw x again:
 $\lambda x \exists e$ present(e), $v(x)(e)$



Your account v is overdrawn, so your rental application is rejected..

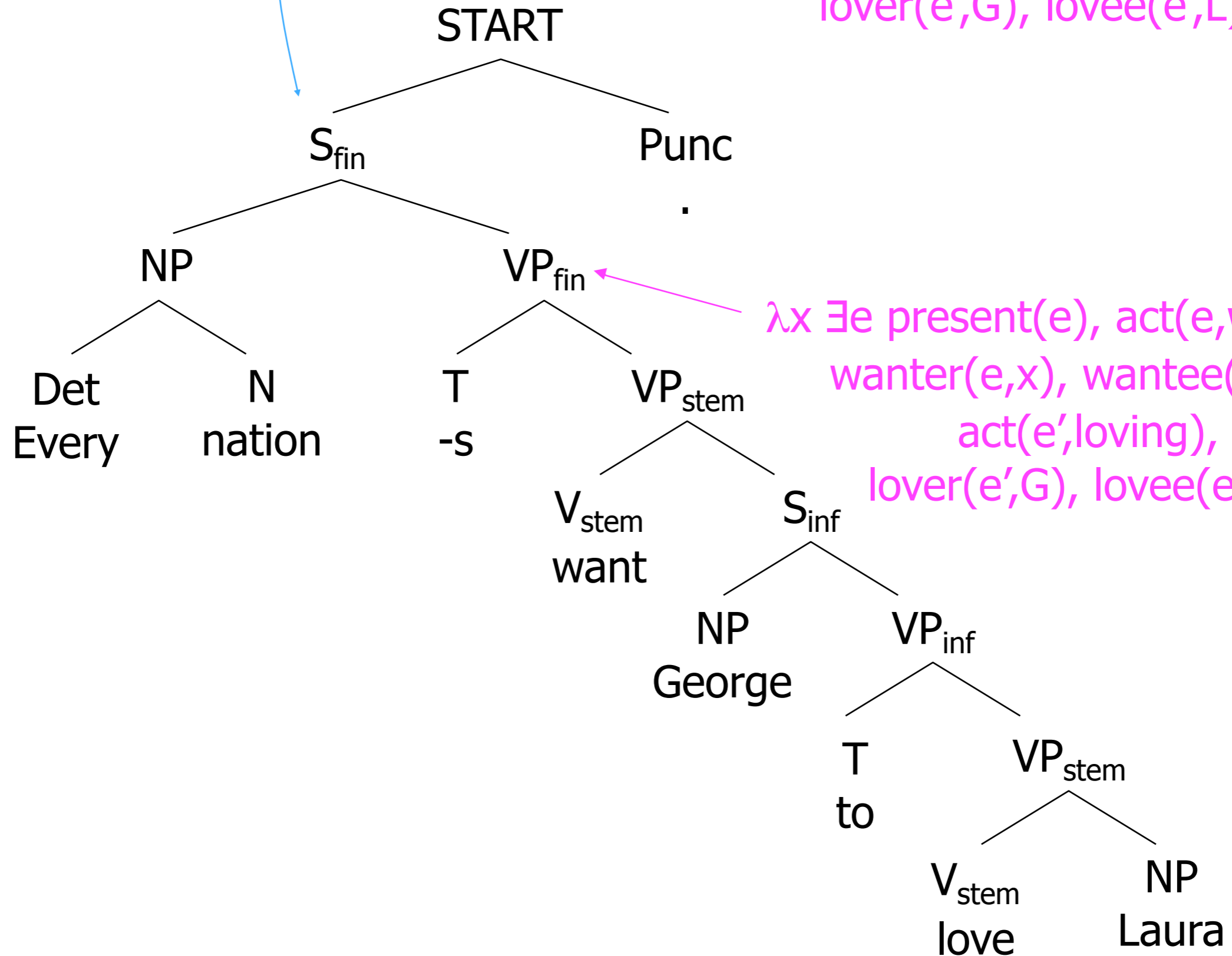
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 $\lambda x \exists e$ present(e), $v(x)(e)$

Better analogy: How would you modify the second object on a stack ($\lambda x, \lambda e, \text{act}...$)?



