Context-Free Parsing: CKY & Earley Algorithms and Probabilistic Parsing

Natural Language Processing
CS 6120—Spring 2014
Northeastern University

David Smith
with some slides
from Jason Eisner & Andrew McCallum

Language structure and meaning

We want to know how meaning is mapped onto what language structures. Commonly in English in ways like this:

[THING The dog] is [PLACE in the garden]

[THING The dog] is [PROPERTY fierce]

[ACTION [THING The dog] is chasing [THING the cat]]

[STATE [THING The dog] was sitting [PLACE in the garden] [TIME yesterday]]

[ACTION [THING We] ran [PATH out into the water]]

[ACTION [THING The dog] barked [PROPERTY/MANNER loudly]]

[ACTION [THING The dog] barked [PROPERTY/AMOUNT nonstop for five hours]]

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[State [Thing The dog] was sitting [Place in the garden] [Time yesterday]]

[Action [Thing We] ran [Path out into the water]]

[Action [Thing The dog] barked [Property/Manner loudly]]

[Action [Thing The dog] barked [Property/Amount nonstop for five hours]]

Part of speech “Substitution Test”

The {sad, intelligent, green, fat, ...} one is in the corner.

Constituency

The idea: Groups of words may behave as a single unit or phrase, called a **consituent**.

E.g. Noun Phrase

Kermit the frog
they
December twenty-sixth
the reason he is running for president

Constituency

Sentences have parts, some of which appear to have subparts. These groupings of words that go together we will call constituents.

(How do we know they go together? Coming in a few slides...)

I hit the man with a cleaver
I hit [the man with a cleaver]
I hit [the man] with a cleaver

You could not go to her party
You [could not] go to her party
You could [not go] to her party

Constituent Phrases

For constituents, we usually name them as phrases based on the word that heads the constituent:

the man from Amherst is a Noun Phrase (NP) because the head man is a noun
extremely clever is an Adjective Phrase (AP) because the head clever is an adjective
down the river is a Prepositional Phrase (PP) because the head down is a preposition
killed the rabbit is a Verb Phrase (VP) because the head killed is a verb

Note that a word is a constituent (a little one). Sometimes words also act as phrases. In:

Joe grew potatoes.
Joe and *potatoes* are both nouns and noun phrases.

Compare with:

The man from Amherst grew beautiful russet potatoes.

We say *Joe* counts as a noun phrase because it appears in a place that a larger noun phrase could have been.

Evidence constituency exists

1. They appear in similar environments (before a verb)

   Kermit the frog comes on stage

   They come to Massachusetts every summer

   December twenty-sixth comes after Christmas

   The reason he is running for president comes out only now.

   But not each individual word in the consituent

   *_The comes our..._ *_is comes out..._ *_for comes out..._

2. The constituent can be placed in a number of different locations

   Consituent = Prepositional phrase: _On December twenty-sixth_

   On December twenty-sixth I'd like to fly to Florida.

   I'd like to fly on December twenty-sixth to Florida.

   I'd like to fly to Florida on December twenty-sixth.

   But not split apart

   *_On December I'd like to fly twenty-sixth to Florida._

   *_On I'd like to fly December twenty-sixth to Florida._

Context-free grammar

The most common way of modeling constituency.

CFG = Context-Free Grammar = Phrase Structure Grammar
= BNF = Backus-Naur Form

The idea of basing a grammar on constituent structure dates back to Wilhem Wundt (1890), but not formalized until Chomsky (1956), and, independently, by Backus (1959).

Context-free grammar

$G = \langle T, N, S, R \rangle$

- T is set of terminals (lexicon)

- N is set of non-terminals For NLP, we usually distinguish out a set $P \subset N$ of *preterminals* which always rewrite as terminals.

- S is start symbol (one of the nonterminals)

- R is rules/productions of the form $X \rightarrow \gamma$, where X is a nonterminal and γ is a sequence of terminals and nonterminals (may be empty).

- A grammar G generates a language L.

An example context-free grammar

$G = \langle T, N, S, R \rangle$

$T = \{\textit{that, this, a, the, man, book, flight, meal, include, read, does}\}$

$N = \{$S, NP, NOM, VP, Det, Noun, Verb, Aux$\}$

$S = S$

$R = \{$

S → NP VP
S → Aux NP VP
S → VP
NP → Det NOM
NOM → Noun
NOM → Noun NOM
VP → Verb
VP → Verb NP

Det → *that* | *this* | *a* | *the*
Noun → *book* | *flight* | *meal* | *man*
Verb → *book* | *include* | *read*
Aux → *does*

$\}$

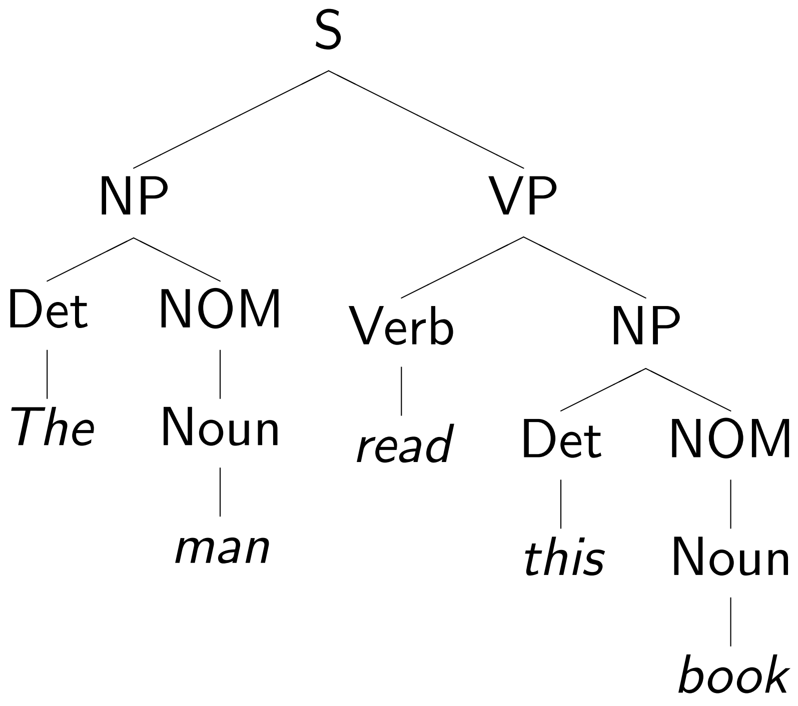
Application of grammar rewrite rules

S → NP VP
S → Aux NP VP
S → VP
NP → Det NOM
NOM → Noun
NOM → Noun NOM
VP → Verb
VP → Verb NP

Det → *that* | *this* | *a* | *the*
Noun → *book* | *flight* | *meal* | *man*
Verb → *book* | *include* | *read*
Aux → *does*

S → NP VP
→ Det NOM VP
→ *The* NOM VP
→ *The* Noun VP
→ *The man* VP
→ *The man* Verb NP
→ *The man read* NP
→ *The man read* Det NOM
→ *The man read this* NOM
→ *The man read this* Noun
→ *The man read this book*

Parse tree



CFGs can capture recursion

Example of seemingly endless recursion of embedded prepositional phrases:
PP → Prep NP
NP → Noun PP

$[_S$ The mailman ate his $[_{NP}$ lunch $[_{PP}$ with his friend $[_{PP}$ from the cleaning staff $[_{PP}$ of the building $[_{PP}$ at the intersection $[_{PP}$ on the north end $[_{PP}$ of town]]]]]]].

(Bracket notation)

Grammaticality

A CFG defines a formal language = the set of all sentences (strings of words) that can be derived by the grammar.

Sentences in this set said to be **grammatical**.

Sentences outside this set said to be **ungrammatical**.

The Chomsky hierarchy

- Type 0 Languages / Grammars
  Rewrite rules $\alpha \rightarrow \beta$
  where α and β are any string of terminals and nonterminals

- Context-sensitive Languages / Grammars
  Rewrite rules $\alpha X \beta \rightarrow \alpha\gamma\beta$
  where X is a non-terminal, and α, β, γ are any string of terminals and nonterminals, (γ must be non-empty).

- Context-free Languages / Grammars
  Rewrite rules $X \rightarrow \gamma$
  where X is a nonterminal and γ is any string of terminals and nonterminals

- Regular Languages / Grammars
  Rewrite rules $X \rightarrow \alpha Y$
  where X, Y are single nonterminals, and α is a string of terminals; Y might be missing.

Parsing regular grammars

(Languages that can be generated by finite-state automata.)
Finite state automaton ↔ regular expression ↔ regular grammar

Space needed to parse: constant

Time needed to parse: linear (in the length of the input string)

Cannot do embedded recursion, e.g. $a^n b^n$. (Context-free grammars can.)
In the language: ab, aaabbb; not in the language: aabbb

The cat likes tuna fish.
The cat the dog chased likes tuna fish
The cat the dog the boy loves chased likes tuna fish.

John, always early to rise, even after a sleepless night filled with the cries of the neighbor's baby, goes running every morning.

John and Mary, always early to rise, even after a sleepless night filled with the cries of the neighbor's baby, go running every morning.

Parsing context-free grammars

(Languages that can be generated by pushdown automata.)

Widely used for surface syntax description (correct word order specification) in natural languages.

Space needed to parse: stack (sometimes a stack of stacks)
In general, proportional to the number of levels of recursion in the data.

Time needed to parse: in general $O(n^3)$.

Can to $a^n b^n$, but cannot do $a^n b^n c^n$.

Chomsky Normal Form

All rules of the form $X \rightarrow YZ$ or $X \rightarrow a$ or $S \rightarrow \epsilon$.
(S is the only non-terminal that can go to ϵ.)
Any CFG can be converted into this form.

How would you convert the rule $W \rightarrow XYaZ$ to Chomsky Normal Form?

Chomsky Normal Form Conversion

These steps are used in the conversion:

1. Make S non-recursive

2. Eliminate all epsilon except the one in S (if there is one)

3. Eliminate all chain rules

4. Remove useless symbols (the ones not used in any production).

How would you convert the following grammar?

$S \rightarrow ABS$

$S \rightarrow \epsilon$

$A \rightarrow \epsilon$

$A \rightarrow xyz$

$B \rightarrow wB$

$B \rightarrow v$

What is parsing?

We want to run the grammar backwards to find the structure.

Parsing can be viewed as a search problem.

We search through the legal rewritings of the grammar.
We want to find *all* structures matching an input string of words (for the moment)

We can do this bottom-up or top-down
This distinction is independent of depth-first versus breadth-first; we can do either both ways.
Doing this we build a *search tree* which is different from the *parse tree*.

Recognizers and parsers

- A **recognizer** is a program for which a given grammar and a given sentence returns YES if the sentence is accepted by the grammar (i.e., the sentence is in the language), and NO otherwise.

- A **parser** in addition to doing the work of a recognizer also returns the set of parse trees for the string.

Soundness and completeness

- A parser is **sound** if every parse it returns is valid/correct.

- A parser **terminates** if it is guaranteed not to go off into an infinite loop.

- A parser is **complete** if for any given grammar and sentence it is sound, produces every valid parse for that sentence, and terminates.

- (For many cases, we settle for sound but incomplete parsers: e.g. probabilistic parsers that return a k-best list.)

Top-down parsing

Top-down parsing is goal-directed.

- A top-down parser starts with a list of constituents to be built.
- It rewrites the goals in the goal list by matching one against the LHS of the grammar rules,
- and expanding it with the RHS,
- ...attempting to match the sentence to be derived.

If a goal can be rewritten in several ways, then there is a choice of which rule to apply (search problem)

Can use depth-first or breadth-first search, and goal ordering.

Top-down parsing example (Breadth-first)

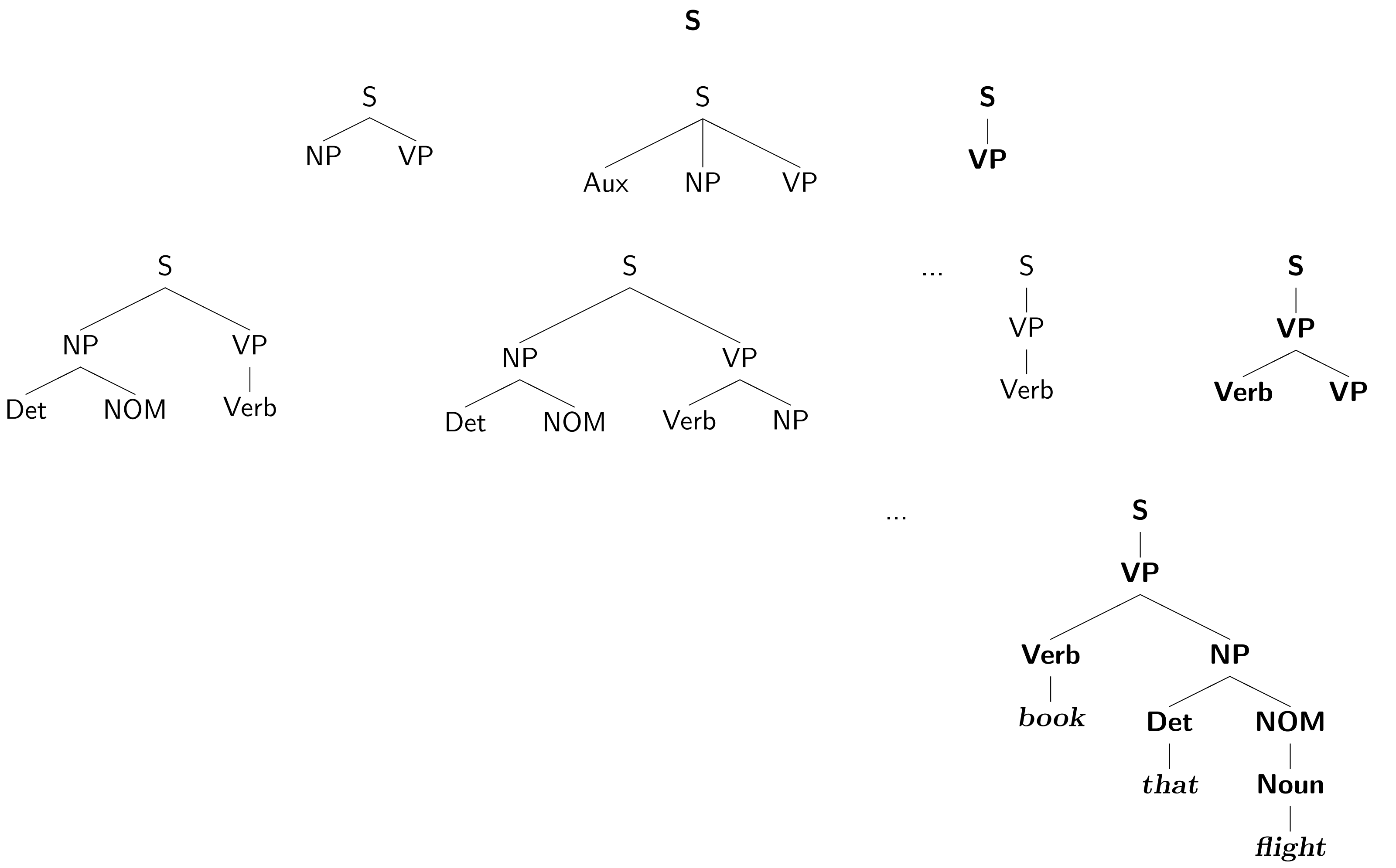
S → NP VP
S → Aux NP VP
S → VP
NP → Det NOM
NOM → Noun
NOM → Noun NOM
VP → Verb
VP → Verb NP

Det → *that* | *this* | *a* | *the*
Noun → *book* | *flight* | *meal* | *man*
Verb → *book* | *include* | *read*
Aux → *does*

Book that flight.

(Work out top-down, breadth-first search on the board...)

Top-down parsing example (Breadth-first)



Problems with top-down parsing

- Left recursive rules... e.g. NP → NP PP... lead to infinite recursion
- Will do badly if there are many different rules for the same LHS. Consider if there are 600 rules for S, 599 of which start with NP, but one of which starts with a V, and the sentence starts with a V.
- Useless work: expands things that are possible top-down but not there (no bottom-up evidence for them).
- Top-down parsers do well if there is useful grammar-driven control: search is directed by the grammar.
- Top-down is hopeless for rewriting parts of speech (preterminals) with words (terminals). In practice that is always done bottom-up as lexical lookup.
- Repeated work: anywhere there is common substructure.

Bottom-up parsing

Top-down parsing is data-directed.

- The initial goal list of a bottom-up parser is the string to be parsed.
- If a sequence in the goal list matches the RHS of a rule, then this sequence may be replaced by the LHS of the rule.
- Parsing is finished when the goal list contains just the start symbol.

If the RHS of several rules match the goal list, then there is a choice of which rule to apply (search problem)

Can use depth-first or breadth-first search, and goal ordering.

The standard presentation is as **shift-reduce** parsing.

Bottom-up parsing example

S → NP VP

S → Aux NP VP

S → VP

NP → Det NOM

NOM → Noun

NOM → Noun NOM

VP → Verb

VP → Verb NP

Det → *that* | *this* | *a* | *the*

Noun → *book* | *flight* | *meal* | *man*

Verb → *book* | *include* | *read*

Aux → *does*

Book that flight.

(Work out bottom-up search on the board...)

Shift-reduce parsing

Stack	Input remaining	Action
()	Book that flight	shift
(Book)	that flight	reduce, Verb → book, (Choice #1 of 2)
(Verb)	that flight	shift
(Verb that)	flight	reduce, Det → that
(Verb Det)	flight	shift
(Verb Det flight)		reduce, Noun → flight
(Verb Det Noun)		reduce, NOM → Noun
(Verb Det NOM)		reduce, NP → Det NOM
(Verb NP)		reduce, VP → Verb NP
(Verb)		reduce, S → V
(S)		SUCCESS!

Ambiguity may lead to the need for backtracking.

Shift Reduce Parser

Start with the sentence to be parsed in an input buffer.

- a ”shift” action correponds to pushing the next input symbol from the buffer onto the stack

- a ”reduce” action occurrs when we have a rule's RHS on top of the stack. To perform the reduction, we pop the rule's RHS off the stack and replace it with the terminal on the LHS of the corresponding rule.

(When either ”shift” or ”reduce” is possible, choose one arbitrarily.)

If you end up with only the START symbol on the stack, then success!
If you don't, and you cannot and no ”shift” or ”reduce” actions are possible, backtrack.

Shift Reduce Parser

In a top-down parser, the main decision was which production rule to pick.
In a bottom-up shift-reduce parser there are two decisions:

1. Should we shift another symbol, or reduce by some rule?

2. If reduce, then reduce by which rule?

both of which can lead to the need to backtrack

Problems with bottom-up parsing

- Unable to deal with empty categories: termination problem, unless rewriting empties as constituents is somehow restricted (but then it's generally incomplete)

- Useless work: locally possible, but globally impossible.

- Inefficient when there is great lexical ambiguity (grammar-driven control might help here). Conversely, it is data-directed: it attempts to parse the words that are there.

- Repeated work: anywhere there is common substructure.

- Both Top-down (LL) and Bottom-up (LR) parsers can (and frequently do) do work exponential in the sentence length on NLP problems.

Principles for success

- Left recursive structures must be found, not predicted.
- Empty categories must be predicted, not found.
- Don't waste effort re-working what was previously parsed before backtracking.

An alternative way to fix things:

- Grammar transformations can fix both left-recursion and epsilon productions.
- Then you parse the same language but with different trees.
- BUT linguists tend to hate you, because the structure of the re-written grammar isn't what they wanted.

From Shift-Reduce to CKY

- Shift-reduce parsing can make wrong turns, needs backtracking
- Shift-reduce must pop the top of the stack, but how many items to pop?
- Time-space tradeoff
- Chomsky normal form

Chomsky Normal Form

- Any CFL can be generated by an equivalent grammar in CNF
- Rules of three types
  - $X \rightarrow Y\ Z$ X, Y, Z nonterminals
  - $X \rightarrow a$ X nonterminal, a terminal
  - $S \rightarrow \varepsilon$ S the start symbol
- NB: the derivation of a given string may change

CNF Conversion

- Create new start symbol
- Remove NTs that can generate epsilon
- Remove NTs that can generate each other, (unary rule cycles)
- Chain rules with RHS > 2
- Related topic: rule Markovization (later)

CKY Algorithm

- Input: string of n words
- Output (of recognizer): grammatical or not
- Dynamic programming in a **chart**:
    - rows labeled 0 to n-1
    - columns labeled 1 to n
    - cell [i,j] lists possible constituents spanning words between i and j

CKY Algorithm

```
for i := 1 to n
    Add to [i-1,i] all (part-of-speech) categories for the $i^{th}$ word
for width := 2 to n
    for start := 0 to n-width
        Define end := start + width
        for mid := start+1 to end-1
            for every constituent X in [start,mid]
                for every constituent Y in [mid,end]
                    for all ways of combining X and Y (if any)
                        Add the resulting constituent to
                        [start,end] ~~if it's not already there.~~
```

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Time complexity?

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Time complexity? $O(Gn^3)$

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```

Time complexity? $O(Gn^3)$

Space complexity?

CKY Algorithm

```
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for width := 2 to n
    for start := 0 to n-width
        Define end := start + width
        for mid := start+1 to end-1
            for every constituent X in [start,mid]
                for every constituent Y in [mid,end]
                    for all ways of combining X and Y (if any)
                        Add the resulting constituent to
                        [start,end] ~~if it's not already there.~~
```

Time complexity? $O(Gn^3)$

Space complexity? $O(Tn^2)$

	time 1	flies 2	like 3	an 4	arrow 5
0	NP 3 Vst 3				
1		NP 4 VP 4			
2			P 2 V 5		
3				Det 1	
4					N 8

NP → time
Vst → time
NP → flies
VP → flies
P → like
V → like
Det → an
N → arrow

1 S → NP VP
6 S → Vst NP
2 S → S PP
1 VP → V NP
2 VP → VP PP
1 NP → Det N
2 NP → NP PP
3 NP → NP NP
0 PP → P NP

	time 1	flies 2	like 3	an 4	arrow 5
0	NP 3 Vst 3				
1		NP 4 VP 4			
2			P 2 V 5		
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1 VP → V NP
2 VP → VP PP

1 NP → Det N
2 NP → NP PP
3 NP → NP NP

0 PP → P NP

	time 1	flies 2	like 3	an 4	arrow 5
0	NP 3 Vst 3	NP 10			
1		NP 4 VP 4			
2			P 2 V 5		
3				Det 1	
4					N 8

1 S → NP VP
6 S → Vst NP
2 S → S PP

1 VP → V NP
2 VP → VP PP

1 NP → Det N
2 NP → NP PP
3 NP → NP NP

0 PP → P NP

time 1 flies 2 like 3 an 4 arrow 5

0	NP 3 Vst 3	NP 10 S 8			
1		NP 4 VP 4			
2			P 2 V 5		
3				Det 1	
4					N 8

1 S → NP VP
6 S → Vst NP
2 S → S PP

1 VP → V NP
2 VP → VP PP

1 NP → Det N
2 NP → NP PP
3 NP → NP NP

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	time 1	flies 2	like 3	an 4	arrow 5
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1		NP 4 VP 4			
2			P 2 V 5		
3				Det 1	
4					N 8

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2 VP → VP PP

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2 NP → NP PP
3 NP → NP NP

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	time 1	flies 2	like 3	an 4	arrow 5
0	NP 3 Vst 3	NP 10 S 8 S 13			
1		NP 4 VP 4	–		
2			P 2 V 5	–	
3				Det 1	
4					N 8

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6 S → Vst NP
2 S → S PP

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2 VP → VP PP

1 NP → Det N
2 NP → NP PP
3 NP → NP NP

0 PP → P NP

time **1** flies **2** like **3** an **4** arrow **5**

0	**NP 3** **Vst 3**	**NP 10** **S 8** **S 13**			
1		**NP 4** **VP 4**	–		
2			**P 2** **V 5**	–	
3				**Det 1**	**NP 10**
4					**N 8**

1 S → NP VP
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2 S → S PP

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time **1** flies **2** like **3** an **4** arrow **5**

0	**NP 3** **Vst 3**	**NP 10** **S 8** **S 13**			
1		**NP 4** **VP 4**	–		
2			**P 2** **V 5**	–	
3				**Det 1**	**NP 10**
4					**N 8**

1 S → NP VP
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1 VP → V NP
2 VP → VP PP

1 NP → Det N
2 NP → NP PP
3 NP → NP NP

0 PP → P NP

	time 1	flies 2	like 3	an 4	arrow 5
0	NP 3 Vst 3	NP 10 S 8 S 13	–		
1		NP 4 VP 4	–	–	
2			P 2 V 5	–	PP 12
3				Det 1	NP 10
4					N 8

1 S → NP VP
6 S → Vst NP
2 S → S PP

1 VP → V NP
2 VP → VP PP

1 NP → Det N
2 NP → NP PP
3 NP → NP NP

0 PP → P NP

	time 1	flies 2	like 3	an 4	arrow 5
0	NP 3 Vst 3	NP 10 S 8 S 13	–		
1		NP 4 VP 4	–	–	
2			P 2 V 5	–	PP 12 VP 16
3				Det 1	NP 10
4					N 8

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6 S → Vst NP
2 S → S PP

1 VP → V NP
2 VP → VP PP

1 NP → Det N
2 NP → NP PP
3 NP → NP NP

0 PP → P NP

	time 1	flies 2	like 3	an 4	arrow 5
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1		NP 4 VP 4	–	–	
2			P 2 V 5	–	PP 12 VP 16
3				Det 1	NP 10
4					N 8

1 S → NP VP
6 S → Vst NP
2 S → S PP

1 VP → V NP
2 VP → VP PP

1 NP → Det N
2 NP → NP PP
3 NP → NP NP

0 PP → P NP

time 1 flies 2 like 3 an 4 arrow 5

0	NP 3 Vst 3	NP 10 S 8 S 13	–	–	
1		NP 4 VP 4	–	–	NP 18
2			P 2 V 5	–	PP 12 VP 16
3				Det 1	NP 10
4					N 8

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6 S → Vst NP
2 S → S PP

1 VP → V NP
2 VP → VP PP

1 NP → Det N
2 NP → NP PP
3 NP → NP NP

0 PP → P NP

time 1 flies 2 like 3 an 4 arrow 5

0	NP 3 Vst 3	NP 10 S 8 S 13	–	–	
1		NP 4 VP 4	–	–	NP 18 S 21
2			P 2 V 5	–	PP 12 VP 16
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1		NP 4 VP 4	–	–	NP 18 S 21 VP 18
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3				Det 1	NP 10
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0	NP 3 Vst 3	NP 10 S 8 S 13	–	–	
1		NP 4 VP 4	–	–	NP 18 S 21 VP 18
2			P 2 V 5	–	PP 12 VP 16
3				Det 1	NP 10
4					N 8

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1 VP → V NP
2 VP → VP PP

1 NP → Det N
2 NP → NP PP
3 NP → NP NP

0 PP → P NP

	time 1	flies 2	like 3	an 4	arrow 5
0	NP 3 Vst 3	NP 10 S 8 S 13	–	–	NP 24
1		NP 4 VP 4	–	–	NP 18 S 21 VP 18
2			P 2 V 5	–	PP 12 VP 16
3				Det 1	NP 10
4					N 8

1 S → NP VP
6 S → Vst NP
2 S → S PP

1 VP → V NP
2 VP → VP PP

1 NP → Det N
2 NP → NP PP
3 NP → NP NP

0 PP → P NP

time **1** flies **2** like **3** an **4** arrow **5**

0	**NP 3** **Vst 3**	**NP 10** **S 8** **S 13**	–	–	**NP 24** **S 22**
1		**NP 4** **VP 4**	–	–	**NP 18** **S 21** **VP 18**
2			**P 2** **V 5**	–	**PP 12** **VP 16**
3				**Det 1**	**NP 10**
4					**N 8**

1 S → NP VP
6 S → Vst NP
2 S → S PP

1 VP → V NP
2 VP → VP PP

1 NP → Det N
2 NP → NP PP
3 NP → NP NP

0 PP → P NP

time 1 flies 2 like 3 an 4 arrow 5

0	NP 3 Vst 3	NP 10 S 8 S 13	–	–	NP 24 S 22 S 27
1		NP 4 VP 4	–	–	NP 18 S 21 VP 18
2			P 2 V 5	–	PP 12 VP 16
3				Det 1	NP 10
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	time 1	flies 2	like 3	an 4	arrow 5
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2 VP → VP PP

1 NP → Det N
2 NP → NP PP
3 NP → NP NP

0 PP → P NP

Follow backpointers …

S

time 1 flies 2 like 3 an 4 arrow 5

0	NP 3 Vst 3	NP 10 S 8 S 13	_	_	NP 24 S 22 S 27 NP 24 S 27 S 22 S 27
1		NP 4 VP 4	_	_	NP 18 S 21 VP 18
2			P 2 V 5	_	PP 12 VP 16
3				Det 1	NP 10
4					N 8

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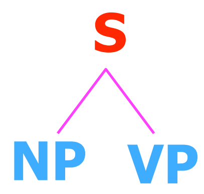
2 S → S PP

1 VP → V NP
2 VP → VP PP

1 NP → Det N
2 NP → NP PP
3 NP → NP NP

0 PP → P NP



	time 1	flies 2	like 3	an 4	arrow 5
0	NP 3 Vst 3	NP 10 S 8 S 13	_	_	NP 24 S 22 S 27 NP 24 S 27 S 22 S 27
1		NP 4 VP 4	_	_	NP 18 S 21 VP 18
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3 NP → NP NP

0 PP → P NP

time 1 flies 2 like 3 an 4 arrow 5

0	NP 3 Vst 3	NP 10 S 8 S 13	_	_	NP 24 S 22 S 27 NP 24 S 27 S 22 S 27
1		NP 4 VP 4	_	_	NP 18 S 21 VP 18
2			P 2 V 5	_	PP 12 VP 16
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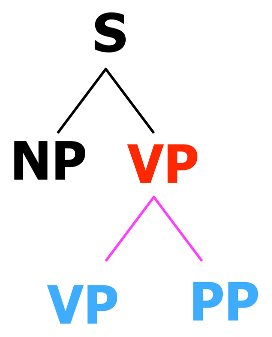
4					N 8



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	time 1	flies 2	like 3	an 4	arrow 5
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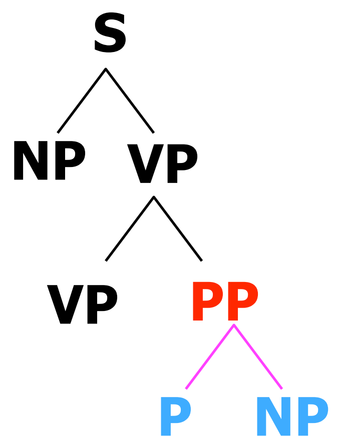
4					N 8



1 S → NP VP
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2 NP → NP PP
3 NP → NP NP

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	time 1	flies 2	like 3	an 4	arrow 5
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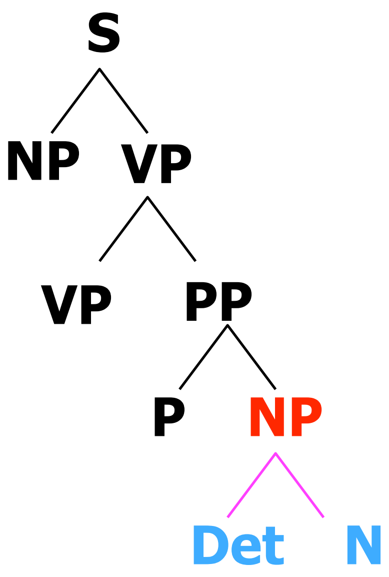

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Parsing as Deduction

- CKY as inference rules
- CKY as Prolog program
- But Prolog is top-down with backtracking
  - i.e., “backward chaining”, CKY is “forward chaining”
- Inference rules as Boolean semiring

Probabilistic CFGs

- Generative process (already familiar)
- It’s context free: Rules are applied independently, therefore we multiply their probabilities
- How to estimate probabilities?
    - Supervised and unsupervised

Questions for PCFGs

- What is the most likely parse for a sentence? (parsing)
- What is the probability of a sentence? (language modeling)
- What rule probabilities maximize the probability of a sentence? (parameter estimation)

Algorithms for PCFGs

- Exact analogues to HMM algorithms
- Parsing: Viterbi CKY
- Language modeling: inside probabilities
- Parameter estimation: inside-outside probabilities with EM

Parsing as Deduction

$$\forall A, B, C \in N, W \in V, 0 \leq i, j, k \leq m$$

$$constit(B, i, j) \wedge constit(C, j, k) \wedge A \rightarrow BC \Rightarrow constit(A, i, k)$$

$$word(W, i) \wedge A \rightarrow W \Rightarrow constit(A, i, i + 1)$$

In Prolog:

```
constit(A, I1, I) :-
  word(I, W),
  (A ---> [W]),
  I1 is I - 1.
```

```
constit(A, I, K) :-
  constit(B, I, J),
  constit(C, J, K),
  (A ---> [B, C]).
```

But Prolog uses top-down search with backtracking...

Parsing as Deduction

$$\forall A, B, C \in N, W \in V, 0 \leq i, j, k \leq m$$

$$constit(B, i, j) \wedge constit(C, j, k) \wedge A \rightarrow BC \Rightarrow constit(A, i, k)$$

$$word(W, i) \wedge A \rightarrow W \Rightarrow constit(A, i, i+1)$$

$$constit(A, i, k) = \bigvee_{B,C,j} constit(B, i, j) \wedge constit(C, j, k) \wedge A \rightarrow B\ C$$

$$constit(A, i, j) = \bigvee_{W} word(W, i, j) \wedge A \rightarrow W$$

Parsing as Deduction

$$constit(A, i, k) = \bigvee_{B,C,j} constit(B, i, j) \wedge constit(C, j, k) \wedge A \rightarrow B\ C$$

$$constit(A, i, j) = \bigvee_{W} word(W, i, j) \wedge A \rightarrow W$$

$$\begin{aligned} score(constit(A, i, k)) = \max_{B,C,j}\ & score(constit(B, i, j)) \\ & \cdot score(constit(C, j, k)) \\ & \cdot score(A \rightarrow B\ C) \end{aligned}$$

$$score(constit(A, i, j)) = \max_{W} score(word(W, i, j)) \cdot score(A \rightarrow W)$$

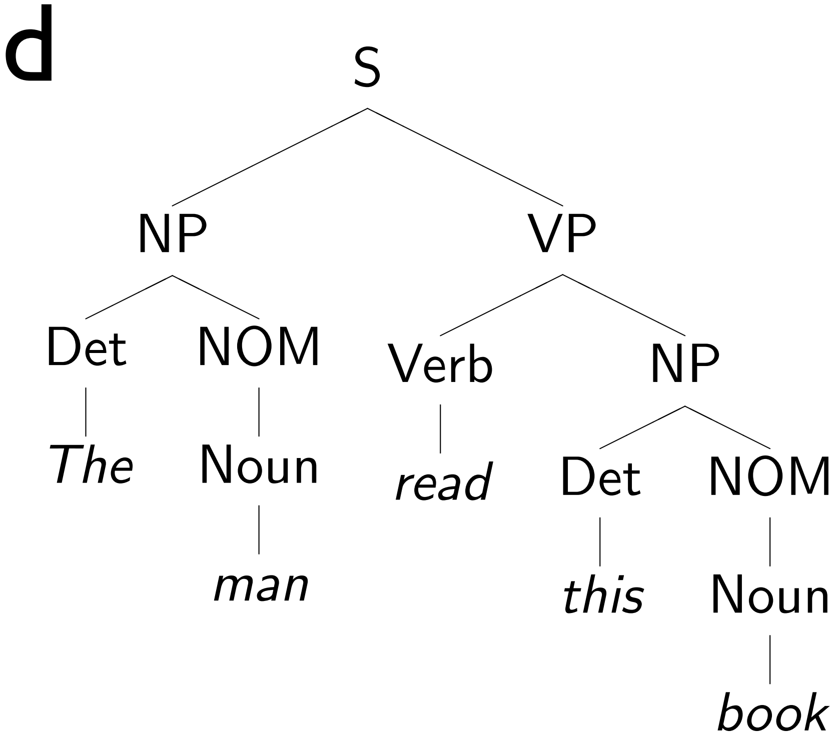
And how about the inside algorithm?

Problems with Inside-Outside EM

- Each sentence at each iteration takes $O(m^3n^3)$
- Local maxima even more problematic than for HMMs: Charniak (1993) found a different maximum for each of 300 trials
- More NTs needed to learn a good model
- NTs don't correspond to intuitions: HMMs are easier to constrain with tag dictionaries

Treebank Grammars

- What rules would you extract from this tree?
- What probabilities would you assign them?



Treebank Grammars

- Penn Treebank
- Lots of rules have high fanout (flat phrases)
- Lots of unary cycles
- How should we evaluate?
- What are the consequences of CNF conversion?

Inside & Viterbi Algorithms

NB: index *between* words;
M&S index words

Let $$\begin{aligned}\beta_A(i,j) &= p(constit(A,i,j)) \\ &= p(w_{ij} \mid \text{nonterminal A from i to j})\end{aligned}$$

$$\beta_A(i,k) = \sum_{B,C,j} \beta_B(i,j) \cdot \beta_C(j,k) \cdot p(A \rightarrow B\ C)$$

Let $$\delta_A(i,j) = p_{best}(constit(A,i,j))$$

$$\delta_A(i,k) = \max_{B,C,j} \delta_B(i,j)) \cdot \delta_C(j,k) \cdot p(A \rightarrow B\ C)$$

$$\beta_S(0,n) = ?$$

$$\delta_S(0,n) = ?$$

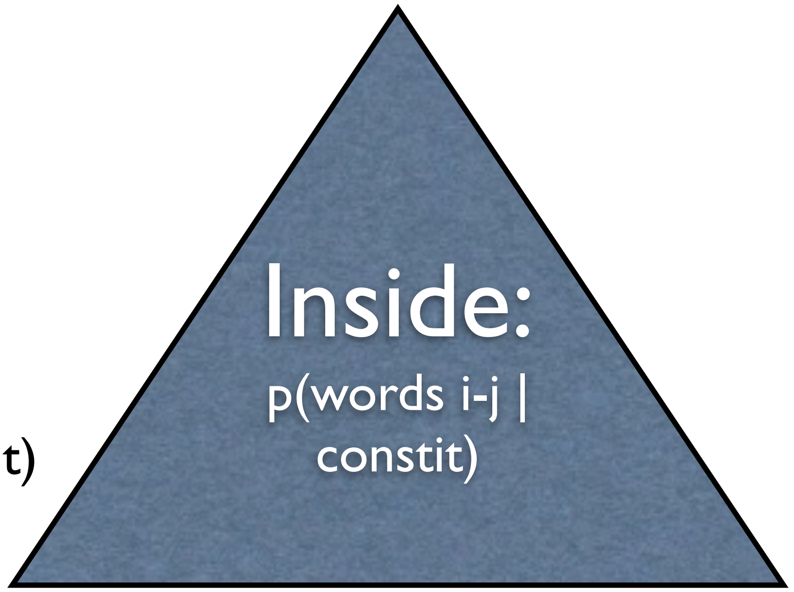
Inside & Outside

constit(A, i, j)

p(words 0-i, words j-n, constit)

w(0, 1) w(i-1, i) w(j, j+1) w(n-1, n)

Inside & Outside



constit(A, i, j)

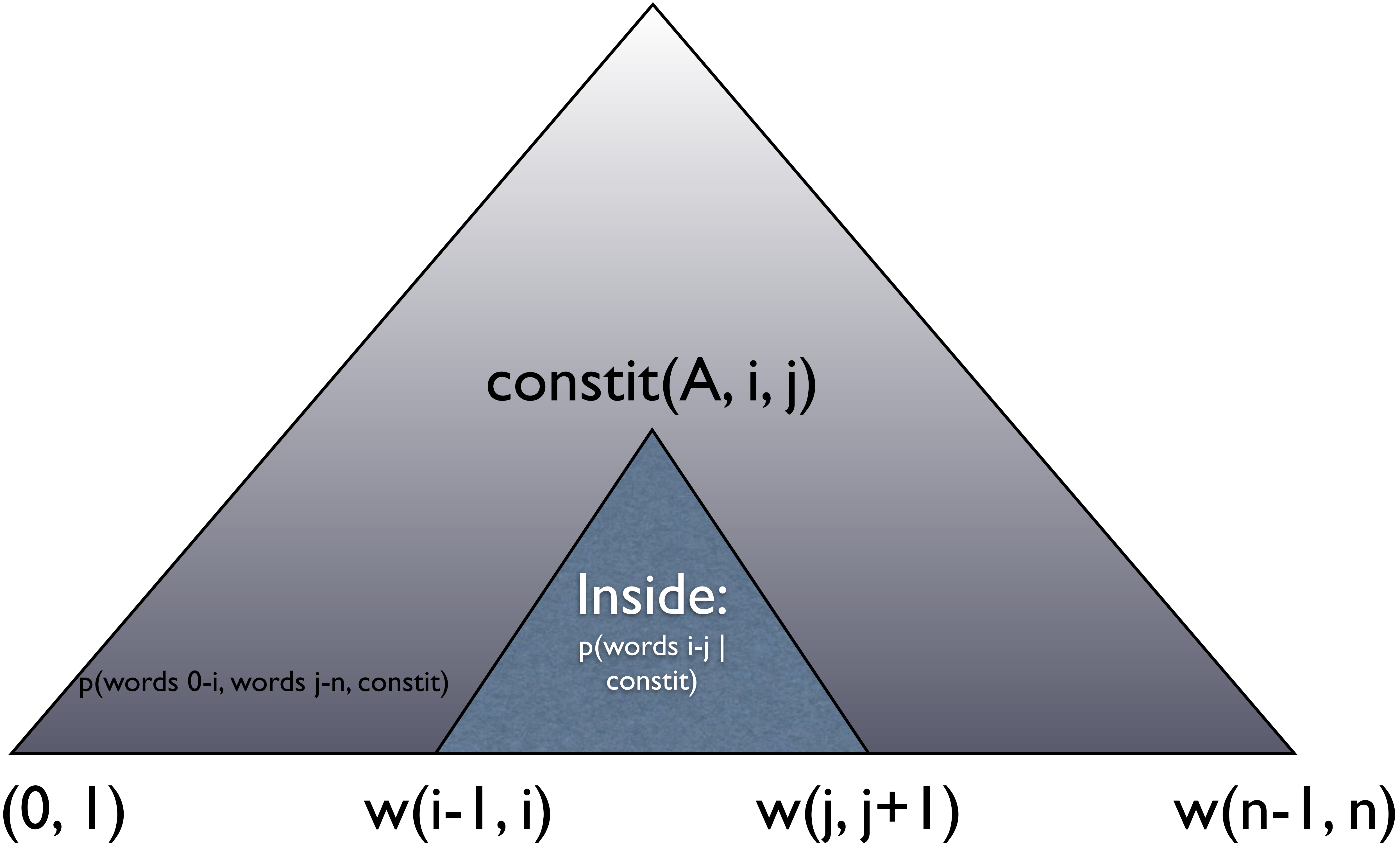
Inside:

p(words i-j | constit)

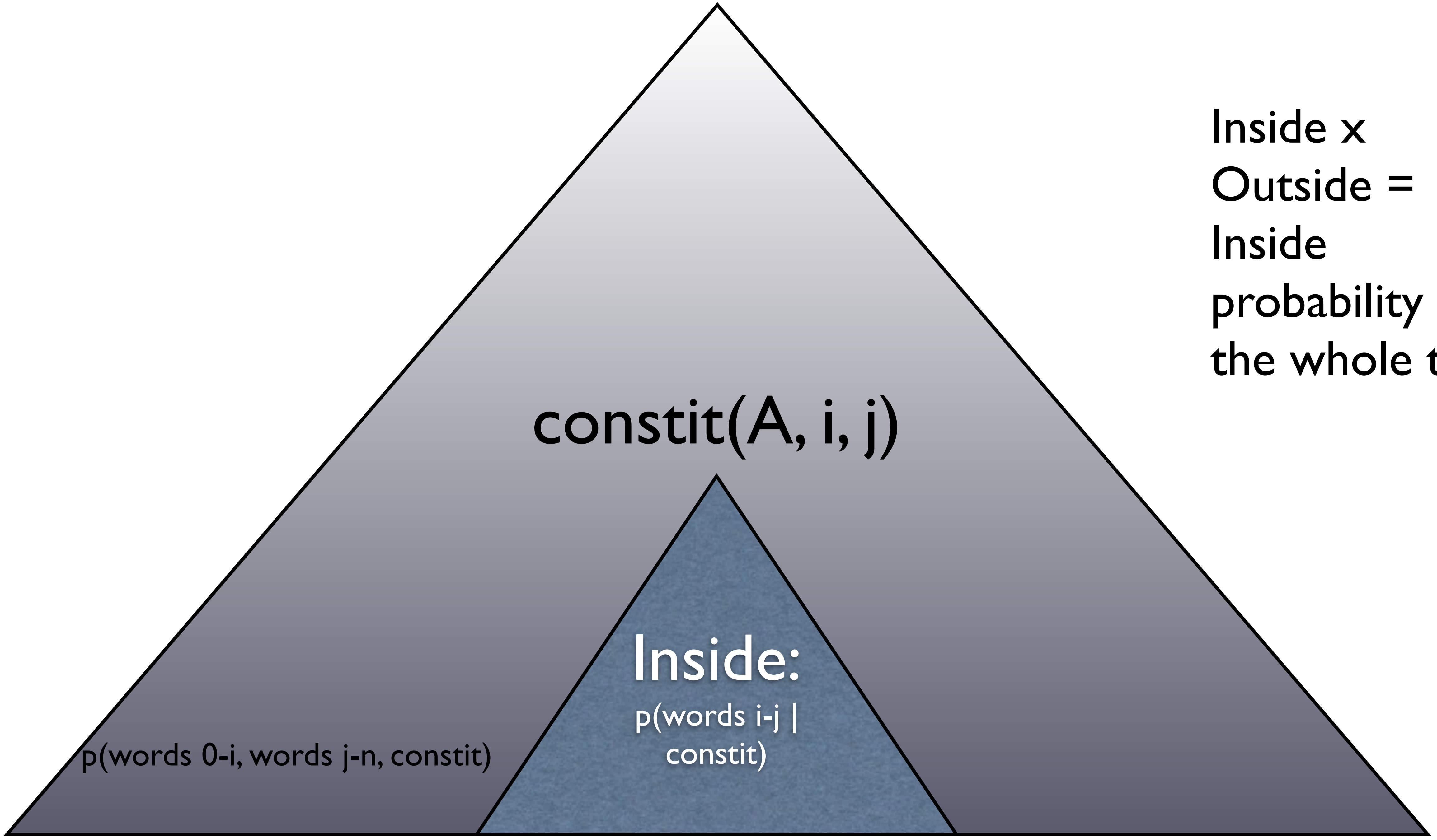
p(words 0-i, words j-n, constit)

w(0, 1) w(i-1, i) w(j, j+1) w(n-1, n)