$\lambda x \exists e$ present(e), act(e,wanting),
 wante(e,x), wantee(e, $\lambda e'$
 act(e',loving),
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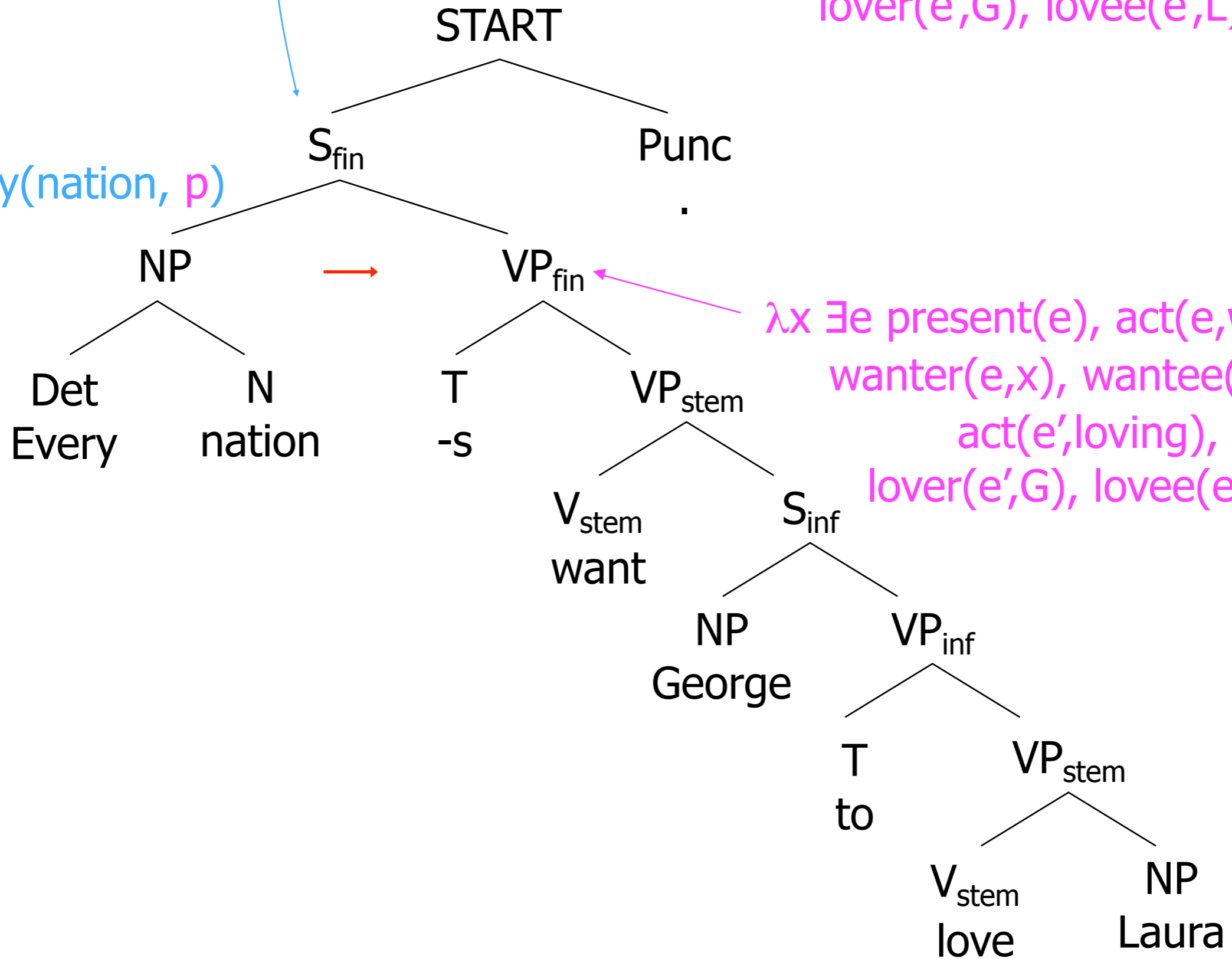
every(nation, $\lambda x \exists e$ present(e),
act(e,wanting), wanter(e,x),
wantee(e, $\lambda e'$ act(e',loving),
lover(e',G), lovee(e',L)))