Inside & Outside



Inside & Outside



constit(A, i, j)

Inside:

p(words i-j | constit)

p(words 0-i, words j-n, constit)

w(0, 1) w(i-1, i) w(j, j+1) w(n-1, n)

Inside x Outside = Inside probability of the whole tree

Outside Algorithm

$$\alpha_A(i,j) = p(w_{0,i}, A_{i,j}, w_{j,n})$$

Uses inside probs.

$$\begin{aligned}\alpha_A(i,j) \quad &= \quad \sum_{B,C,k=j}^{n} \alpha_B(i,k) \cdot \beta_C(j,k) \cdot p(B \rightarrow A\ C) \\ &+ \sum_{B,C,k=0}^{i} \alpha_B(k,j) \cdot \beta_C(k,i) \cdot p(B \rightarrow C\ \ A)\end{aligned}$$

Some resemblance to derivative product rule

$$\alpha_S(0,n) = ?$$

$$\alpha_{PP}(0,n) = ?$$

Top-Down/Bottom-Up

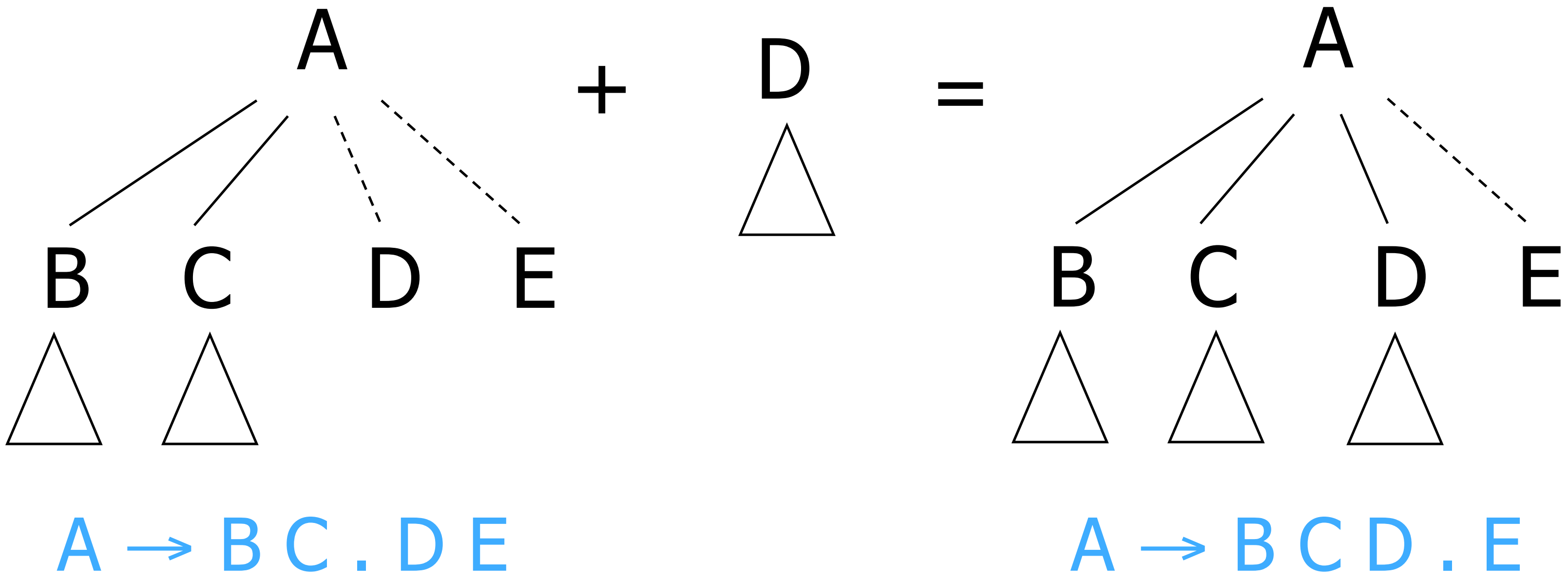
- Top-down parsers
  - Can get caught in infinite loops
  - Take exponential time backtracking
- CKY
  - Needs Chomsky normal form
  - Builds all possible constituents

Earley Parser (1970)

- Nice combination of
  - dynamic programming
  - incremental interpretation
  - avoids infinite loops
  - no restrictions on the form of the context-free grammar.
    A → B C the D of causes no problems
  - O(n^3) worst case, but faster for many grammars
  - Uses left context and optionally right context to constrain search.

Earley’s Overview

- Finds constituents and partial constituents in input
  - A → B C . D E is partial: only the first half of the A



Earley's Overview

- Proceeds incrementally left-to-right
  - Before it reads word 5, it has already built all hypotheses that are consistent with first 4 words
  - Reads word 5 & attaches it to immediately preceding hypotheses. Might yield new constituents that are then attached to hypotheses immediately preceding *them* …
  - E.g., attaching D to A → B C . D E gives A → B C D . E
  - Attaching E to that gives A → B C D E .
  - Now we have a complete A that we can attach to hypotheses immediately preceding the A, etc.

The Parse Table

- Columns 0 through n corresponding to the gaps between words
- Entries in column 5 look like (3, NP → NP . PP)

  (but we'll omit the → etc. to save space)

  - Built while processing word 5
  - Means that the input substring from 3 to 5 matches the initial NP portion of a **NP → NP PP** rule
  - Dot shows how much we've matched as of column 5
  - Perfectly fine to have entries like **(3, VP → is it . true that S)**

The Parse Table

- Entries in column 5 look like (3, NP → NP . PP)
- What will it mean that we have this entry?
  - *Unknown right context: Doesn't* mean we'll necessarily be able to find a VP starting at column 5 to complete the S.
  - *Known left context: Does* mean that some dotted rule back in column 3 is looking for an S that starts at 3.
    - So if we actually do find a VP starting at column 5, allowing us to complete the S, then we'll be able to attach the S to something.
    - And when that something is complete, it too will have a customer to *its* left …
    - In short, a top-down (i.e., goal-directed) parser: it chooses to start building a constituent not because of the input but because that's what the left context needs. In **the spoon**, won't build **spoon** as a verb because there's no way to use a verb there.
    - So any hypothesis in column 5 *could* get used in the correct parse, if words 1-5 are continued in just the right way by words 6-n.

Earley's as a Recognizer

- Add **ROOT → . S** to column 0.
- For each j from 0 to n:
  - – For each dotted rule in column j,
    (including those we add as we go!)
    look at what's after the dot:
    - If it's a word w, SCAN:
      - – If w matches the input word between j and j+1, advance the dot and add the resulting rule to column j+1
    - If it's a non-terminal X, PREDICT:
      - – Add all rules for X to the bottom of column j, wth the dot at the start: e.g. **X → . Y Z**
    - If there's nothing after the dot, ATTACH:
      - – We've finished some constituent, A, that started in column I<j. So for each rule in column j that has A after the dot: Advance the dot and add the result to the bottom of column j.
- Output "yes" just if last column has **ROOT → S .**
- **NOTE: Don't add an entry to a column if it's already there!**

Earley's Summary

- Process all hypotheses one at a time in order. (Current hypothesis is shown in blue.)
- This may add new hypotheses to the end of the to-do list, or try to add old hypotheses again.

- Process a hypothesis according to what follows the dot:
  - If a word, **scan** input and see if it matches
  - If a nonterminal, **predict** ways to match it
    - (we'll predict blindly, but could reduce # of predictions by *looking ahead* k symbols in the input and only making predictions that are compatible with this limited *right context*)
  - If nothing, then we have a complete constituent, so **attach** it to all its customers

A (Whimsical) Grammar

S → NP VP
NP → Det N
NP → NP PP
VP → V NP
VP → VP PP
PP → P NP

NP → Papa
N → caviar
N → spoon
V → ate
P → with
Det → the
Det → a

An Input Sentence

Papa ate the caviar with a spoon.

0
0 ROOT . S

initialize

Remember this stands for (0, ROOT → . S)

0
0 ROOT . S
0 S . NP VP

predict the kind of S we are looking for

Remember this stands for (0, S → . NP VP)

0
0 ROOT . S
0 S . NP VP
0 NP . Det N
0 NP . NP PP
0 NP . Papa

predict the kind of NP we are looking for
(actually we'll look for 3 kinds: any of the 3 will do)

0
0 ROOT . S
0 S . NP VP
0 NP . Det N
0 NP . NP PP
0 NP . Papa
0 Det . the
0 Det . a

predict the kind of Det we are looking for *(2 kinds)*

0
0 ROOT . S
0 S . NP VP
0 NP . Det N
0 NP . NP PP
0 NP . Papa
0 Det . the
0 Det . a

predict the kind of NP we’re looking for
but we were already looking for these so don’t add duplicate goals! Note that this happened when we were processing a left-recursive rule.

0 Papa	1
0 ROOT . S	0 NP Papa .
0 S . NP VP	
0 NP . Det N	
0 NP . NP PP	
0 NP . Papa	
0 Det . the	
0 Det . a	

scan: the desired word is in the input!

0 Papa 1	
0 ROOT . S	0 NP Papa .
0 S . NP VP	
0 NP . Det N	
0 NP . NP PP	
0 NP . Papa	
0 Det . the	
0 Det . a	

scan: failure

0 Papa 1	
0 ROOT . S	0 NP Papa .
0 S . NP VP	
0 NP . Det N	
0 NP . NP PP	
0 NP . Papa	
0 Det . the	
0 Det . a	

scan: failure

0 Papa 1	
0 ROOT . S	0 NP Papa .
0 S . NP VP	0 S NP . VP
0 NP . Det N	0 NP NP . PP
0 NP . NP PP	
0 NP . Papa	
0 Det . the	
0 Det . a	

attach the newly created NP (which starts at 0) to its customers (incomplete constituents that *end* at 0 and have NP after the dot)

0 Papa 1	
0 ROOT . S	0 NP Papa .
0 S . NP VP	0 S NP . VP
0 NP . Det N	0 NP NP . PP
0 NP . NP PP	1 VP . V NP
0 NP . Papa	1 VP . VP PP
0 Det . the	
0 Det . a	

predict

0 Papa 1	
0 ROOT . S	0 NP Papa .
0 S . NP VP	0 S NP . VP
0 NP . Det N	0 NP NP . PP
0 NP . NP PP	1 VP . V NP
0 NP . Papa	1 VP . VP PP
0 Det . the	1 PP . P NP
0 Det . a	

predict

0 Papa 1	
0 ROOT . S	0 NP Papa .
0 S . NP VP	0 S NP . VP
0 NP . Det N	0 NP NP . PP
0 NP . NP PP	1 VP . V NP
0 NP . Papa	1 VP . VP PP
0 Det . the	1 PP . P NP
0 Det . a	1 V . ate

predict

0 Papa 1	
0 ROOT . S	0 NP Papa .
0 S . NP VP	0 S NP . VP
0 NP . Det N	0 NP NP . PP
0 NP . NP PP	1 VP . V NP
0 NP . Papa	1 VP . VP PP
0 Det . the	1 PP . P NP
0 Det . a	1 V . ate

predict

0 Papa 1	
0 ROOT . S	0 NP Papa .
0 S . NP VP	0 S NP . VP
0 NP . Det N	0 NP NP . PP
0 NP . NP PP	1 VP . V NP
0 NP . Papa	1 VP . VP PP
0 Det . the	1 PP . P NP
0 Det . a	1 V . ate
	1 P . with

predict

0 Papa 1 ate 2		
0 ROOT . S	0 NP Papa .	1 V ate .
0 S . NP VP	0 S NP . VP	
0 NP . Det N	0 NP NP . PP	
0 NP . NP PP	1 VP . V NP	
0 NP . Papa	1 VP . VP PP	
0 Det . the	1 PP . P NP	
0 Det . a	1 V . ate	
	1 P . with	

scan: success!

0 Papa 1 ate 2		
0 ROOT . S	0 NP Papa .	1 V ate .
0 S . NP VP	0 S NP . VP	
0 NP . Det N	0 NP NP . PP	
0 NP . NP PP	1 VP . V NP	
0 NP . Papa	1 VP . VP PP	
0 Det . the	1 PP . P NP	
0 Det . a	1 V . ate	
	1 P . with	

scan: failure

0 Papa	1 ate	2
0 ROOT . S	0 NP Papa .	1 V ate .
0 S . NP VP	0 S NP . VP	1 VP V . NP
0 NP . Det N	0 NP NP . PP	
0 NP . NP PP	1 VP . V NP	
0 NP . Papa	1 VP . VP PP	
0 Det . the	1 PP . P NP	
0 Det . a	1 V . ate	
	1 P . with	

attach

0 Papa 1 ate 2		
0 ROOT . S	0 NP Papa .	1 V ate .
0 S . NP VP	0 S NP . VP	1 VP V . NP
0 NP . Det N	0 NP NP . PP	2 NP . Det N
0 NP . NP PP	1 VP . V NP	2 NP . NP PP
0 NP . Papa	1 VP . VP PP	2 NP . Papa
0 Det . the	1 PP . P NP	
0 Det . a	1 V . ate	
	1 P . with	

predict

0 Papa 1 ate 2		
0 ROOT . S	0 NP Papa .	1 V ate .
0 S . NP VP	0 S NP . VP	1 VP V . NP
0 NP . Det N	0 NP NP . PP	2 NP . Det N
0 NP . NP PP	1 VP . V NP	2 NP . NP PP
0 NP . Papa	1 VP . VP PP	2 NP . Papa
0 Det . the	1 PP . P NP	2 Det . the
0 Det . a	1 V . ate	2 Det . a
	1 P . with	

predict *(these next few steps should look familiar)*

0 Papa 1 ate 2		
0 ROOT . S	0 NP Papa .	1 V ate .
0 S . NP VP	0 S NP . VP	1 VP V . NP
0 NP . Det N	0 NP NP . PP	2 NP . Det N
0 NP . NP PP	1 VP . V NP	2 NP . NP PP
0 NP . Papa	1 VP . VP PP	2 NP . Papa
0 Det . the	1 PP . P NP	2 Det . the
0 Det . a	1 V . ate	2 Det . a
	1 P . with	

predict

0 Papa 1 ate 2		
0 ROOT . S	0 NP Papa .	1 V ate .
0 S . NP VP	0 S NP . VP	1 VP V . NP
0 NP . Det N	0 NP NP . PP	2 NP . Det N
0 NP . NP PP	1 VP . V NP	2 NP . NP PP
0 NP . Papa	1 VP . VP PP	2 NP . Papa
0 Det . the	1 PP . P NP	2 Det . the
0 Det . a	1 V . ate	2 Det . a
	1 P . with	

scan *(this time we fail since Papa is not the next word)*

0 Papa	1 ate	2 the	3
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .
0 S . NP VP	0 S NP . VP	1 VP V . NP	
0 NP . Det N	0 NP NP . PP	2 NP . Det N	
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	
0 NP . Papa	1 VP . VP PP	2 NP . Papa	
0 Det . the	1 PP . P NP	2 Det . the	
0 Det . a	1 V . ate	2 Det . a	
	1 P . with		

scan: success!

0 Papa 1 ate 2 the 3

0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .
0 S . NP VP	0 S NP . VP	1 VP V . NP	
0 NP . Det N	0 NP NP . PP	2 NP . Det N	
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	
0 NP . Papa	1 VP . VP PP	2 NP . Papa	
0 Det . the	1 PP . P NP	2 Det . the	
0 Det . a	1 V . ate	2 Det . a	
	1 P . with		

0 Papa 1 ate 2 the 3			
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N
0 NP . Det N	0 NP NP . PP	2 NP . Det N	
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	
0 NP . Papa	1 VP . VP PP	2 NP . Papa	
0 Det . the	1 PP . P NP	2 Det . the	
0 Det . a	1 V . ate	2 Det . a	
	1 P . with		

0 Papa	1 ate	2 the	3
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon
0 NP . Papa	1 VP . VP PP	2 NP . Papa	
0 Det . the	1 PP . P NP	2 Det . the	
0 Det . a	1 V . ate	2 Det . a	
	1 P . with		

0 Papa 1 ate 2 the 3 caviar 4				
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	
0 NP . Papa	1 VP . VP PP	2 NP . Papa		
0 Det . the	1 PP . P NP	2 Det . the		
0 Det . a	1 V . ate	2 Det . a		
	1 P . with			

0 Papa 1 ate 2 the 3 caviar 4				
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	
0 NP . Papa	1 VP . VP PP	2 NP . Papa		
0 Det . the	1 PP . P NP	2 Det . the		
0 Det . a	1 V . ate	2 Det . a		
	1 P . with			

0 Papa 1 ate 2 the 3 caviar 4				
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	
0 NP . Papa	1 VP . VP PP	2 NP . Papa		
0 Det . the	1 PP . P NP	2 Det . the		
0 Det . a	1 V . ate	2 Det . a		
	1 P . with			

attach

0 Papa 1 ate 2 the 3 caviar 4				
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP
0 NP . Papa	1 VP . VP PP	2 NP . Papa		
0 Det . the	1 PP . P NP	2 Det . the		
0 Det . a	1 V . ate	2 Det . a		
	1 P . with			

attach
(again!)

0 Papa 1 ate 2 the 3 caviar 4				
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP
0 NP . Papa	1 VP . VP PP	2 NP . Papa		0 S NP VP .
0 Det . the	1 PP . P NP	2 Det . the		1 VP VP . PP
0 Det . a	1 V . ate	2 Det . a		
	1 P . with			

attach

(again!)

0 Papa 1 ate 2 the 3 caviar 4				
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP
0 NP . Papa	1 VP . VP PP	2 NP . Papa		0 S NP VP .
0 Det . the	1 PP . P NP	2 Det . the		1 VP VP . PP
0 Det . a	1 V . ate	2 Det . a		4 PP . P NP
	1 P . with			

0 Papa 1 ate 2 the 3 caviar 4				
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP
0 NP . Papa	1 VP . VP PP	2 NP . Papa		0 S NP VP .
0 Det . the	1 PP . P NP	2 Det . the		1 VP VP . PP
0 Det . a	1 V . ate	2 Det . a		4 PP . P NP
	1 P . with			0 ROOT S .

attach

(again!)

0 Papa 1 ate 2 the 3 caviar 4				
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP
0 NP . Papa	1 VP . VP PP	2 NP . Papa		0 S NP VP .
0 Det . the	1 PP . P NP	2 Det . the		1 VP VP . PP
0 Det . a	1 V . ate	2 Det . a		4 PP . P NP
	1 P . with			0 ROOT S .

0 Papa	1 ate	2 the	3 caviar	4
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP
0 NP . Papa	1 VP . VP PP	2 NP . Papa		0 S NP VP .
0 Det . the	1 PP . P NP	2 Det . the		1 VP VP . PP
0 Det . a	1 V . ate	2 Det . a		4 PP . P NP
	1 P . with			0 ROOT S .
				4 P . with

0 Papa 1 ate 2 the 3 caviar 4				
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP
0 NP . Papa	1 VP . VP PP	2 NP . Papa		0 S NP VP .
0 Det . the	1 PP . P NP	2 Det . the		1 VP VP . PP
0 Det . a	1 V . ate	2 Det . a		4 PP . P NP
	1 P . with			0 ROOT S .
				4 P . with

0 Papa	1 ate	2 the	3 caviar	4 with	5
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .	4 P with .
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .	
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .	
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP	
0 NP . Papa	1 VP . VP PP	2 NP . Papa		0 S NP VP .	
0 Det . the	1 PP . P NP	2 Det . the		1 VP VP . PP	
0 Det . a	1 V . ate	2 Det . a		4 PP . P NP	
	1 P . with			0 ROOT S .	
				4 P . with	

0 Papa 1 ate 2 the 3 caviar 4 with 5					
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .	4 P with .
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .	4 PP P . NP
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .	
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP	
0 NP . Papa	1 VP . VP PP	2 NP . Papa		0 S NP VP .	
0 Det . the	1 PP . P NP	2 Det . the		1 VP VP . PP	
0 Det . a	1 V . ate	2 Det . a		4 PP . P NP	
	1 P . with			0 ROOT S .	
				4 P . with	

0 Papa 1 ate 2 the 3 caviar 4 with 5					
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .	4 P with .
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .	4 PP P . NP
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .	5 NP . Det N
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP	5 NP . NP PP
0 NP . Papa	1 VP . VP PP	2 NP . Papa		0 S NP VP .	5 NP . Papa
0 Det . the	1 PP . P NP	2 Det . the		1 VP VP . PP	
0 Det . a	1 V . ate	2 Det . a		4 PP . P NP	
	1 P . with			0 ROOT S .	
				4 P . with	

0 Papa	1 ate	2 the	3 caviar	4 with	5
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .	4 P with .
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .	4 PP P . NP
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .	5 NP . Det N
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP	5 NP . NP PP
0 NP . Papa	1 VP . VP PP	2 NP . Papa		0 S NP VP .	5 NP . Papa
0 Det . the	1 PP . P NP	2 Det . the		1 VP VP . PP	5 Det . the
0 Det . a	1 V . ate	2 Det . a		4 PP . P NP	5 Det . a
	1 P . with			0 ROOT S .	
				4 P . with	

0 Papa	1 ate	2 the	3 caviar	4 with	5
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .	4 P with .
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .	4 PP P . NP
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .	5 NP . Det N
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP	5 NP . NP PP
0 NP . Papa	1 VP . VP PP	2 NP . Papa		0 S NP VP .	5 NP . Papa
0 Det . the	1 PP . P NP	2 Det . the		1 VP VP . PP	5 Det . the
0 Det . a	1 V . ate	2 Det . a		4 PP . P NP	5 Det . a
	1 P . with			0 ROOT S .	
				4 P . with	

0 Papa 1 ate 2 the 3 caviar 4 with 5					
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .	4 P with .
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .	4 PP P . NP
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .	5 NP . Det N
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP	5 NP . NP PP
0 NP . Papa	1 VP . VP PP	2 NP . Papa		0 S NP VP .	5 NP . Papa
0 Det . the	1 PP . P NP	2 Det . the		1 VP VP . PP	5 Det . the
0 Det . a	1 V . ate	2 Det . a		4 PP . P NP	5 Det . a
	1 P . with			0 ROOT S .	
				4 P . with	

0 Papa	1 ate	2 the	3 caviar	4 with	5
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .	4 P with .
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .	4 PP P . NP
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .	5 NP . Det N
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP	5 NP . NP PP
0 NP . Papa	1 VP . VP PP	2 NP . Papa		0 S NP VP .	5 NP . Papa
0 Det . the	1 PP . P NP	2 Det . the		1 VP VP . PP	5 Det . the
0 Det . a	1 V . ate	2 Det . a		4 PP . P NP	5 Det . a
	1 P . with			0 ROOT S .	
				4 P . with	

e	2 the	3 caviar	4 with	5 a	6
	1 V ate .	2 Det the .	3 N caviar .	4 P with .	5 Det a .
	1 VP V . NP	2 NP Det . N	2 NP Det N .	4 PP P . NP	
P	2 NP . Det N	3 N . caviar	1 VP V NP .	5 NP . Det N	
	2 NP . NP PP	3 N . spoon	2 NP NP . PP	5 NP . NP PP	
P	2 NP . Papa		0 S NP VP .	5 NP . Papa	
	2 Det . the		1 VP VP . PP	5 Det . the	
	2 Det . a		4 PP . P NP	5 Det . a	
			0 ROOT S .		
			4 P . with		

e	2 the	3 caviar	4 with	5 a 6	
	1 V ate .	2 Det the .	3 N caviar .	4 P with .	5 Det a .
	1 VP V . NP	2 NP Det . N	2 NP Det N .	4 PP P . NP	5 NP Det . N
P	2 NP . Det N	3 N . caviar	1 VP V NP .	5 NP . Det N	
	2 NP . NP PP	3 N . spoon	2 NP NP . PP	5 NP . NP PP	
P	2 NP . Papa		0 S NP VP .	5 NP . Papa	
	2 Det . the		1 VP VP . PP	5 Det . the	
	2 Det . a		4 PP . P NP	5 Det . a	
			0 ROOT S .		
			4 P . with		

e	2 the	3 caviar	4 with	5 a 6	
	1 V ate .	2 Det the .	3 N caviar .	4 P with .	5 Det a .
	1 VP V . NP	2 NP Det . N	2 NP Det N .	4 PP P . NP	5 NP Det . N
P	2 NP . Det N	3 N . caviar	1 VP V NP .	5 NP . Det N	6 N . caviar
	2 NP . NP PP	3 N . spoon	2 NP NP . PP	5 NP . NP PP	6 N . spoon
P	2 NP . Papa		0 S NP VP .	5 NP . Papa	
	2 Det . the		1 VP VP . PP	5 Det . the	
	2 Det . a		4 PP . P NP	5 Det . a	
			0 ROOT S .		
			4 P . with		

e	2 the	3 caviar	4 with	5 a	6
	1 V ate .	2 Det the .	3 N caviar .	4 P with .	5 Det a .
	1 VP V . NP	2 NP Det . N	2 NP Det N .	4 PP P . NP	5 NP Det . N
P	2 NP . Det N	3 N . caviar	1 VP V NP .	5 NP . Det N	6 N . caviar
	2 NP . NP PP	3 N . spoon	2 NP NP . PP	5 NP . NP PP	6 N . spoon
P	2 NP . Papa		0 S NP VP .	5 NP . Papa	
	2 Det . the		1 VP VP . PP	5 Det . the	
	2 Det . a		4 PP . P NP	5 Det . a	
			0 ROOT S .		
			4 P . with		

e	2 the	3 caviar	4 with	5 a	6 spoon 7	
	1 V ate .	2 Det the .	3 N caviar .	4 P with .	5 Det a .	6 N spoon .
	1 VP V . NP	2 NP Det . N	2 NP Det N .	4 PP P . NP	5 NP Det . N	
P	2 NP . Det N	3 N . caviar	1 VP V NP .	5 NP . Det N	6 N . caviar	
	2 NP . NP PP	3 N . spoon	2 NP NP . PP	5 NP . NP PP	6 N . spoon	
P	2 NP . Papa		0 S NP VP .	5 NP . Papa		
	2 Det . the		1 VP VP . PP	5 Det . the		
	2 Det . a		4 PP . P NP	5 Det . a		
			0 ROOT S .			
			4 P . with			

e	2 the	3 caviar	4 with	5 a	6 spoon 7	
	1 V ate .	2 Det the .	3 N caviar .	4 P with .	5 Det a .	6 N spoon .
	1 VP V . NP	2 NP Det . N	2 NP Det N .	4 PP P . NP	5 NP Det . N	5 NP Det N .
P	2 NP . Det N	3 N . caviar	1 VP V NP .	5 NP . Det N	6 N . caviar	
	2 NP . NP PP	3 N . spoon	2 NP NP . PP	5 NP . NP PP	6 N . spoon	
P	2 NP . Papa		0 S NP VP .	5 NP . Papa		
	2 Det . the		1 VP VP . PP	5 Det . the		
	2 Det . a		4 PP . P NP	5 Det . a		
			0 ROOT S .			
			4 P . with			

e 2 the 3 caviar 4 with 5 a 6 spoon 7

	1 V ate .	2 Det the .	3 N caviar .	4 P with .	5 Det a .	6 N spoon .
	1 VP V . NP	2 NP Det . N	2 NP Det N .	4 PP P . NP	5 NP Det . N	5 NP Det N .
P	2 NP . Det N	3 N . caviar	1 VP V NP .	5 NP . Det N	6 N . caviar	4 PP P NP .
	2 NP . NP PP	3 N . spoon	2 NP NP . PP	5 NP . NP PP	6 N . spoon	5 NP NP . PP
P	2 NP . Papa		0 S NP VP .	5 NP . Papa		
	2 Det . the		1 VP VP . PP	5 Det . the		
	2 Det . a		4 PP . P NP	5 Det . a		
			0 ROOT S .			
			4 P . with			

0 Papa	1 ate	2 the	3 caviar	4 with a spoon 7		
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .	…	6 N spoon .
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .		5 NP Det N .
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .		4 PP P NP .
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP		5 NP NP . PP
0 NP . Papa	1 VP . VP PP	2 NP . Papa		0 S NP VP .		2 NP NP PP .
0 Det . the	1 PP . P NP	2 Det . the		1 VP VP . PP		1 VP VP PP .
0 Det . a	1 V . ate	2 Det . a		4 PP . P NP		
	1 P . with			0 ROOT S .		
				4 P . with		

0 Papa	1 ate	2 the	3 caviar	4 with a spoon		7
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .	…	6 N spoon .
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .		5 NP Det N .
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .		4 PP P NP .
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP		5 NP NP . PP
0 NP . Papa	1 VP . VP PP	2 NP . Papa		0 S NP VP .		2 NP NP PP .
0 Det . the	1 PP . P NP	2 Det . the		1 VP VP . PP		1 VP VP PP .
0 Det . a	1 V . ate	2 Det . a		4 PP . P NP		7 PP . P NP
	1 P . with			0 ROOT S .		
				4 P . with		

0 Papa 1 ate 2 the 3 caviar 4 with a spoon 7

0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .	…	6 N spoon .
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .		5 NP Det N .
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .		4 PP P NP .
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP		5 NP NP . PP
0 NP . Papa	1 VP . VP PP	2 NP . Papa		0 S NP VP .		2 NP NP PP .
0 Det . the	1 PP . P NP	2 Det . the		1 VP VP . PP		1 VP VP PP .
0 Det . a	1 V . ate	2 Det . a		4 PP . P NP		7 PP . P NP
	1 P . with			0 ROOT S .		1 VP V NP .
				4 P . with		2 NP NP . PP

0 Papa	1 ate	2 the	3 caviar	4 with a spoon 7		
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .	...	6 N spoon .
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .		5 NP Det N .
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .		4 PP P NP .
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP		5 NP NP . PP
0 NP . Papa	1 VP . VP PP	2 NP . Papa		0 S NP VP .		2 NP NP PP .
0 Det . the	1 PP . P NP	2 Det . the		1 VP VP . PP		1 VP VP PP .
0 Det . a	1 V . ate	2 Det . a		4 PP . P NP		7 PP . P NP
	1 P . with			0 ROOT S .		1 VP V NP .
				4 P . with		2 NP NP . PP
						0 S NP VP .
						1 VP VP . PP

0 Papa	1 ate	2 the	3 caviar	4 with a spoon 7		
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .	…	6 N spoon .
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .		5 NP Det N .
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .		4 PP P NP .
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP		5 NP NP . PP
0 NP . Papa	1 VP . VP PP	2 NP . Papa		0 S NP VP .		2 NP NP PP .
0 Det . the	1 PP . P NP	2 Det . the		1 VP VP . PP		1 VP VP PP .
0 Det . a	1 V . ate	2 Det . a		4 PP . P NP		7 PP . P NP
	1 P . with			0 ROOT S .		1 VP V NP .
				4 P . with		2 NP NP . PP
						0 S NP VP .
						1 VP VP . PP
						7 P . with

0 Papa 1 ate 2 the 3 caviar 4 with a spoon 7

0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .	…	6 N spoon .
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .		5 NP Det N .
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .		4 PP P NP .
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP		5 NP NP . PP
0 NP . Papa	1 VP . VP PP	2 NP . Papa		0 S NP VP .		2 NP NP PP .
0 Det . the	1 PP . P NP	2 Det . the		1 VP VP . PP		1 VP VP PP .
0 Det . a	1 V . ate	2 Det . a		4 PP . P NP		7 PP . P NP
	1 P . with			0 ROOT S .		1 VP V NP .
				4 P . with		2 NP NP . PP
						0 S NP VP .
						1 VP VP . PP
						7 P . with

0 Papa 1 ate 2 the 3 caviar 4 with a spoon 7

0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .	…	6 N spoon .
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .		5 NP Det N .
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .		4 PP P NP .
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP		5 NP NP . PP
0 NP . Papa	1 VP . VP PP	2 NP . Papa		0 S NP VP .		2 NP NP PP .
0 Det . the	1 PP . P NP	2 Det . the		1 VP VP . PP		1 VP VP PP .
0 Det . a	1 V . ate	2 Det . a		4 PP . P NP		7 PP . P NP
	1 P . with			0 ROOT S .		1 VP V NP .
				4 P . with		2 NP NP . PP
						0 S NP VP .
						1 VP VP . PP
						7 P . with

0 Papa	1 ate	2 the	3 caviar	4 with a spoon 7		
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .	…	6 N spoon .
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .		5 NP Det N .
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .		4 PP P NP .
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP		5 NP NP . PP
0 NP . Papa	1 VP . VP PP	2 NP . Papa		0 S NP VP .		2 NP NP PP .
0 Det . the	1 PP . P NP	2 Det . the		1 VP VP . PP		1 VP VP PP .
0 Det . a	1 V . ate	2 Det . a		4 PP . P NP		7 PP . P NP
	1 P . with			0 ROOT S .		1 VP V NP .
				4 P . with		2 NP NP . PP
						0 S NP VP .
						1 VP VP . PP
						7 P . with
						0 ROOT S .

0 **Papa** 1 ate 2 the 3 caviar 4 with a spoon 7

0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .	…	6 N spoon .
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .		5 NP Det N .
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .		4 PP P NP .
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP		5 NP NP . PP
0 NP . Papa	1 VP . VP PP	2 NP . Papa		0 S NP VP .		2 NP NP PP .
0 Det . the	1 PP . P NP	2 Det . the		1 VP VP . PP		1 VP VP PP .
0 Det . a	1 V . ate	2 Det . a		4 PP . P NP		7 PP . P NP
	1 P . with			0 ROOT S .		1 VP V NP .
				4 P . with		2 NP NP . PP
						0 S NP VP .
						1 VP VP . PP
						7 P . with
						0 ROOT S .

0 Papa	1 ate	2 the	3 caviar	4 with a spoon 7		
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .	…	6 N spoon .
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .		5 NP Det N .
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .		4 PP P NP .
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP		5 NP NP . PP
0 NP . Papa	1 VP . VP PP	2 NP . Papa		0 S NP VP .		2 NP NP PP .
0 Det . the	1 PP . P NP	2 Det . the		1 VP VP . PP		1 VP VP PP .
0 Det . a	1 V . ate	2 Det . a		4 PP . P NP		7 PP . P NP
	1 P . with			0 ROOT S .		1 VP V NP .
				4 P . with		2 NP NP . PP
						0 S NP VP .
						1 VP VP . PP
						7 P . with
						0 ROOT S .

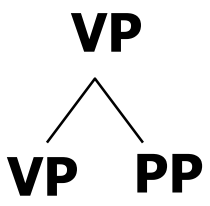
0 Papa	1 ate	2 the	3 caviar	4 with a spoon	7	
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .	...	6 N spoon .
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .		5 NP Det N .
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .		4 PP P NP .
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP		5 NP NP . PP
0 NP . Papa	1 VP . VP PP	2 NP . Papa		0 S NP VP .		2 NP NP PP .
0 Det . the	1 PP . P NP	2 Det . the		1 VP VP . PP		1 VP VP PP .
0 Det . a	1 V . ate	2 Det . a		4 PP . P NP		7 PP . P NP
	1 P . with			0 ROOT S .		1 VP V NP .
				4 P . with		2 NP NP . PP
						0 S NP VP .
						1 VP VP . PP
						7 P . with
						0 ROOT S .

0 Papa 1 ate 2 the 3 caviar 4 with a spoon 7						
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .	…	6 N spoon .
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .		5 NP Det N .
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .		4 PP P NP .
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP		5 NP NP . PP
0 NP . Papa	1 VP . VP PP	2 NP . Papa		0 S NP VP .		2 NP NP PP .
0 Det . the	1 PP . P NP	2 Det . the		1 VP VP . PP		1 VP VP PP .
0 Det . a	1 V . ate	2 Det . a		4 PP . P NP		7 PP . P NP
	1 P . with			0 ROOT S .		1 VP V NP .
				4 P . with		2 NP NP . PP
						0 S NP VP .
						1 VP VP . PP
						7 P . with
						0 ROOT S .

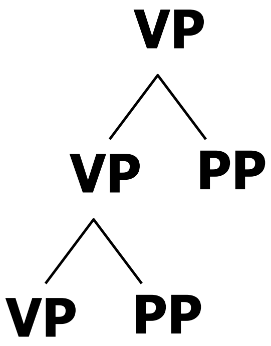
Left Recursion Kills Pure Top-Down Parsing …

VP

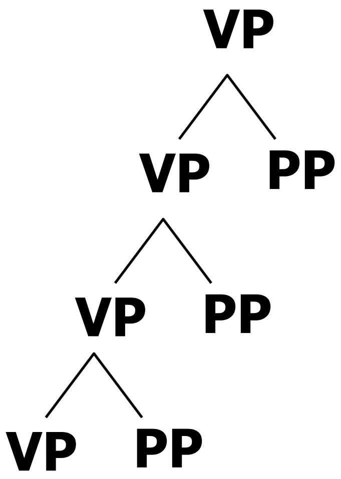
Left Recursion Kills Pure Top-Down Parsing …



Left Recursion Kills Pure Top-Down Parsing …



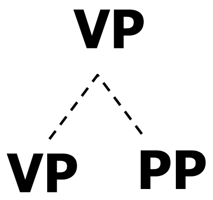
Left Recursion Kills Pure Top-Down Parsing …



makes new hypotheses ad infinitum before we've seen the PPs at all

hypotheses try to predict in advance how many PP's will arrive in input

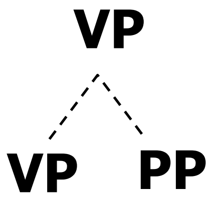
… but Earley's Alg is Okay!



1 VP → . VP PP

(in column 1)

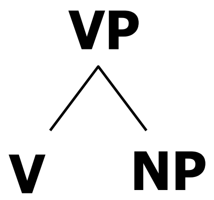
… but Earley's Alg is Okay!


VP

VP PP

1 VP → . VP PP

(in column 1)


VP

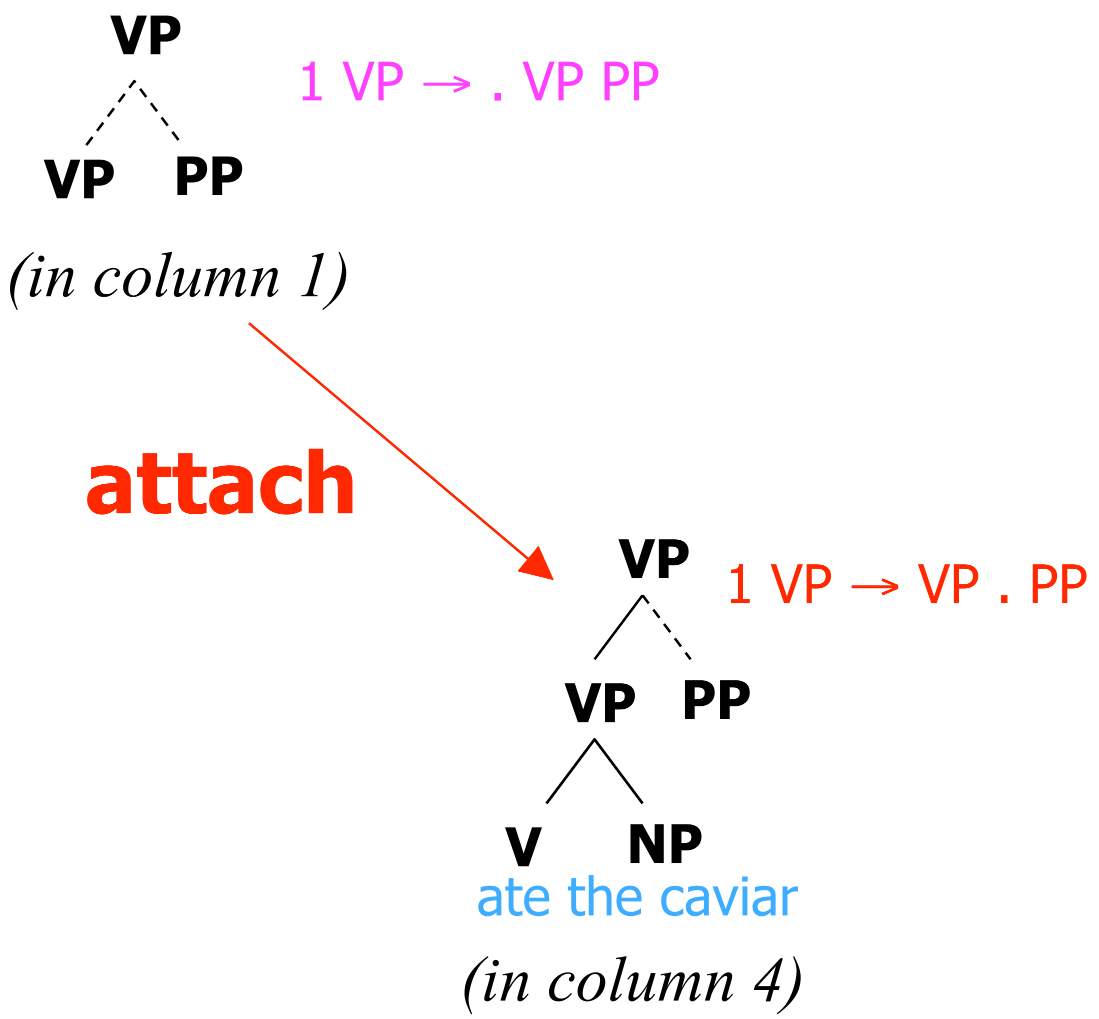
V NP

ate the caviar

1 VP → V NP .

(in column 4)

… but Earley's Alg is Okay!



VP

VP PP

1 VP → . VP PP

(in column 1)

attach

VP

1 VP → VP . PP

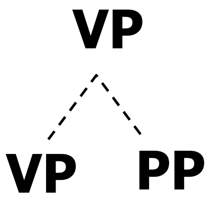
VP PP

V NP

ate the caviar

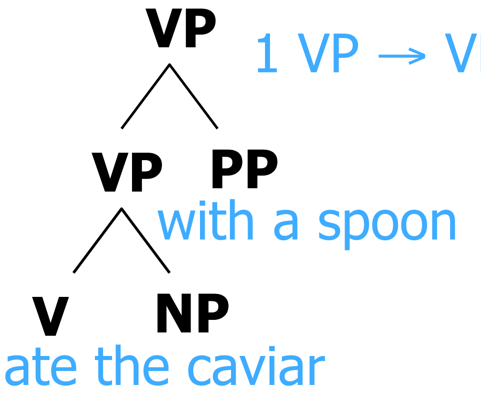
(in column 4)

… but Earley's Alg is Okay!



1 VP → . VP PP

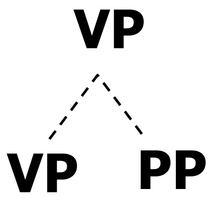
(in column 1)



1 VP → VP PP .

(in column 7)

… but Earley's Alg is Okay!