$\lambda x \exists e$ present(e), act(e,wanting),
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act(e,wanting), wanter(e,x),
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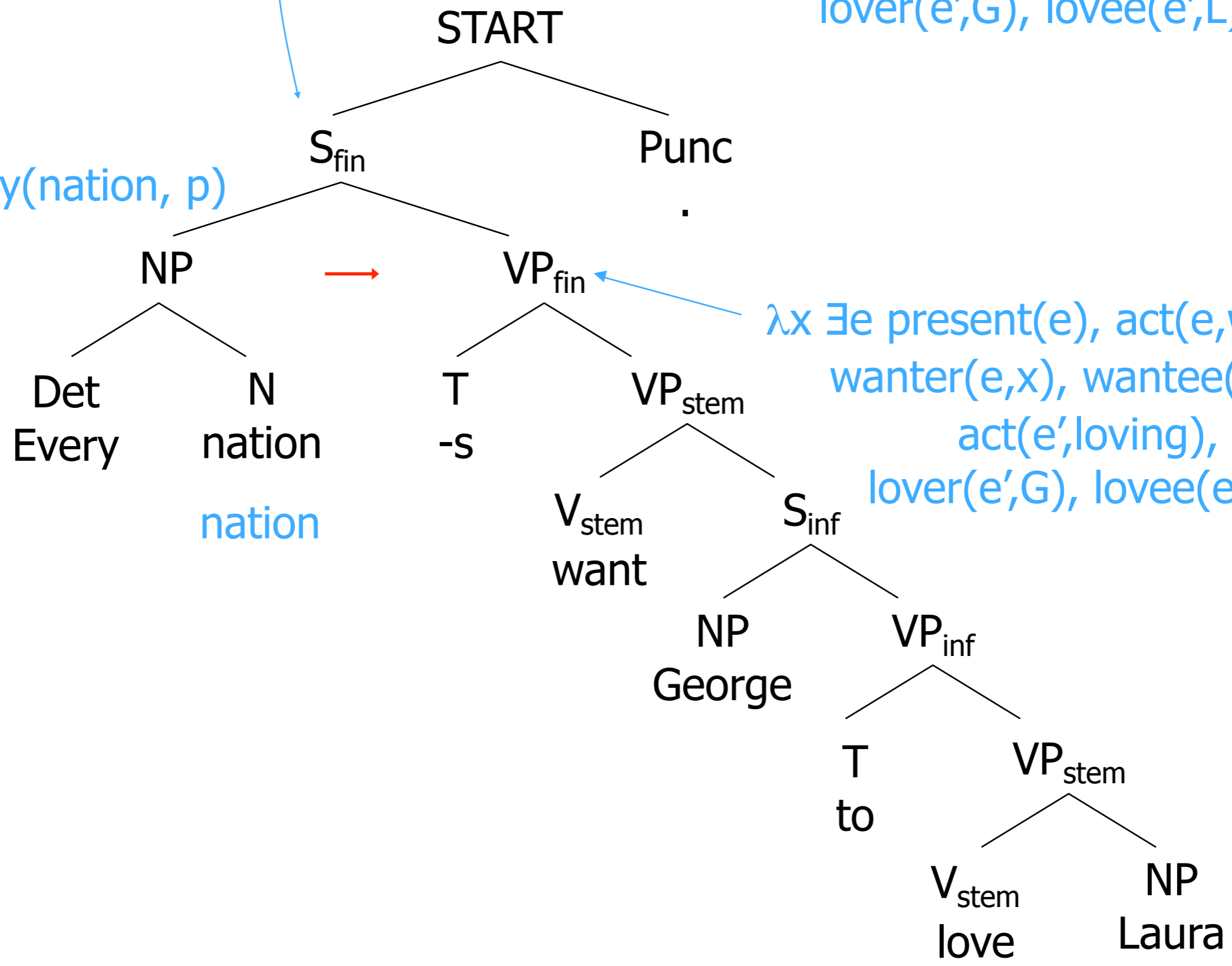
λp every(nation, p)