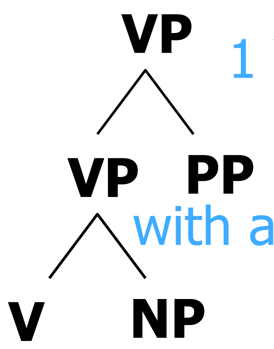
VP

VP PP

1 VP → . VP PP

can be reused

(in column 1)



VP

VP PP

V NP

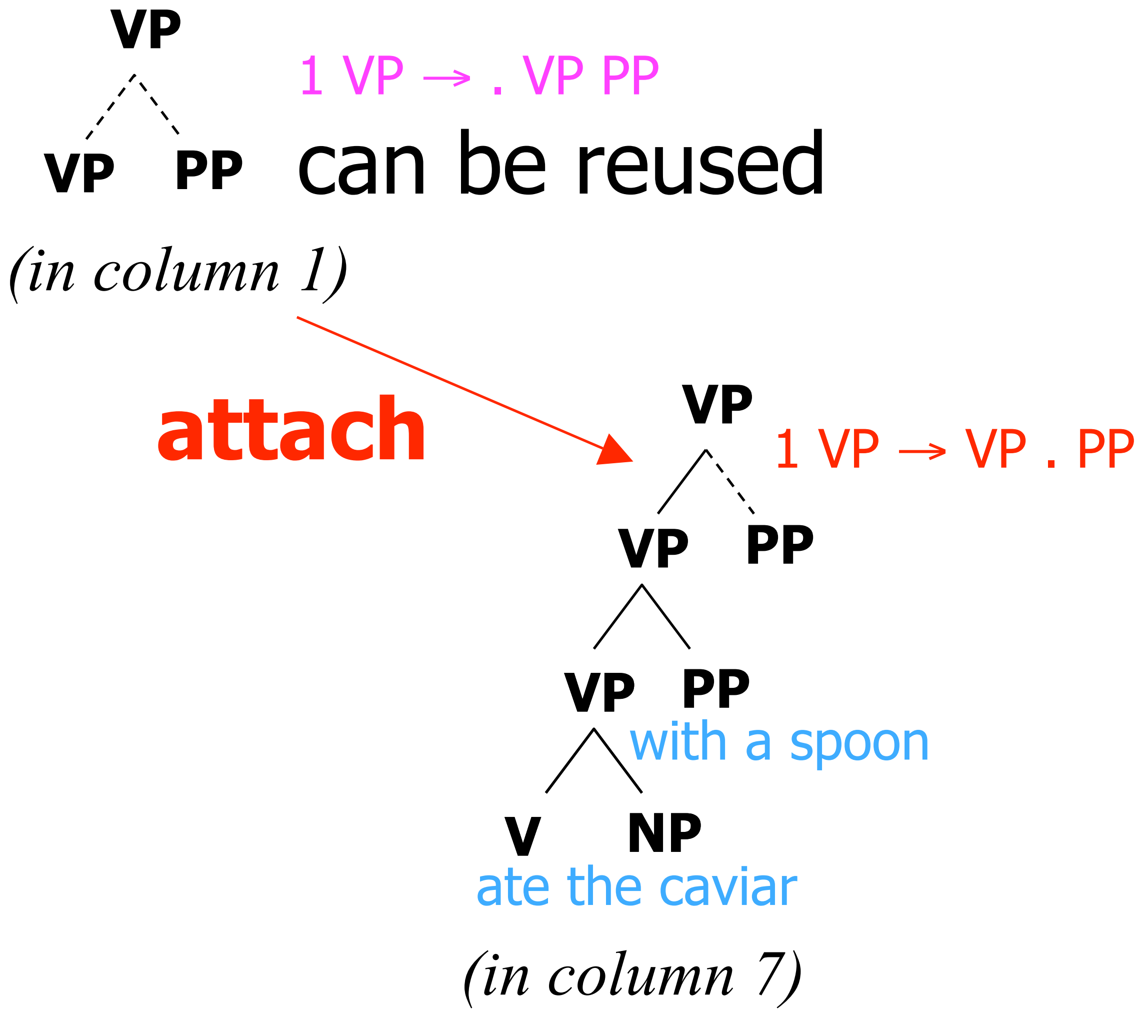
1 VP → VP PP .

with a spoon

ate the caviar

(in column 7)

… but Earley's Alg is Okay!



1 VP → . VP PP

can be reused

(in column 1)

attach

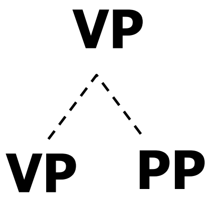
1 VP → VP . PP

with a spoon

ate the caviar

(in column 7)

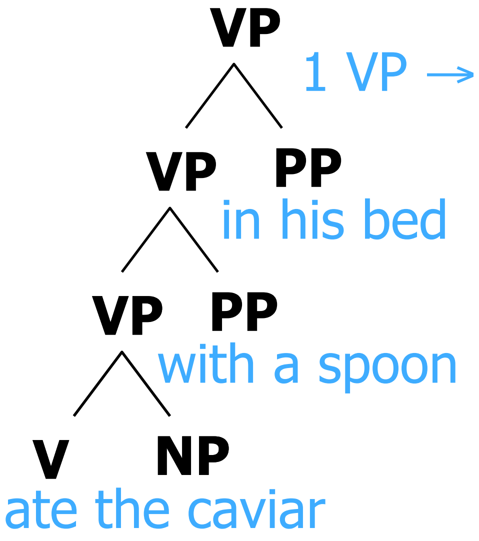
… but Earley's Alg is Okay!



1 VP → . VP PP

can be reused

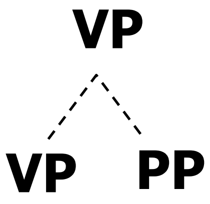
(in column 1)



1 VP → VP PP .

(in column 10)

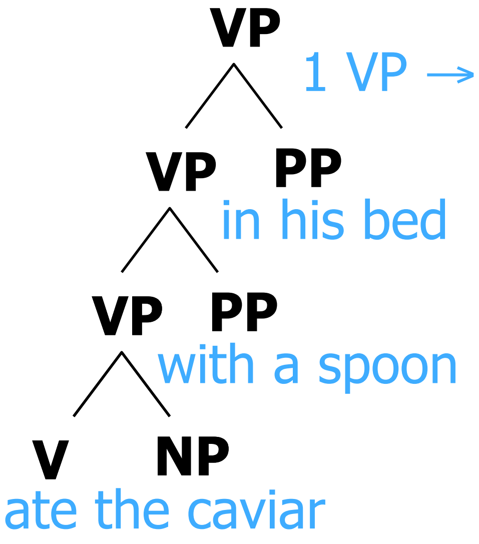
… but Earley's Alg is Okay!



1 VP → . VP PP

can be reused again

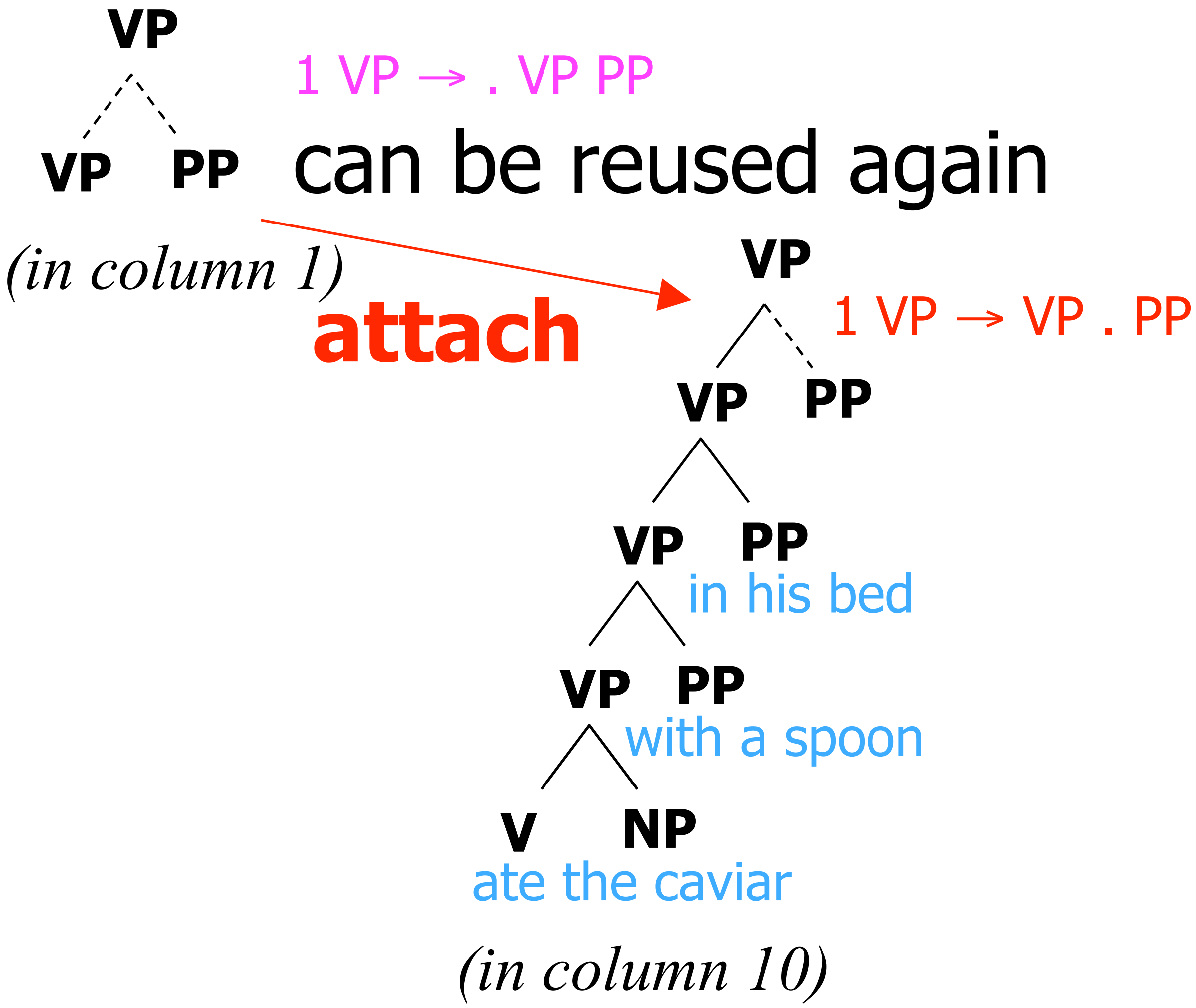
(in column 1)



1 VP → VP PP .

(in column 10)

… but Earley's Alg is Okay!



0 Papa	**1 ate**	**2 the**	**3 caviar**	**4 with a spoon**		**7**
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .	…	6 N spoon .
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .		5 NP Det N .
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .		4 PP P NP .
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP		5 NP NP . PP
0 NP . Papa	**1 VP . VP PP**	2 NP . Papa		0 S NP VP .		2 NP NP PP .
0 Det . the	1 PP . P NP	2 Det . the		**1 VP VP . PP**		1 VP VP PP .
0 Det . a	1 V . ate	2 Det . a		4 PP . P NP		7 PP . P NP
	1 P . with			0 ROOT S .		1 VP V NP .
				4 P . with		2 NP NP . PP
						0 S NP VP .
						1 VP VP . PP
						7 P . with
						0 ROOT S .

completed a VP in col 4
col 1 lets us use it in a VP PP structure

0 Papa	1 ate	2 the	3 caviar	4 with a spoon		7
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .	…	6 N spoon .
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .		5 NP Det N .
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .		4 PP P NP .
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP		5 NP NP . PP
0 NP . Papa	**1 VP . VP PP**	2 NP . Papa		0 S NP VP .		2 NP NP PP .
0 Det . the	1 PP . P NP	2 Det . the		1 VP VP . PP		1 VP VP PP .
0 Det . a	1 V . ate	2 Det . a		4 PP . P NP		7 PP . P NP
	1 P . with			0 ROOT S .		1 VP V NP .
				4 P . with		2 NP NP . PP
						0 S NP VP .
						1 VP VP . PP
						7 P . with
						0 ROOT S .

completed that VP = VP PP in col 7
col 1 would let us use *it* in a VP PP structure
can reuse col 1 as often as we need

Beyond Recognition

- So far, we've described an Earley *recognizer*
- Note what we did when we tried to create entries that already existed
- What should we do when combining items?
- How to derive outside algorithm?

Parsing Tricks

Left-Corner Parsing

- Technique for 1 word of lookahead in algorithms like Earley's

- (can also do multi-word lookahead but it's harder)

Basic Earley's Algorithm

0 Papa 1	
0 ROOT . S	0 NP Papa .
0 S . NP VP	0 S NP . VP
0 NP . Det N	0 NP NP . PP
0 NP . NP PP	
0 NP . Papa	
0 Det . the	
0 Det . a	

attach

0 Papa	1
0 ROOT . S	0 NP Papa .
0 S . NP VP	0 S NP . VP
0 NP . Det N	0 NP NP . PP
0 NP . NP PP	1 VP . V NP
0 NP . Papa	1 VP . VP PP
0 Det . the	
0 Det . a	

predict

0 Papa 1	
0 ROOT . S	0 NP Papa .
0 S . NP VP	0 S NP . VP
0 NP . Det N	0 NP NP . PP
0 NP . NP PP	1 VP . V NP
0 NP . Papa	1 VP . VP PP
0 Det . the	1 PP . P NP
0 Det . a	

predict

0 Papa 1	
0 ROOT . S	0 NP Papa .
0 S . NP VP	0 S NP . VP
0 NP . Det N	0 NP NP . PP
0 NP . NP PP	1 VP . V NP
0 NP . Papa	1 VP . VP PP
0 Det . the	1 PP . P NP
0 Det . a	1 V . ate
	1 V . drank
	1 V . snorted

predict

0 Papa 1	
0 ROOT . S	0 NP Papa .
0 S . NP VP	0 S NP . VP
0 NP . Det N	0 NP NP . PP
0 NP . NP PP	1 VP . V NP
0 NP . Papa	1 VP . VP PP
0 Det . the	1 PP . P NP
0 Det . a	1 V . ate
	1 V . drank
	1 V . snorted

predict

0 Papa 1	
0 ROOT . S	0 NP Papa .
0 S . NP VP	0 S NP . VP
0 NP . Det N	0 NP NP . PP
0 NP . NP PP	1 VP . V NP
0 NP . Papa	1 VP . VP PP
0 Det . the	1 PP . P NP
0 Det . a	1 V . ate
	1 V . drank
	1 V . snorted

predict

- .V makes us add all the verbs in the vocabulary!
- **Slow** – we'd like a shortcut.

0 **Papa** 1	
0 ROOT . S	0 NP Papa .
0 S . NP VP	0 S NP . VP
0 NP . Det N	0 NP NP . PP
0 NP . NP PP	1 VP . V NP
0 NP . Papa	1 VP . VP PP
0 Det . the	1 PP . P NP
0 Det . a	1 V . ate
	1 V . drank
	1 V . snorted

predict

0 Papa 1	
0 ROOT . S	0 NP Papa .
0 S . NP VP	0 S NP . VP
0 NP . Det N	0 NP NP . PP
0 NP . NP PP	1 VP . V NP
0 NP . Papa	1 VP . VP PP
0 Det . the	1 PP . P NP
0 Det . a	1 V . ate
	1 V . drank
	1 V . snorted

predict

0 **Papa** 1	
0 ROOT . S	0 NP Papa .
0 S . NP VP	0 S NP . VP
0 NP . Det N	0 NP NP . PP
0 NP . NP PP	1 VP . V NP
0 NP . Papa	1 VP . VP PP
0 Det . the	1 PP . P NP
0 Det . a	1 V . ate
	1 V . drank
	1 V . snorted

predict

- Every .VP adds all VP → ... rules again.
- Before adding a rule, check it's not a duplicate.
- **Slow** if there are > 700 VP → ... rules, so what will you do in Homework 3?

0 Papa 1	
0 ROOT . S	0 NP Papa .
0 S . NP VP	0 S NP . VP
0 NP . Det N	0 NP NP . PP
0 NP . NP PP	1 VP . V NP
0 NP . Papa	1 VP . VP PP
0 Det . the	1 PP . P NP
0 Det . a	1 V . ate
	1 V . drank
	1 V . snorted
	1 P . with

predict

0 Papa 1	
0 ROOT . S	0 NP Papa .
0 S . NP VP	0 S NP . VP
0 NP . Det N	0 NP NP . PP
0 NP . NP PP	1 VP . V NP
0 NP . Papa	1 VP . VP PP
0 Det . the	1 PP . P NP
0 Det . a	1 V . ate
	1 V . drank
	1 V . snorted
	1 P . with

predict

- .P makes us add all the prepositions …

1-word lookahead would help

0 Papa 1 ate

0 ROOT . S	0 NP Papa .
0 S . NP VP	0 S NP . VP
0 NP . Det N	0 NP NP . PP
0 NP . NP PP	1 VP . V NP
0 NP . Papa	1 VP . VP PP
0 Det . the	1 PP . P NP
0 Det . a	1 V . ate
	1 V . drank
	1 V . snorted
	1 P . with

1-word lookahead would help

0 Papa	1 ate
0 ROOT . S	0 NP Papa .
0 S . NP VP	0 S NP . VP
0 NP . Det N	0 NP NP . PP
0 NP . NP PP	1 VP . V NP
0 NP . Papa	1 VP . VP PP
0 Det . the	1 PP . P NP
0 Det . a	1 V . ate
	~~1 V . drank~~
	~~1 V . snorted~~
	~~1 P . with~~

No point in adding words other than ate

1-word lookahead would help

0 Papa	1
0 ROOT . S	0 NP Papa .
0 S . NP VP	0 S NP . VP
0 NP . Det N	~~0 NP NP . PP~~
0 NP . NP PP	1 VP . V NP
0 NP . Papa	1 VP . VP PP
0 Det . the	~~1 PP . P NP~~
0 Det . a	1 V . ate
	~~1 V . drank~~
	~~1 V . snorted~~
	~~1 P . with~~

ate

In fact, no point in adding any constituent that can't start with ate

Don't bother adding PP, P, etc.

No point in adding words other than ate

With Left-Corner Filter

0 Papa	1
0 ROOT . S	0 NP Papa .
0 S . NP VP	0 S NP . VP
0 NP . Det N	~~0 NP NP . PP~~
0 NP . NP PP	
0 NP . Papa	
0 Det . the	
0 Det . a	

ate

attach

With Left-Corner Filter

0 Papa	1
0 ROOT . S	0 NP Papa .
0 S . NP VP	0 S NP . VP
0 NP . Det N	~~0 NP NP . PP~~
0 NP . NP PP	
0 NP . Papa	
0 Det . the	
0 Det . a	

ate

attach

PP can’t start with ate

With Left-Corner Filter

0 Papa	1	ate
0 ROOT . S	0 NP Papa .	
0 S . NP VP	0 S NP . VP	
0 NP . Det N	~~0 NP NP . PP~~	
0 NP . NP PP		
0 NP . Papa		
0 Det . the		
0 Det . a		

attach

PP can’t start with ate

Pruning– now we won’t predict

1 PP . P NP

1 PP . ate

With Left-Corner Filter

0 Papa 1 ate

0 ROOT . S	0 NP Papa .
0 S . NP VP	0 S NP . VP
0 NP . Det N	~~0 NP NP . PP~~
0 NP . NP PP	
0 NP . Papa	
0 Det . the	
0 Det . a	

attach

PP can’t start with ate

Pruning– now we won’t predict

1 PP . P NP

1 PP . ate

either!

With Left-Corner Filter

0 Papa 1 ate

0 ROOT . S	0 NP Papa .
0 S . NP VP	0 S NP . VP
0 NP . Det N	~~0 NP NP . PP~~
0 NP . NP PP	
0 NP . Papa	
0 Det . the	
0 Det . a	

attach

PP can’t start with ate

Pruning– now we won’t predict

1 PP . P NP

1 PP . ate

either!

0 Papa	1
0 ROOT . S	0 NP Papa .
0 S . NP VP	0 S NP . VP
0 NP . Det N	~~0 NP NP . PP~~
0 NP . NP PP	1 VP . V NP
0 NP . Papa	1 VP . VP PP
0 Det . the	
0 Det . a	

ate

predict

0 Papa	1
0 ROOT . S	0 NP Papa .
0 S . NP VP	0 S NP . VP
0 NP . Det N	~~0 NP NP . PP~~
0 NP . NP PP	1 VP . V NP
0 NP . Papa	1 VP . VP PP
0 Det . the	1 V . ate
0 Det . a	~~1 V . drank~~
	~~1 V . snorted~~

ate

predict

0 Papa 1 ate

0 ROOT . S	0 NP Papa .
0 S . NP VP	0 S NP . VP
0 NP . Det N	~~0 NP NP . PP~~
0 NP . NP PP	1 VP . V NP
0 NP . Papa	1 VP . VP PP
0 Det . the	1 V . ate
0 Det . a	~~1 V . drank~~
	~~1 V . snorted~~

predict

0 Papa	1	ate
0 ROOT . S	0 NP Papa .	
0 S . NP VP	0 S NP . VP	
0 NP . Det N	~~0 NP NP . PP~~	
0 NP . NP PP	1 VP . V NP	
0 NP . Papa	1 VP . VP PP	
0 Det . the	1 V . ate	
0 Det . a	~~1 V . drank~~	
	~~1 V . snorted~~	

predict

Merging Right-Hand Sides

- Grammar might have rules

  X → A G H P

  X → B G H P

- Could end up with both of these in chart:

  (2, X → A . G H P) in column 5

  (2, X → B . G H P) in column 5

- But these are now interchangeable: if one produces X then so will the other
- To avoid this redundancy, can always use dotted rules of this form: **X → ... G H P**

Merging Right-Hand Sides

- Similarly, grammar might have rules

  X → A G H P

  X → A G H Q

- Could end up with both of these in chart:

  (2, X → A . G H P) in column 5

  (2, X → A . G H Q) in column 5

- Not interchangeable, but we'll be processing them in parallel for a while …
- Solution: write grammar as **X → A G H (P|Q)**

Merging Right-Hand Sides

- Combining the two previous cases:

  X → A G H P

  X → A G H Q

  X → B G H P

  X → B G H Q

becomes

  X → (A | B) G H (P | Q)

- And often nice to write stuff like

  NP → (Det | ε) Adj* N

Merging Right-Hand Sides

Merging Right-Hand Sides

X → (A | B) G H (P | Q)

NP → (Det | ε) Adj* N

Merging Right-Hand Sides

X → (A | B) G H (P | Q)

NP → (Det | ε) Adj* N

- These are regular expressions!