$\lambda x \exists e$ present(e), act(e,wanting),
wanter(e,x), wantee(e, $\lambda e'$
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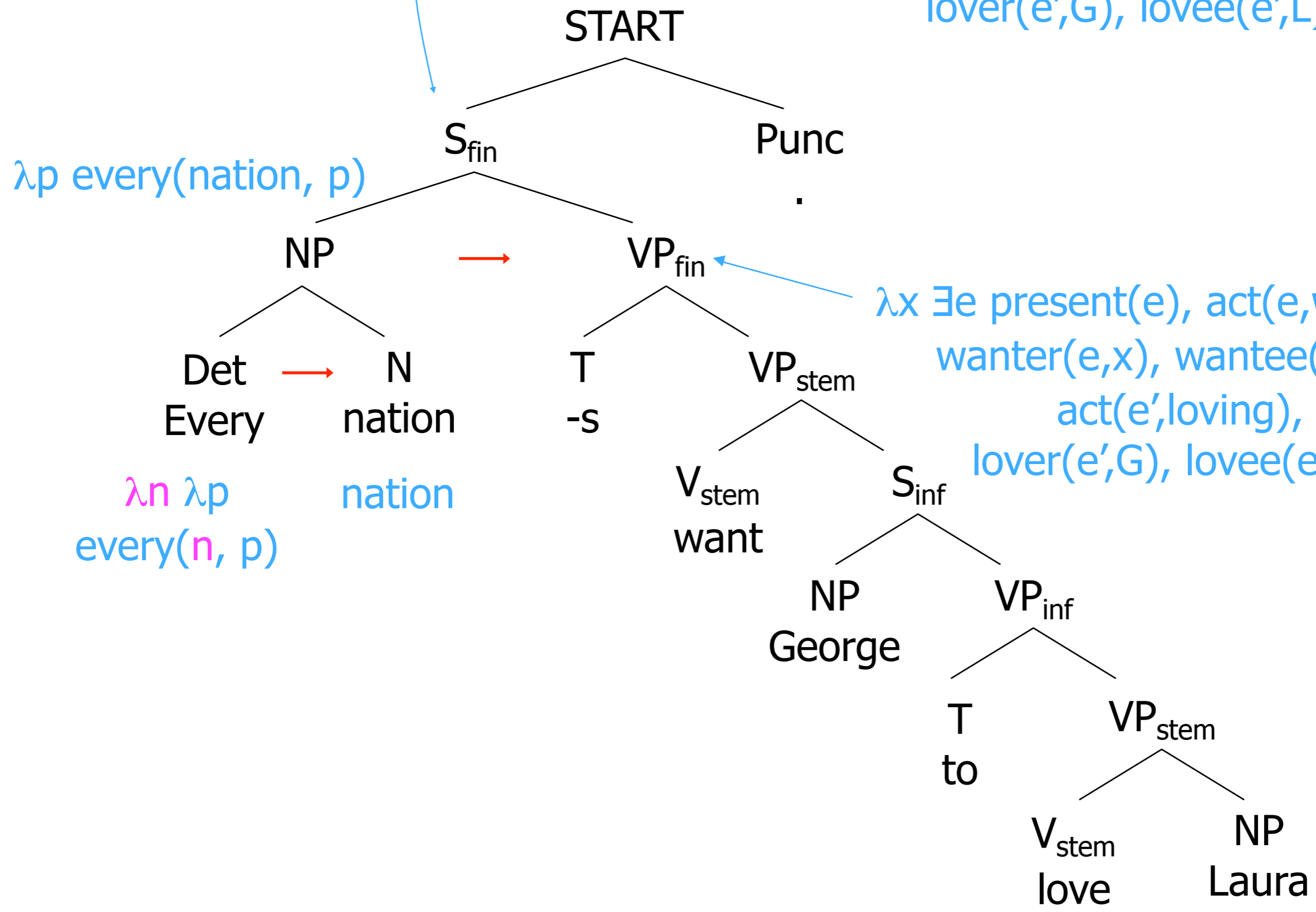
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every(nation, $\lambda x \exists e$ present(e),
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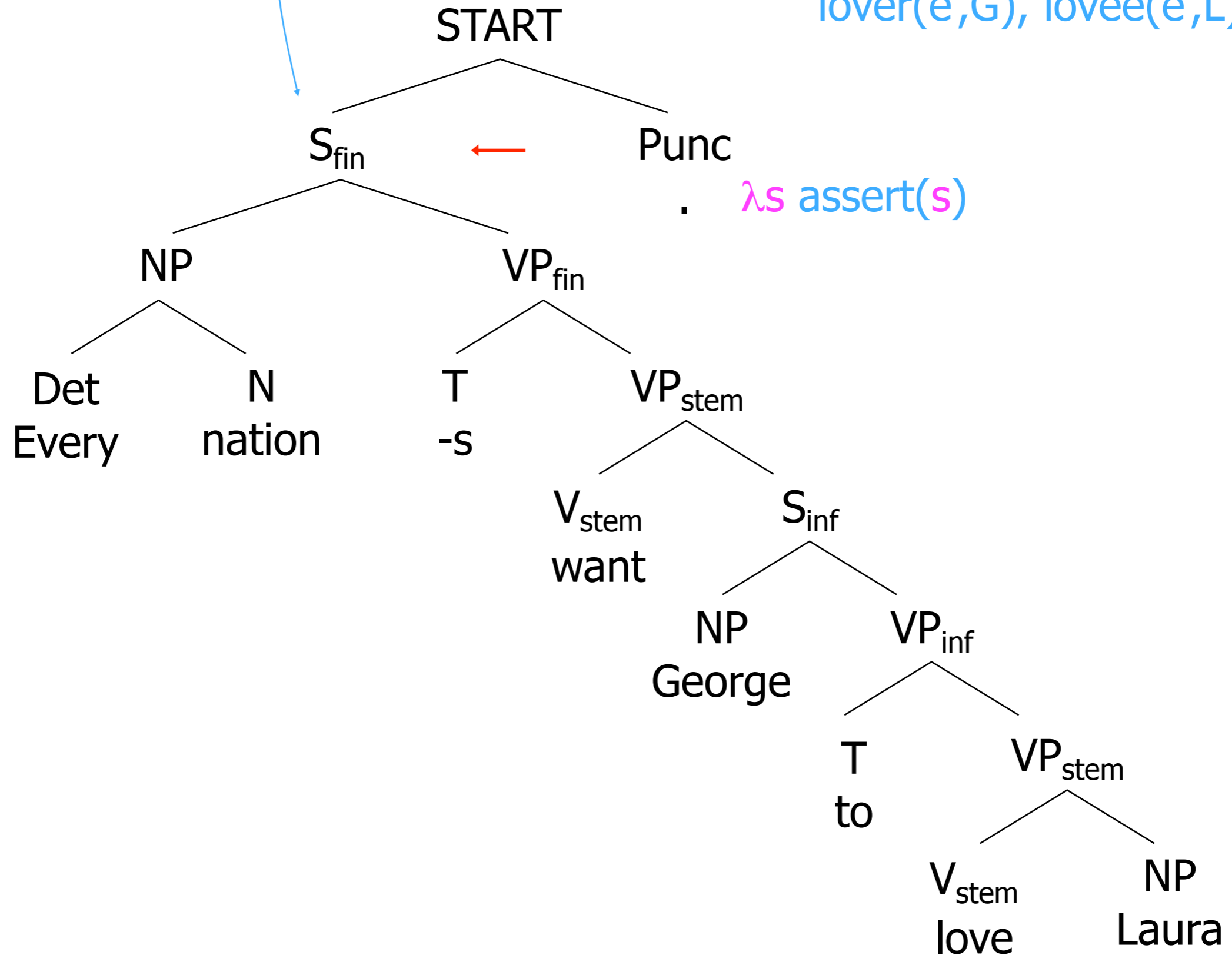
λp every(nation, p)