Merging Right-Hand Sides

X → (A | B) G H (P | Q)

NP → (Det | ε) Adj* N

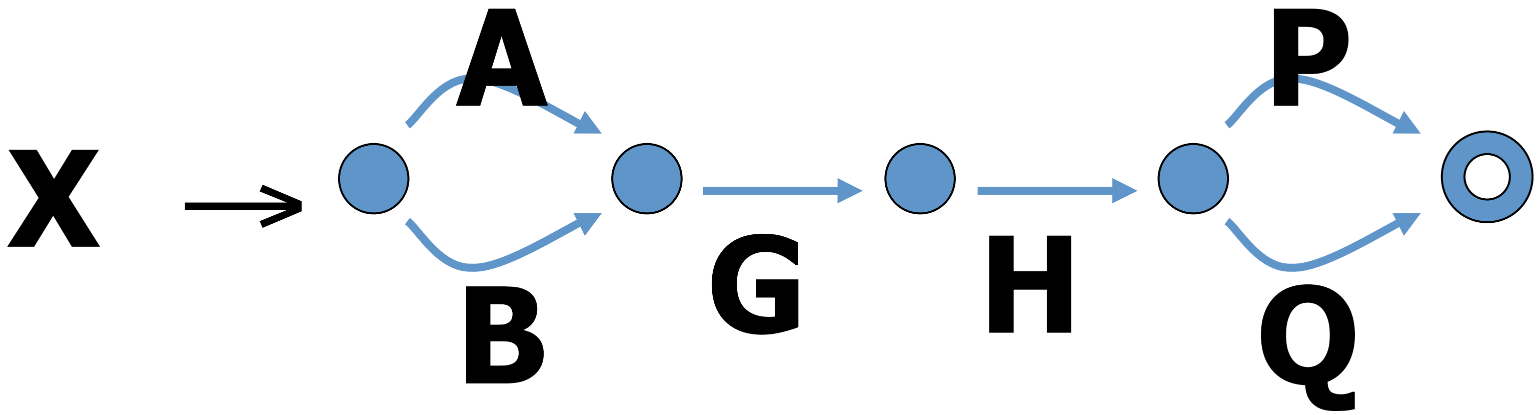
- These are regular expressions!
- Build their minimal DFAs:

Merging Right-Hand Sides

X → (A | B) G H (P | Q)

NP → (Det | ε) Adj* N

- These are regular expressions!
- Build their minimal DFAs:

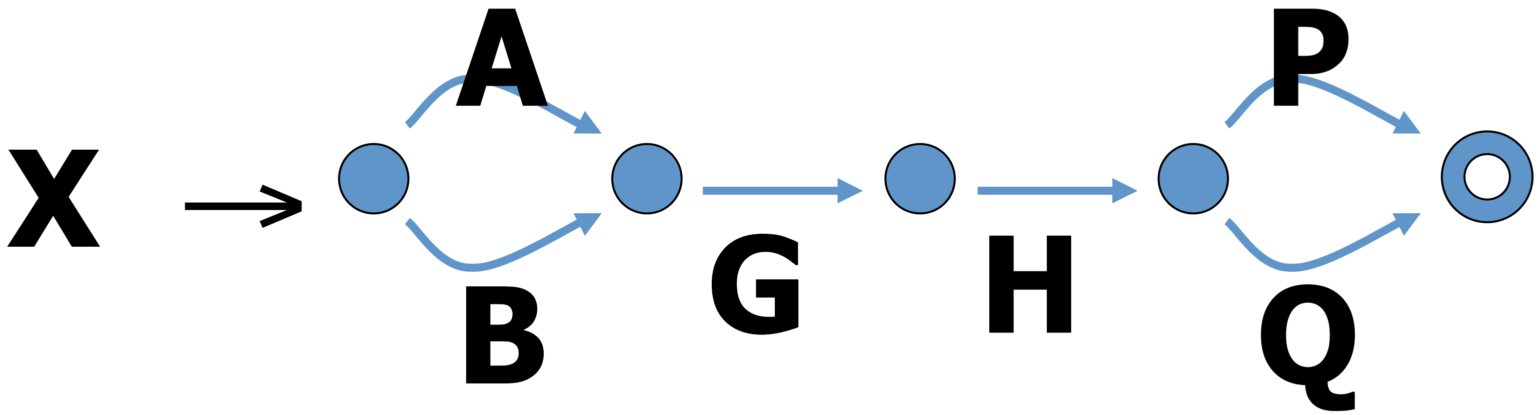


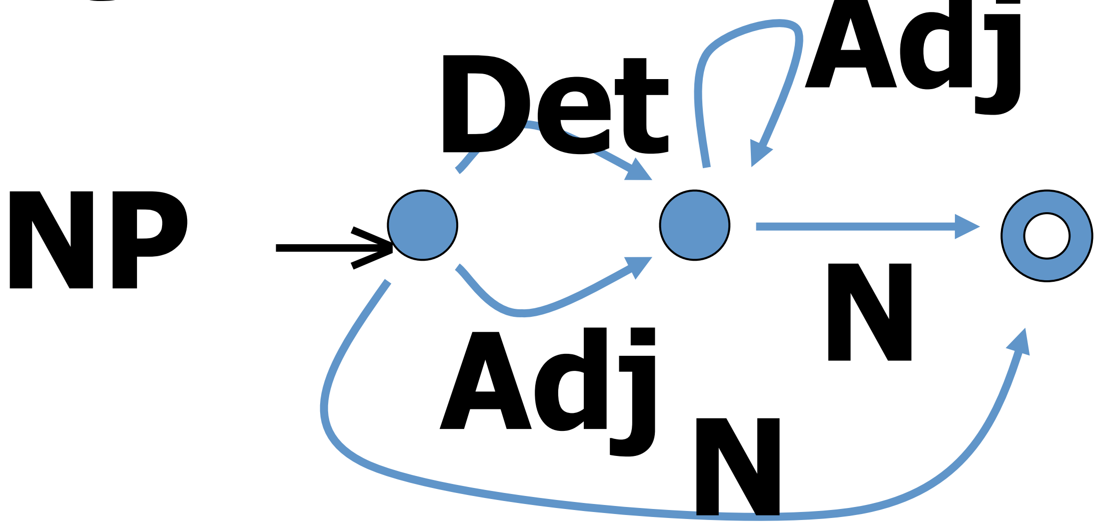
Merging Right-Hand Sides

X → (A | B) G H (P | Q)

NP → (Det | ε) Adj* N

- These are regular expressions!
- Build their minimal DFAs:





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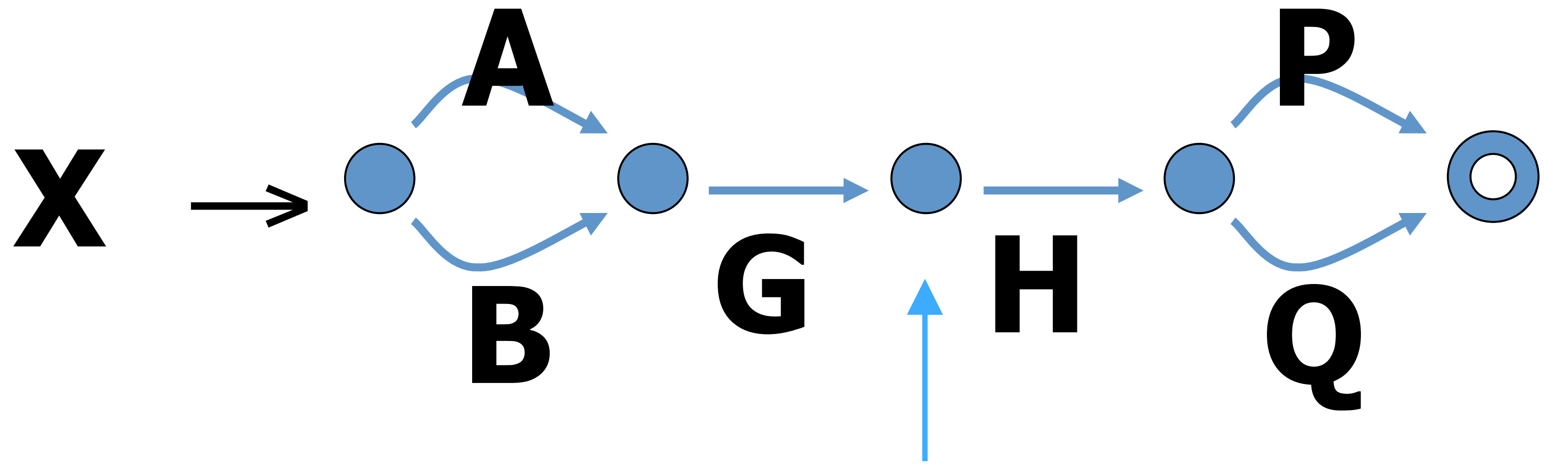
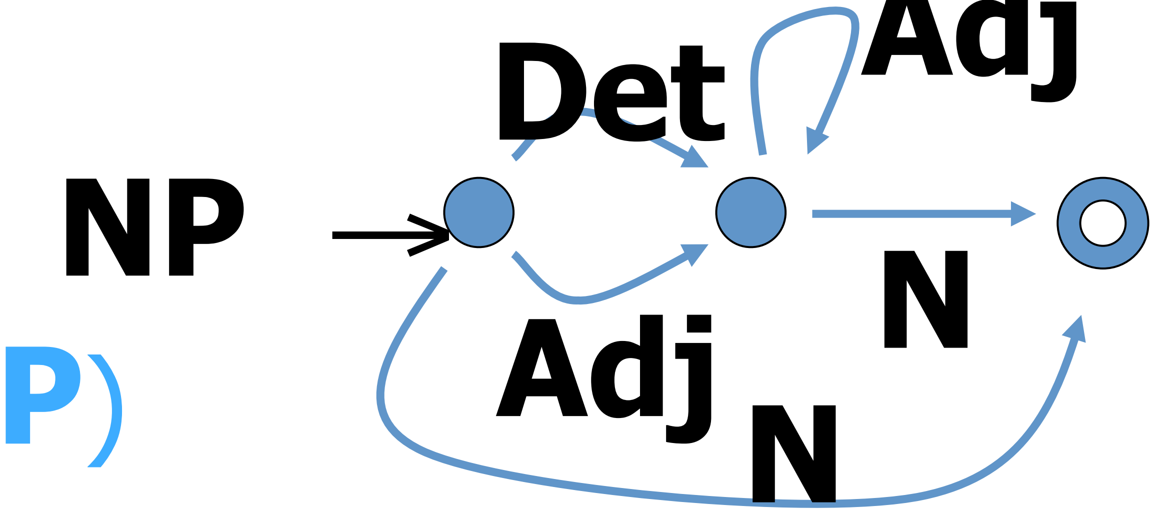


- Automaton states replace dotted rules (**X → A G . H P**)



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Indeed, *all* **NP** → rules can be unioned into a single DFA!

NP → ADJP ADJP JJ JJ NN NNS
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NP → ADJP JJ NN
NP → ADJP JJ NN NNS
NP → ADJP JJ NNS
NP → ADJP NN
NP → ADJP NN NN
NP → ADJP NN NNS
NP → ADJP NNS
NP → ADJP NPR
NP → ADJP NPRS
NP → DT
NP → DT ADJP
NP → DT ADJP , JJ NN
NP → DT ADJP ADJP NN
NP → DT ADJP JJ JJ NN
NP → DT ADJP JJ NN
NP → DT ADJP JJ NN NN

etc

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```
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regular expression

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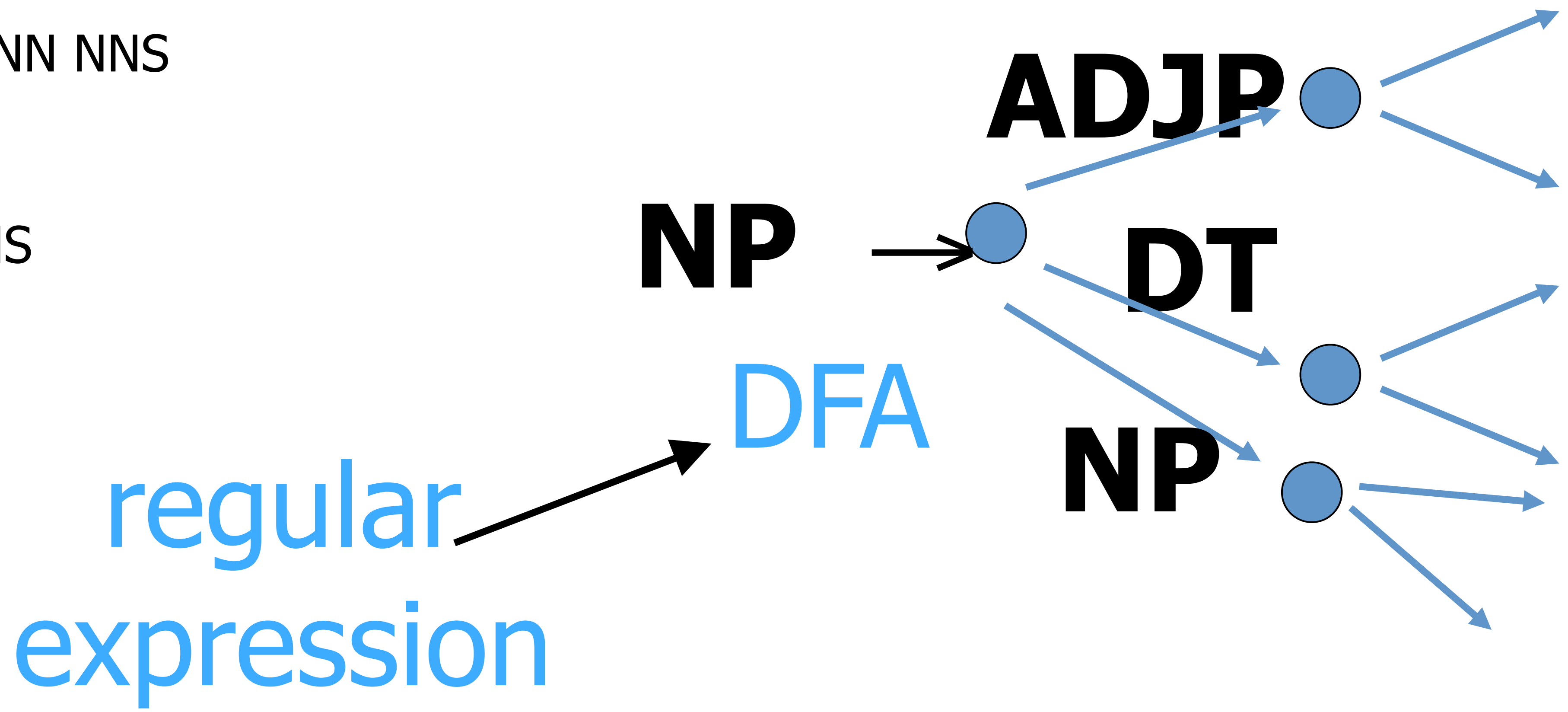

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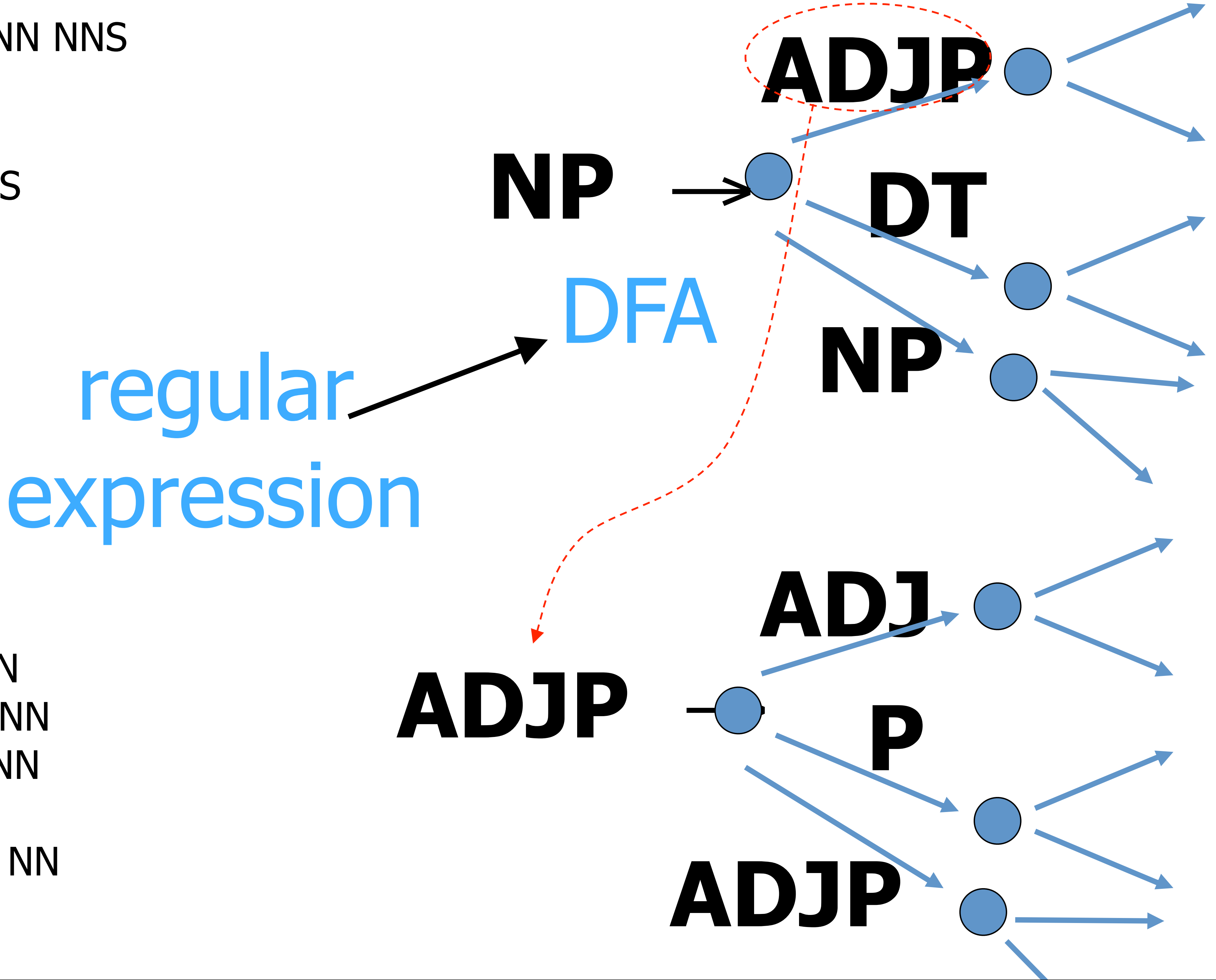

regular expression → DFA

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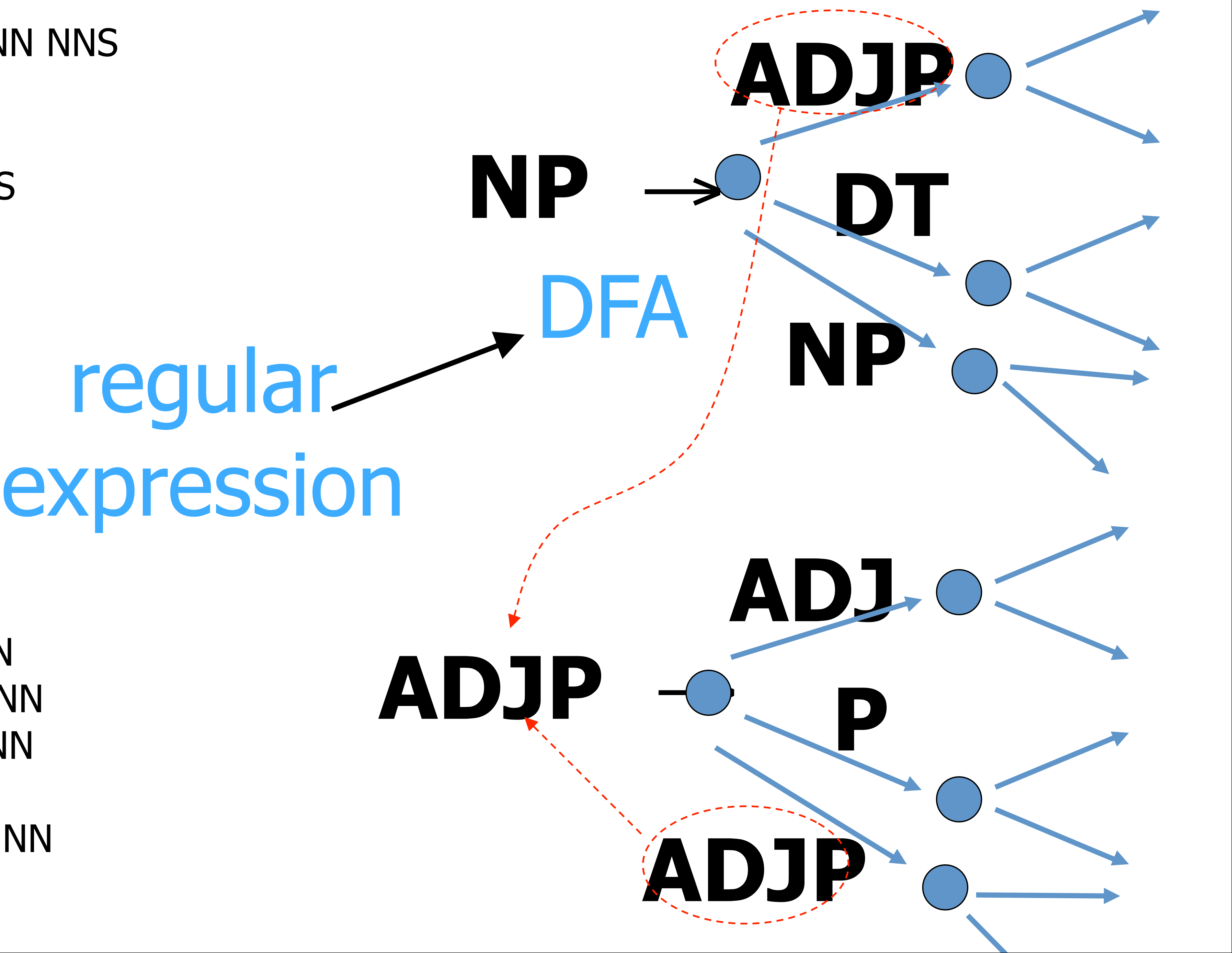
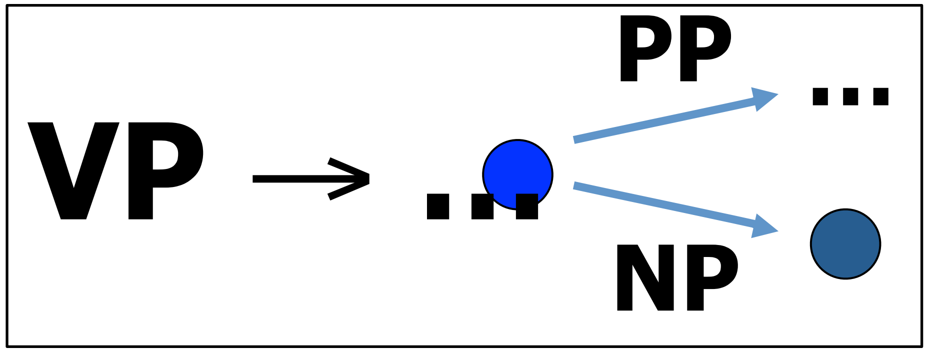


Earley's Algorithm on DFAs

- What does Earley's algorithm now look like?



Column 4
…
(2,●)

predict

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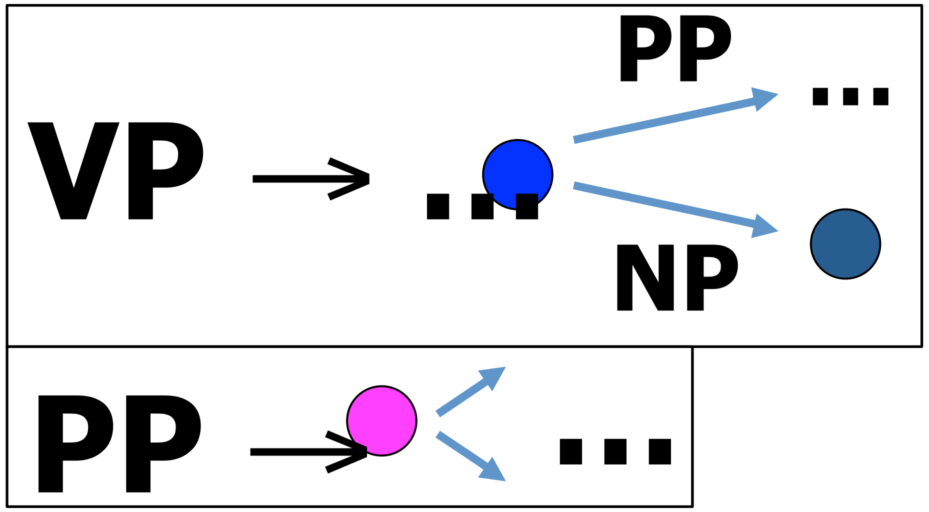

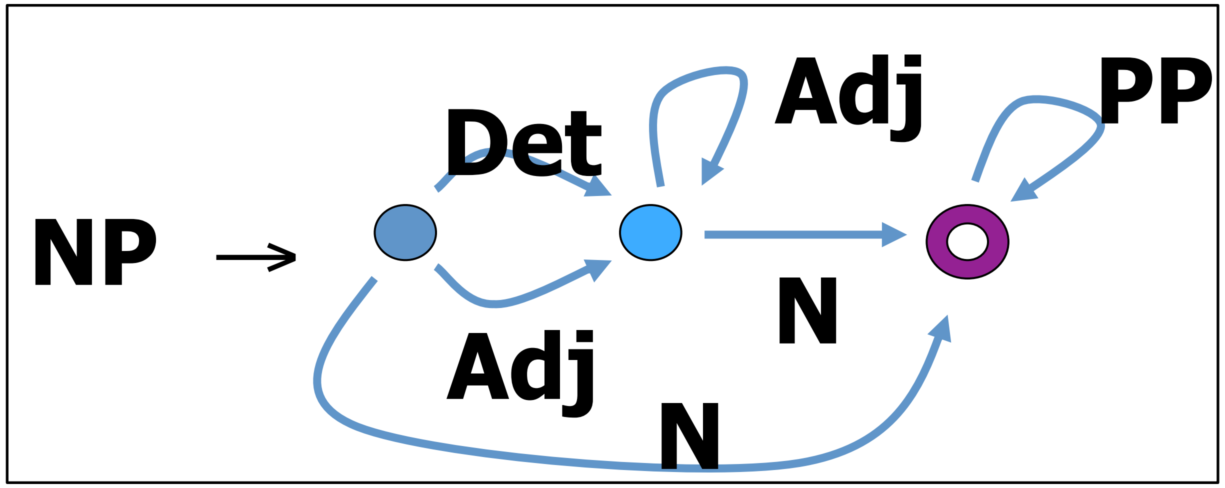

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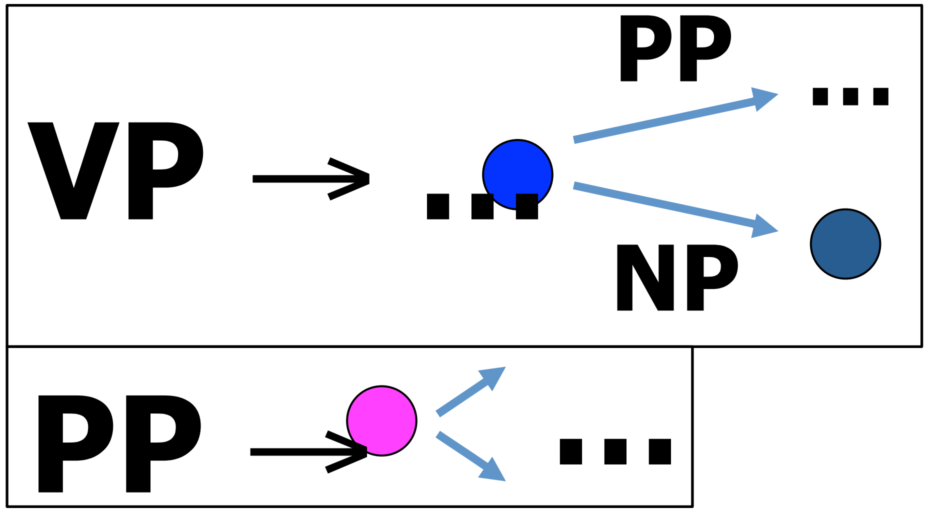

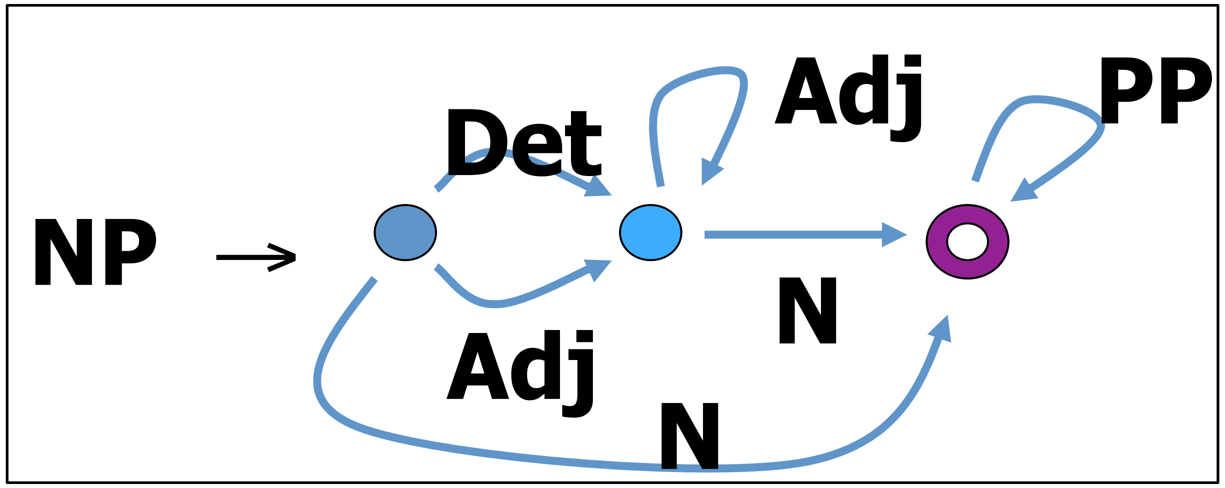

Column 4	Column 5	…	Column 7
…	…		
(2,●)			(4,◉)
(4,●)			
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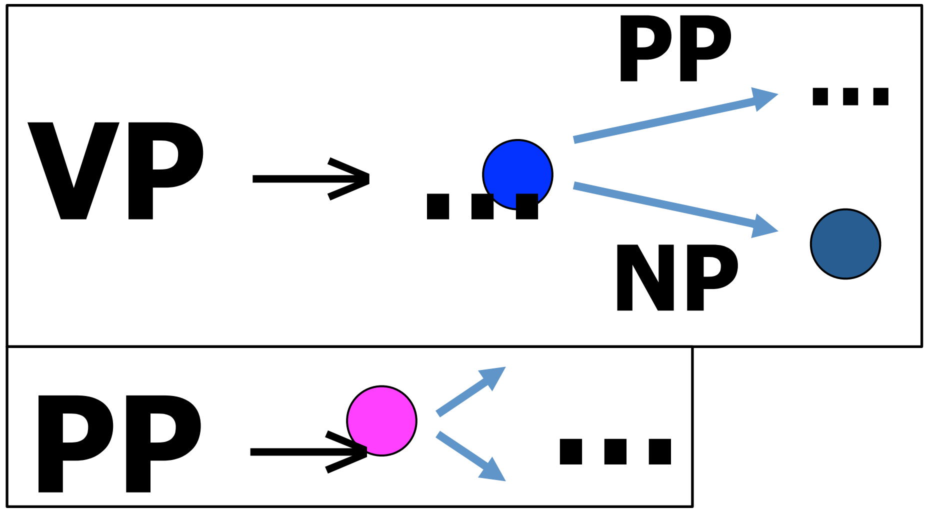

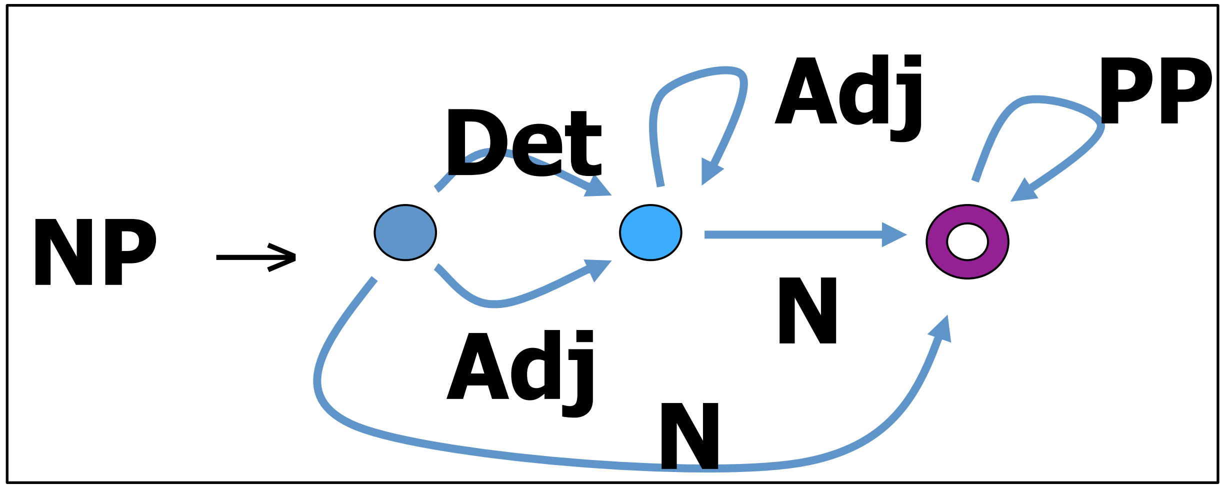

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Both!

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    for some y that spans the same set of words
  - Throw x away if $p(x)*q(x)$ is small, where $q(x)$ is an estimate of probability of all rules needed to combine x with the other words in the sentence

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  - if priorities are defined carefully, obtain an A* algorithm
- Put each constit on a priority queue (heap)
- Repeatedly pop and process best constituent.
  - CKY style: combine w/ previously popped neighbors.
  - Earley style: scan/predict/attach as usual. What else?

Preprocessing

Preprocessing

- First "tag" the input with parts of speech:
  - Guess the correct preterminal for each word, using faster methods we'll learn later
  - Now only allow one part of speech per word
  - This eliminates a lot of crazy constituents!
  - But if you tagged wrong you could be hosed

- Raise the stakes:
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- Raise the stakes:
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- Safer to allow a few possible tags per word, not just one …

Center-Embedding

```
if x
then
    if y
    then
    if a
    then b
    endif
    else b
    endif
else b
endif
```

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STATEMENT → if EXPR then
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STATEMENT → if EXPR then STATEMENT else STATEMENT endif

Center-Embedding

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- This is the rat that ate the malt.

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- This is the rat that ate the malt.
- This is the malt that the rat ate.

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Center-Embedding

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- This is the dog that chased the cat that bit the rat that ate the malt.

Center-Embedding

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- This is the cat that bit the rat that ate the malt.
- This is the malt that the rat that the cat bit ate.

- This is the dog that chased the cat that bit the rat that ate the malt.
- This is the malt that [the rat that [the cat that [the dog chased] bit] ate].

More Center-Embedding

[What did you disguise

[those handshakes that you greeted

[the people we bought

[the bench

[Billy was read to]

on]

with]

with]

for]?

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with]
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[Which mantelpiece did you put
[the idol I sacrificed
[the fellow we sold
[the bridge you threw
[the bench
[Billy was read to]

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Take that, English teachers!

Center Recursion vs. Tail Recursion

[What did you disguise
[those handshakes that you greeted
[the people we bought
[the bench
[Billy was read to]
on]
with]
with]
for]?

[For what did you disguise
[those handshakes with which you greeted
[the people with which we bought
[the bench on which
[Billy was read to]?

“pied piping” –
NP moves leftward,
preposition follows along

Disallow Center-Embedding?

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- Center-embedding seems to be in the grammar, but people have trouble processing more than 1 level of it.
- You can limit # levels of center-embedding via features: e.g., S[S_DEPTH=n+1] → A S[S_DEPTH=n] B

- If a CFG limits # levels of embedding, then it can be compiled into a finite-state machine – we don't need a stack at all!
  - Finite-state recognizers run in linear time.
  - However, it's tricky to turn them into parsers for the original CFG from which the recognizer was compiled.

Overview

- Treebanks and evaluation
- Lexicalized parsing (with heads)
    - Examples: Collins

Treebanks

- Pure Grammar Induction Approaches tend not to produce the parse trees that people want

- Solution
  - Give a some example of parse trees that we want
  - Make a learning tool learn a grammar

- Treebank
  - A collection of such example parses
  - PennTreebank is most widely used

Treebanks

- Penn Treebank
  - Trees are represented via bracketing
  - Fairly flat structures for Noun Phrases

    (NP Arizona real estate loans)
  - Tagged with grammatical and semantic functions

    (-SBJ , –LOC, …)
  - Use empty nodes(*) to indicate understood subjects and extraction gaps

```
(  ( S ( NP-SBJ  The move)
    ( VP  followed
        ( NP  ( NP a round )
            ( PP  of
               (NP  ( NP similar increases )
                   ( PP by
                      ( NP other lenders ) )
                   ( PP against
                      ( NP Arizona real estate loans )))))
        ,
        ( S-ADV ( NP-SBJ * )
              ( VP  reflecting
                   ( NP a continuing decline )
                   ( PP-LOC  in
                           (NP  that market ))))))
  . )
```

Treebanks

- Many people have argued that it is better to have linguists constructing treebanks than grammars

- Because it is easier
  - to work out the correct parse of sentences
- than
  - to try to determine what all possible manifestations of a certain rule or grammatical construct are

Parser Evaluation

Evaluation

Ultimate goal is to build system for IE, QA, MT

- People are rarely interested in syntactic analysis for its own sake
- Evaluate the system for evaluate the parser

For Simplicity and modularization, and Convenience

- Compare parses from a parser with the result of hand parsing of a sentence(gold standard)

What is objective criterion that we are trying to maximize?

Evaluation

Tree Accuracy (Exact match)

- It is a very tough standard!!!
- But in many ways it is a sensible one to use

PARSEVAL Measures

- For some purposes, partially correct parses can be useful
- Originally for non-statistical parsers
- Evaluate the component pieces of a parse
- Measures : Precision, Recall, Crossing brackets

Evaluation

(Labeled) Precision

How many brackets in the parse match those in the correct tree (Gold standard)?

(Labeled) Recall

How many of the brackets in the correct tree are in the parse?

Crossing brackets

Average of how many constituents in one tree cross over constituent boundaries in the other tree

```
B1                     (                              )
B2             (                      )
B3    (                                                       )
B4                                             (              )
          w1      w2      w3      w4      w5      w6      w7      w8
```

Problems with PARSEVAL

Even vanilla PCFG performs quite well

It measures success at the level of individual decisions

You must make many consecutive decisions correctly to be correct on the entire tree.

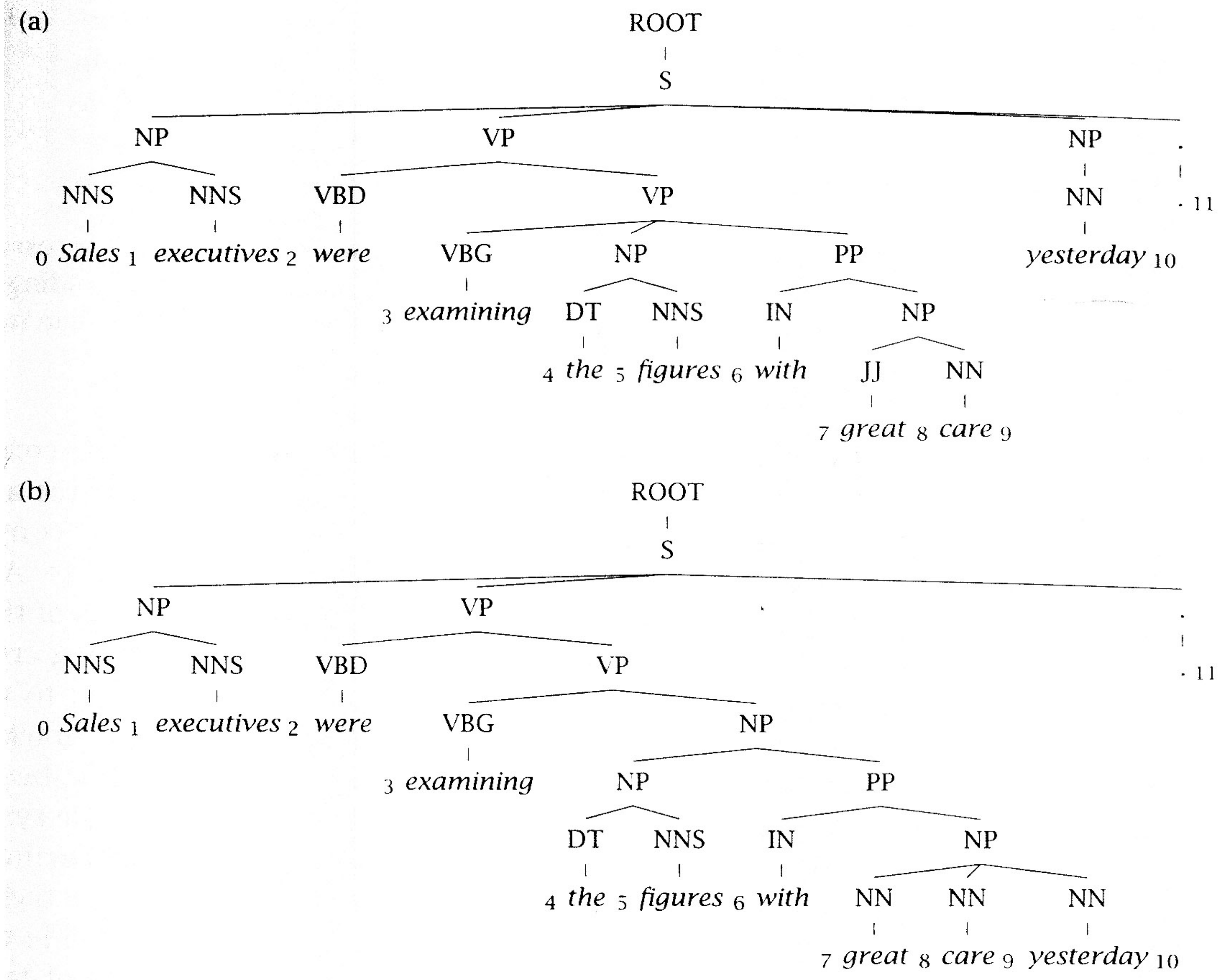
Problems with PARSEVAL (2)

Behind story

- The structure of Penn Treebank
  - Flat → Few brackets → Low Crossing brackets
  - Troublesome brackets are avoided
    - → High Precision/Recall
  - The errors in precision and recall are minimal

In some cases wrong PP attachment penalizes Precision, Recall and Crossing Bracket Accuracy minimally.

On the other hand, attaching low instead of high, then every node in the right-branching tree will be wrong: serious harm



(c) Brackets in gold standard tree (a.):
S-(0:11), NP-(0:2), VP-(2:9), VP-(3:9), **NP-(4:6),** PP-(6-9), NP-(7,9), *NP-(9:10)

(d) Brackets in candidate parse (b.):
S-(0:11), NP-(0:2), VP-(2:10), VP-(3:10), NP-(4:10), **NP-(4:6),** PP-(6-10), NP-(7,10) ?

(e)	Precision:	3/8 = 37.5%	Crossing Brackets:	3
	Recall:	3/8 = 37.5%	Crossing Accuracy:	62%
	Labeled Precision:	3/8 = 37.5%	Tagging Accuracy:	10/11 = 90.9%
	Labeled Recall:	3/8 = 37.5%		

Evaluation

Do PARSEVAL measures succeed in real tasks?

- Many small parsing mistakes might not affect tasks of semantic interpretation
  - (Bonnema 1996,1997)
    - Tree Accuracy of the Parser : 62%
    - Correct Semantic Interpretations : 88%
  - (Hermajakob and Mooney 1997)
    - English to German translation

At the moment, people feel PARSEVAL measures are adequate for the comparing parsers

Lexicalized Parsing

Limitations of PCFGs

- PCFGs assume:
  - Place invariance
  - Context free: P(rule) independent of
    - words outside span
    - *also, words with overlapping derivation*
  - Ancestor free: P(rule) independent of
    - *Non-terminals above.*

- Lack of sensitivity to lexical information
- Lack of sensitivity to structural frequencies

Lack of Lexical Dependency

Means that

P(VP → V NP NP)

is independent of the particular verb involved!

... but much more likely with ditransitive verbs (like ***gave***).