$\lambda x \exists e$ present(e), act(e,wanting),
 wanter(e,x), wantee(e, $\lambda e'$
 act(e',loving),
 lover(e',G), lovee(e',L))

$\lambda n \lambda p$
 every(n, p)

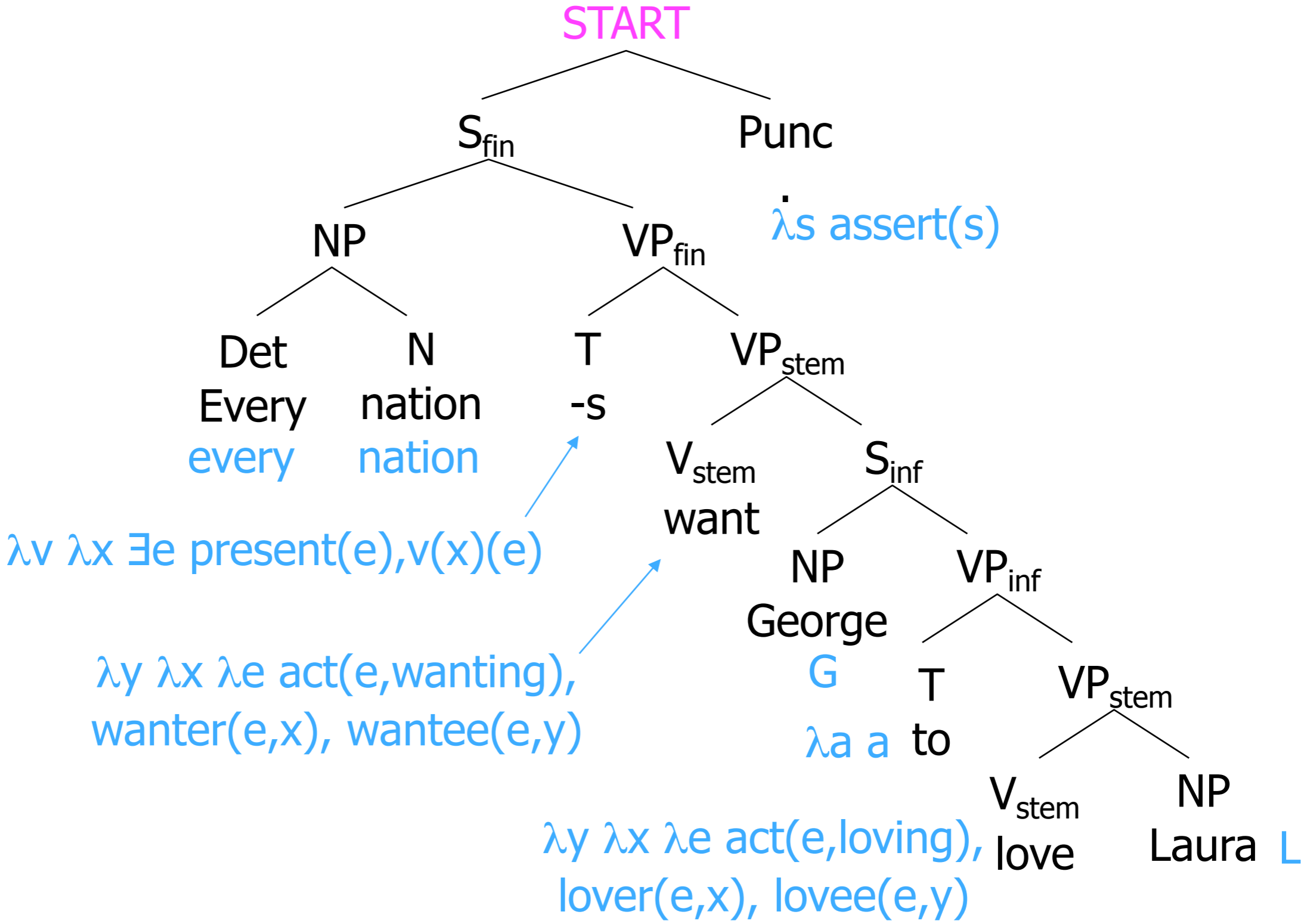


every(nation, $\lambda x \exists e$ present(e),
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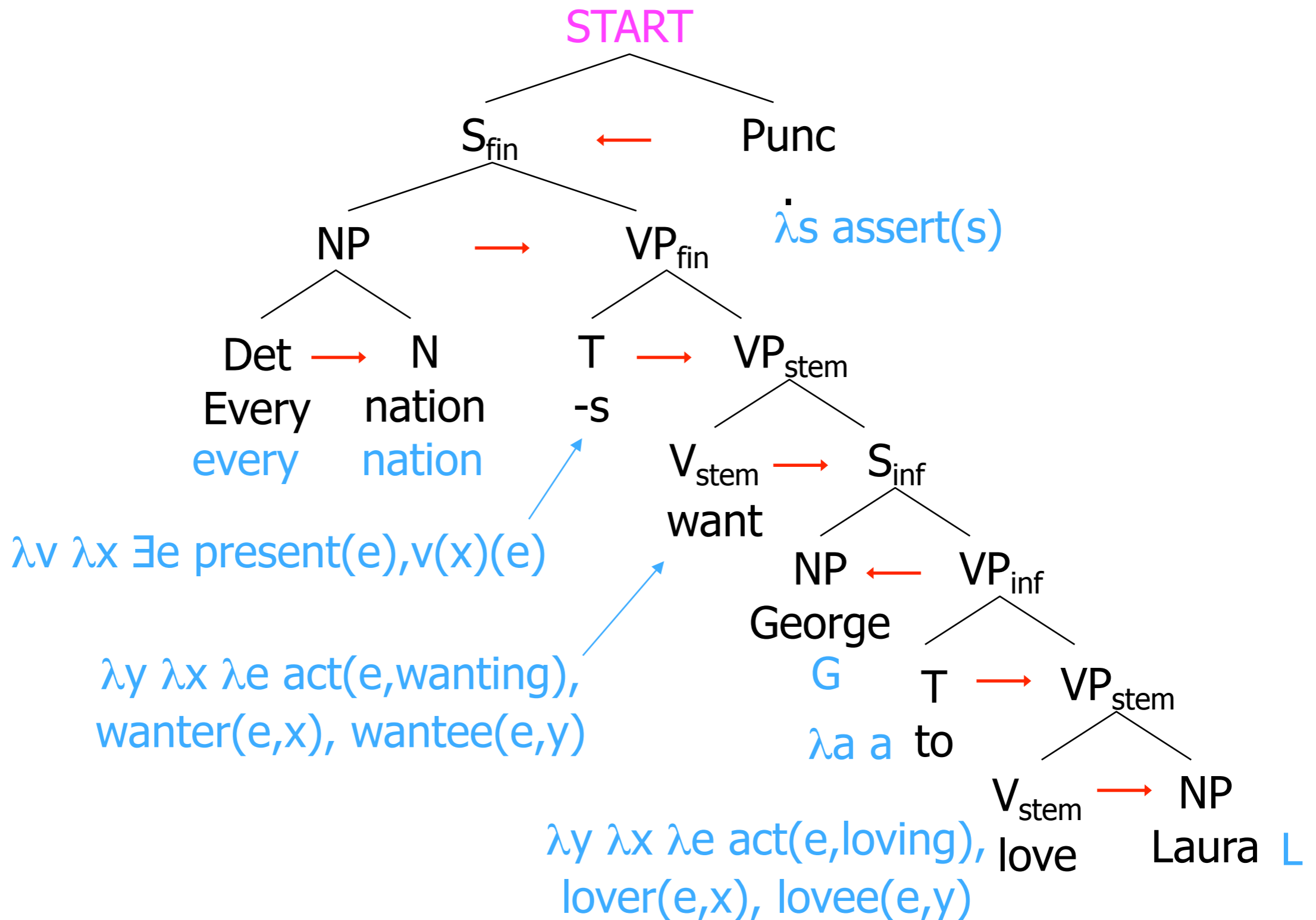


λs assert(s)

In Summary: From the Words

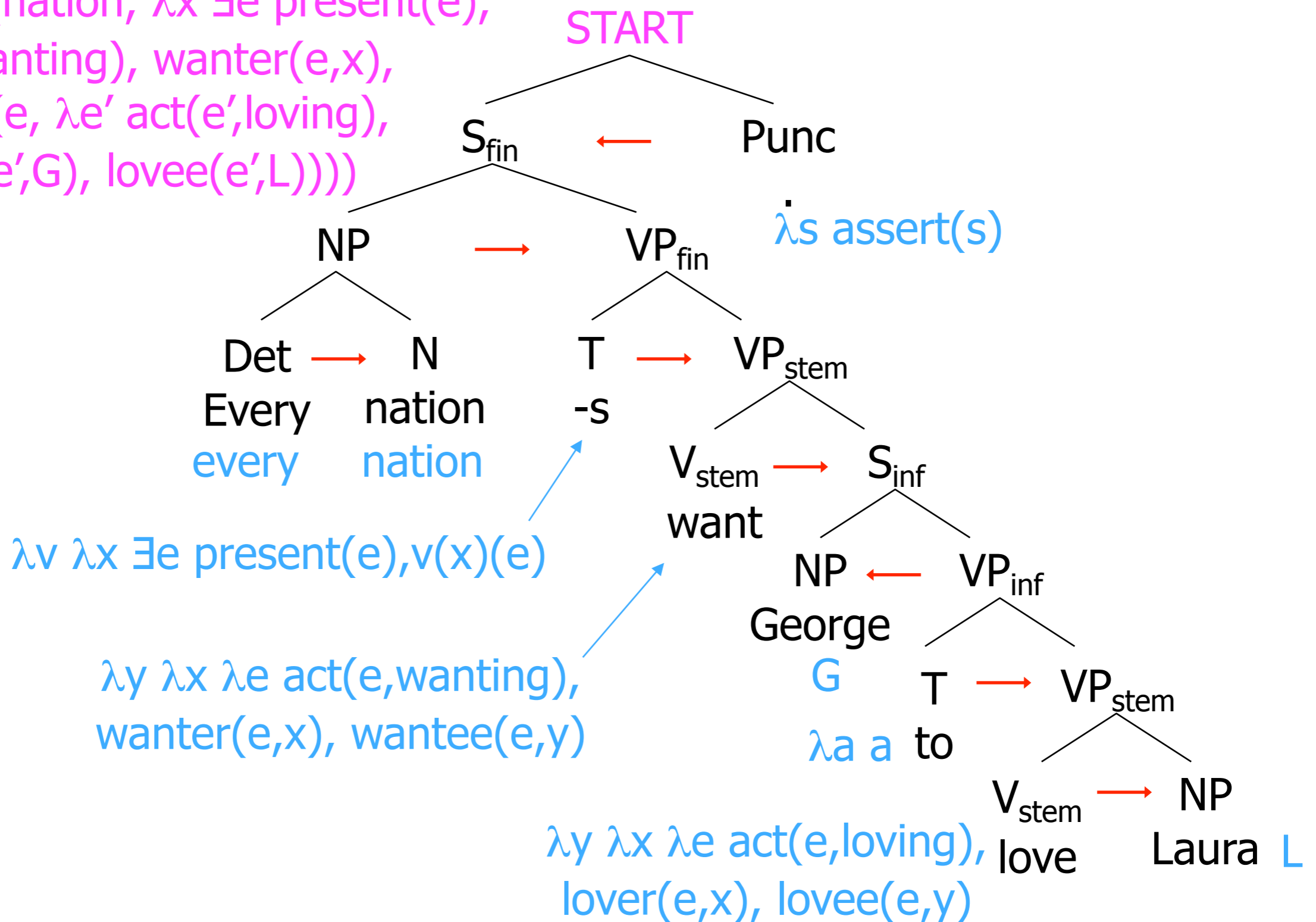


In Summary: From the Words



In Summary: From the Words

assert(every(nation, $\lambda x \exists e$ present(e),
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 wantee(e, $\lambda e'$ act(e',loving),
 lover(e',G), lovee(e',L))))



Other Fun Semantic Stuff:

A Few Much-Studied Miscellany

■ Temporal logic

- Gilly had swallowed eight goldfish before Milly reached the bowl
- Billy said Jilly was pregnant
- Billy said, "Jilly is pregnant."

■ Generics

- Typhoons arise in the Pacific
- Children must be carried

■ Presuppositions

- The king of France is bald.
- Have you stopped beating your wife?

■ Pronoun-Quantifier Interaction ("bound anaphora")

- Every farmer who owns a donkey beats it.
- If you have a dime, put it in the meter.
- The woman who every Englishman loves is his mother.
- I love my mother and so does Billy.

In Summary

- How do we judge a good meaning representation?
- How can we represent sentence meaning with first-order logic?
- How can logical representations of sentences be **composed** from logical forms of words?
- Next time: can we train models to recover logical forms?