He ***gave*** *the boy a ball.*

He ***ran*** *to the store.*

The Need for Lexical Dependency

Probabilities dependent on Lexical words

Problem 1 : Verb subcategorization

VP expansion is independent of the choice of verb

However …

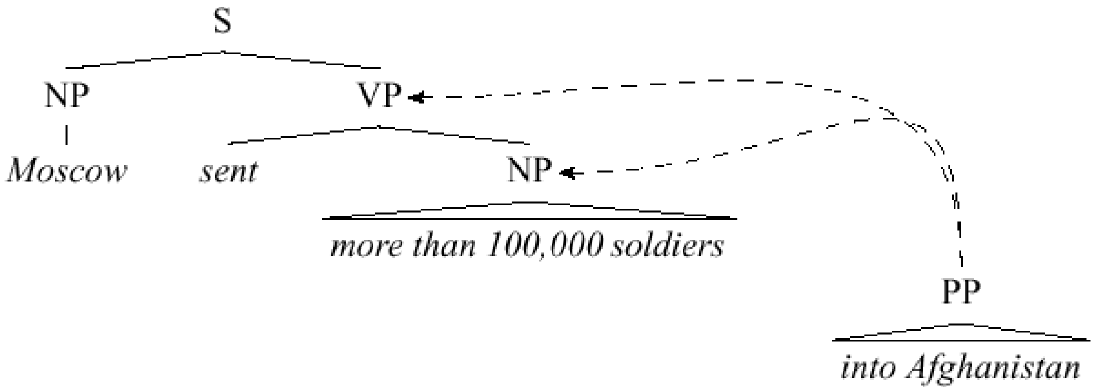
	verb			
	come	take	think	want
VP -> V	9.5%	2.6%	4.6%	5.7%
VP -> V NP	1.1%	32.1%	0.2%	13.9%
VP -> V PP	34.5%	3.1%	7.1%	0.3%
VP -> V SBAR	6.6%	0.3%	73.0%	0.2%
VP -> V S	2.2%	1.3%	4.8%	70.8%

Including actual words information when making decisions about tree structure is necessary

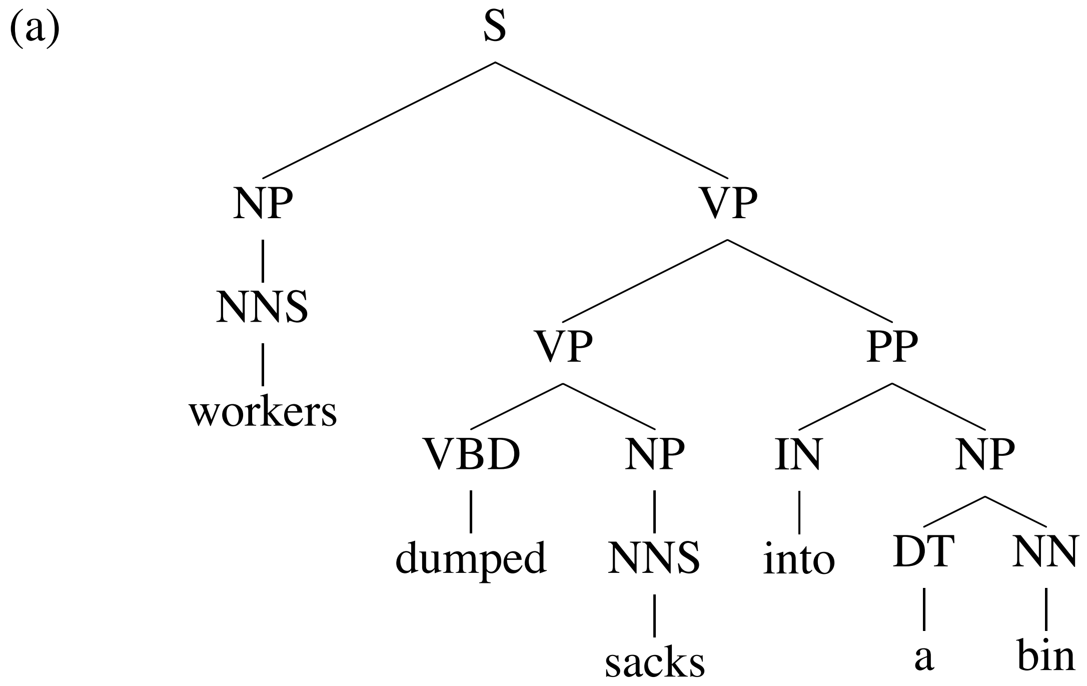
Weakening the independence assumption of PCFG

Probabilities dependent on Lexical words

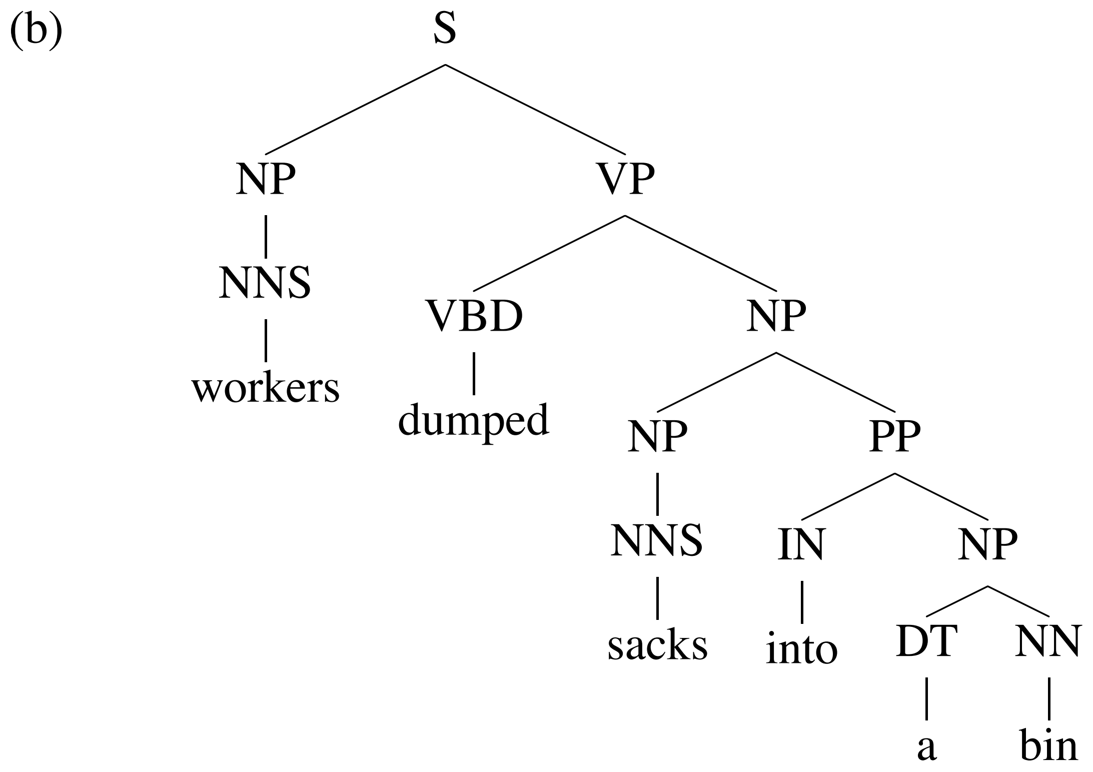
- Problem 2 : Phrasal Attachment
  - Lexical content of phrases provide information for decision
  - Syntactic category of the phrases provide very little information
  - Standard PCFG is worse than n-gram models



Another case of PP attachment ambiguity



Another case of PP attachment ambiguity



Another case of PP attachment ambiguity

(a)

Rules
S → NP VP
NP → NNS
VP → VP PP
VP → VBD NP
NP → NNS
PP → IN NP
NP → DT NN
NNS → workers
VBD → dumped
NNS → sacks
IN → into
DT → a
NN → bin

(b)

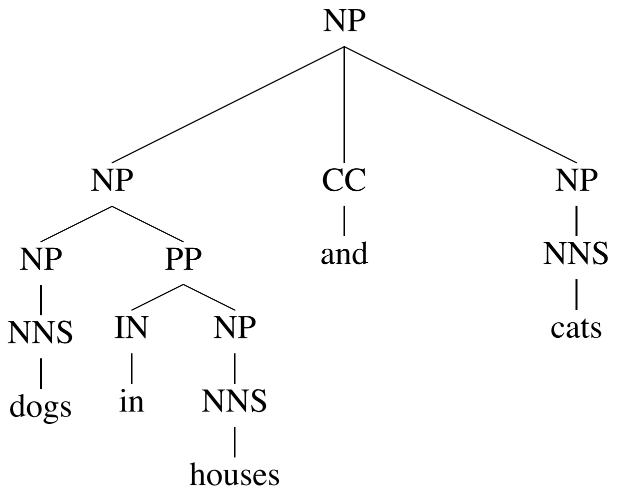
Rules
S → NP VP
NP → NNS
NP → NP PP
VP → VBD NP
NP → NNS
PP → IN NP
NP → DT NN
NNS → workers
VBD → dumped
NNS → sacks
IN → into
DT → a
NN → bin

If $P(\text{NP} \rightarrow \text{NP PP} \mid \text{NP}) > P(\text{VP} \rightarrow \text{VP PP} \mid \text{VP})$ then (b) is more probable, else (a) is more probable.

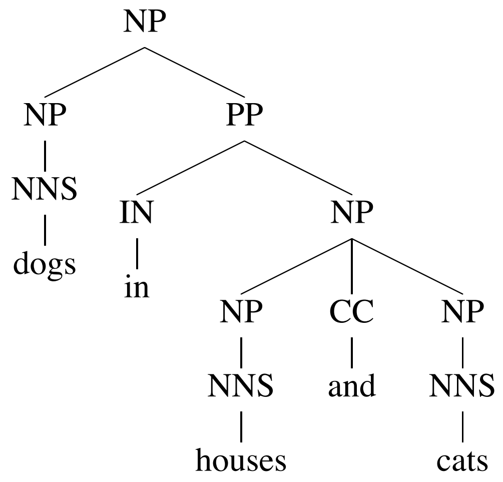
Attachment decision is completely independent of the words

A case of coordination ambiguity

(a)



(b)



(a)

Rules
NP → NP CC NP
NP → NP PP
NP → NNS
PP → IN NP
NP → NNS
NP → NNS
NNS → dogs
IN → in
NNS → houses
CC → and
NNS → cats

(b)

Rules
NP → NP CC NP
NP → NP PP
NP → NNS
PP → IN NP
NP → NNS
NP → NNS
NNS → dogs
IN → in
NNS → houses
CC → and
NNS → cats

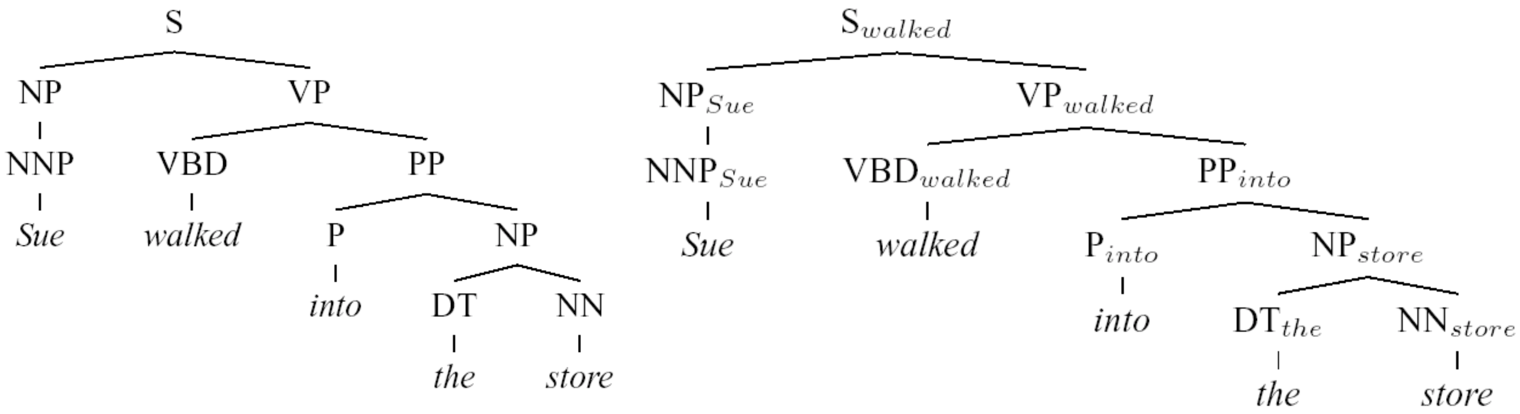
Here the two parses have identical rules, and therefore have identical probability under any assignment of PCFG rule probabilities

Weakening the independence assumption of PCFG

Probabilities dependent on Lexical words

Solution

Lexicalize CFG : Each phrasal node with its head word



Background idea

Strong lexical dependencies between heads and their dependents

Heads in Context-Free Rules

Add annotations specifying the “head” of each rule:

S	⇒	NP	**VP**
VP	⇒	**Vi**	
VP	⇒	**Vt**	NP
VP	⇒	**VP**	PP
NP	⇒	DT	**NN**
NP	⇒	**NP**	PP
PP	⇒	**IN**	NP

Vi	⇒	sleeps
Vt	⇒	saw
NN	⇒	man
NN	⇒	woman
NN	⇒	telescope
DT	⇒	the
IN	⇒	with
IN	⇒	in

Note: S=sentence, VP=verb phrase, NP=noun phrase, PP=prepositional phrase, DT=determiner, Vi=intransitive verb, Vt=transitive verb, NN=noun, IN=preposition

More about heads

- Each context-free rule has one “special” child that is the head of the rule. e.g.,

S	⇒	NP	VP		(VP is the head)
VP	⇒	Vt	NP		(Vt is the head)
NP	⇒	DT	NN	NN	(NN is the head)

- A core idea in linguistics
  (X-bar Theory, Head-Driven Phrase Structure Grammar)

- Some intuitions:
  - The central sub-constituent of each rule.
  - The semantic predicate in each rule.

Rules which recover heads: Example rules for NPs

If the rule contains NN, NNS, or NNP:
Choose the rightmost NN, NNS, or NNP

Else If the rule contains an NP: Choose the leftmost NP

Else If the rule contains a JJ: Choose the rightmost JJ

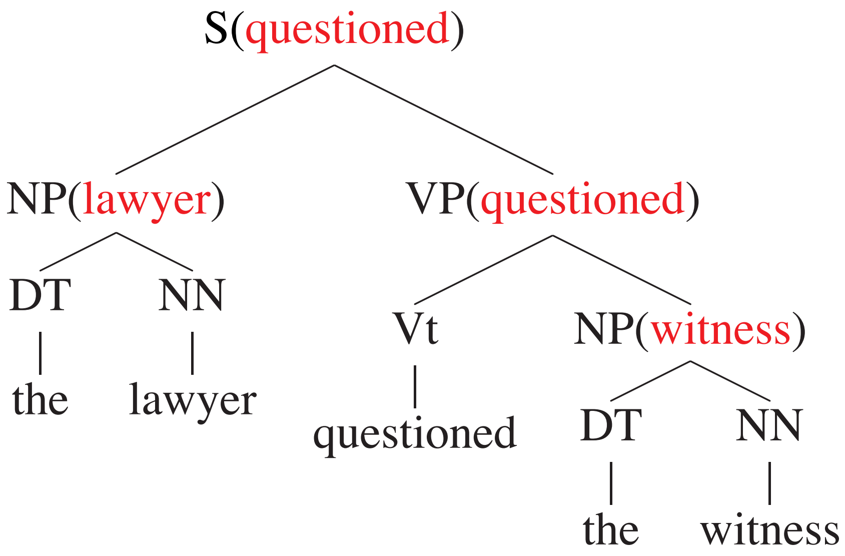
Else If the rule contains a CD: Choose the rightmost CD

Else Choose the rightmost child

e.g.,

NP ⇒ DT NNP NN
NP ⇒ DT NN NNP
NP ⇒ NP PP
NP ⇒ DT JJ
NP ⇒ DT

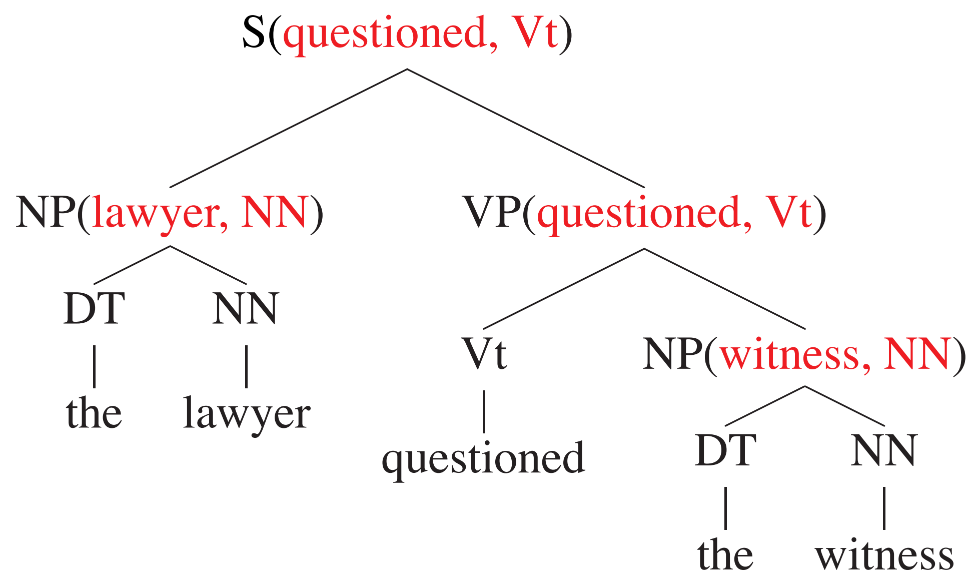
Adding Headwords to Trees



- A constituent receives its headword from its **head child**.

S	⇒	NP	VP		(S receives headword from VP)
VP	⇒	Vt	NP		(VP receives headword from Vt)
NP	⇒	DT		NN	(NP receives headword from NN)

Adding Headtags to Trees



- Also propogate **part-of-speech tags** up the trees

Explosion of number of rules

New rules might look like:

VP[gave] → V[gave] NP[man] NP[book]

But this would be a massive explosion in number of rules (and parameters)

Sparseness and the Penn Treebank

- The Penn Treebank – 1 million words of parsed English *WSJ* – has been a key resource (because of the widespread reliance on supervised learning)
- But 1 million words is like nothing:
  - 965,000 constituents, but only 66 WHADJP, of which only 6 aren't *how much* or *how many*, but there is an infinite space of these (*how clever/original/incompetent (at risk assessment and evaluation*))
- Most of the probabilities that you would like to compute, you can't compute

Sparseness and the Penn Treebank

- Most intelligent processing depends on bilexical statistics: likelihoods of relationships between pairs of words.
- Extremely sparse, even on topics central to the *WSJ*:

stocks plummeted	2 occurrences
stocks stabilized	1 occurrence
stocks skyrocketed	0 occurrences
$^{\#}$stocks discussed	0 occurrences

- So far there has been very modest success augmenting the Penn Treebank with extra unannotated materials or using semantic classes or clusters (cf. Charniak 1997, Charniak 2000) – as soon as there are more than tiny amounts of annotated training data.

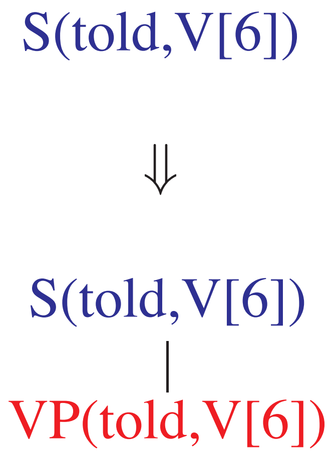
Lexicalized, Markov out from head

Collins 1997: Markov model out from head

- Charniak (1997) expands each phrase structure tree in a single step.
- This is good for capturing dependencies between child nodes
- But it is bad because of data sparseness
- A pure dependency, one child at a time, model is worse
- But one can do better by in between models, such as generating the children as a Markov process on both sides of the head (Collins 1997; Charniak 2000)

Modeling Rule Productions as Markov Processes

- Step 1: generate category of head child



S(told,V[6])

⇓

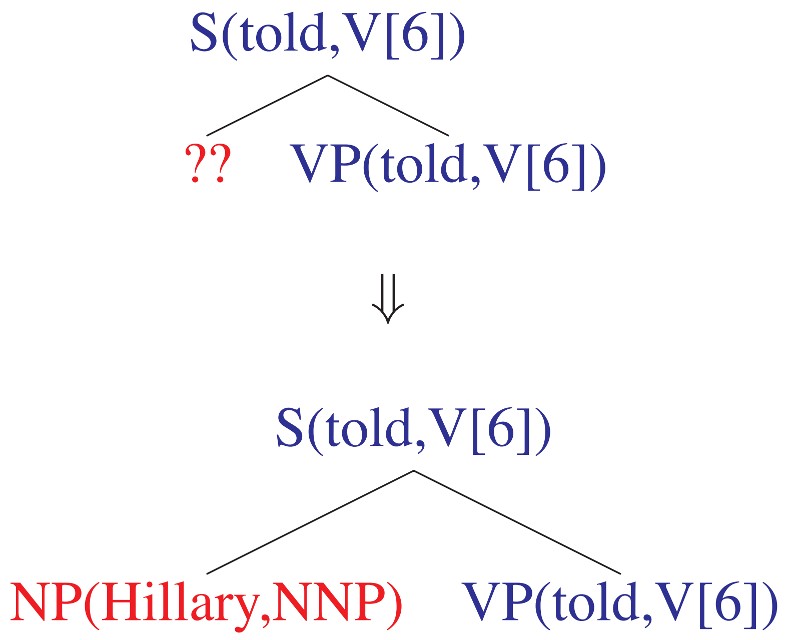
S(told,V[6])

VP(told,V[6])

$P_h(\text{VP} \mid \text{S, told, V[6]})$

Modeling Rule Productions as Markov Processes

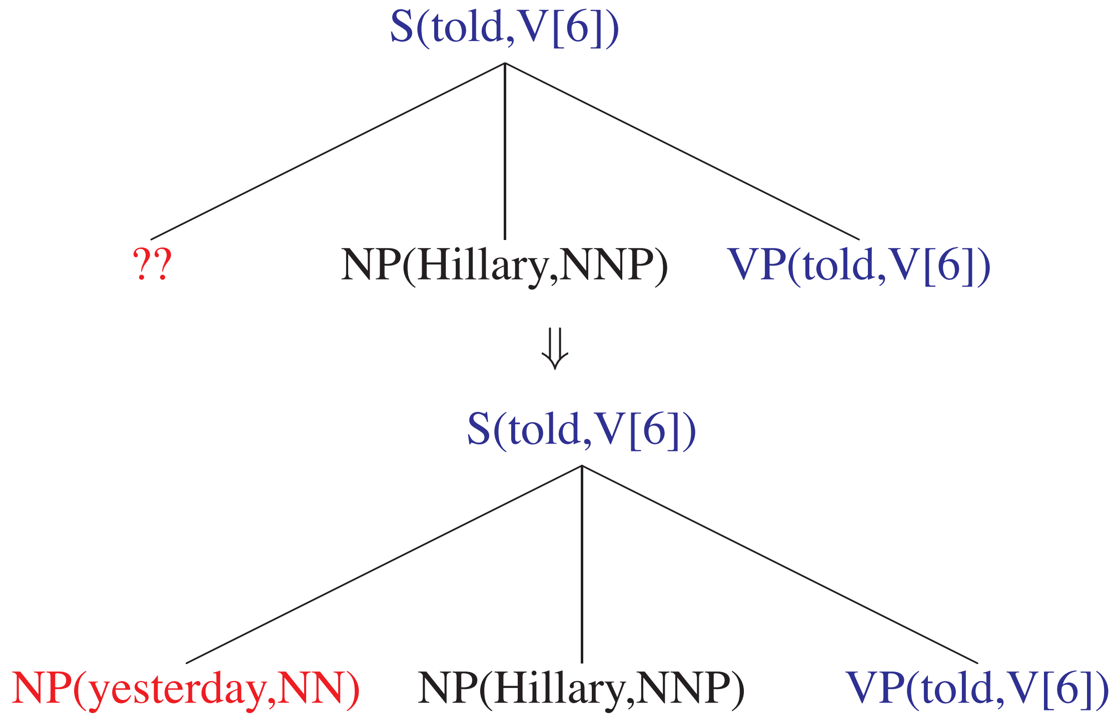
- Step 2: generate left modifiers in a Markov chain



$P_h(\text{VP} \mid \text{S, told, V[6]}) \times P_d(\text{NP(Hillary,NNP)} \mid \text{S,VP,told,V[6],LEFT})$

Modeling Rule Productions as Markov Processes

- Step 2: generate left modifiers in a Markov chain



S(told,V[6])

?? NP(Hillary,NNP) VP(told,V[6])

⇓

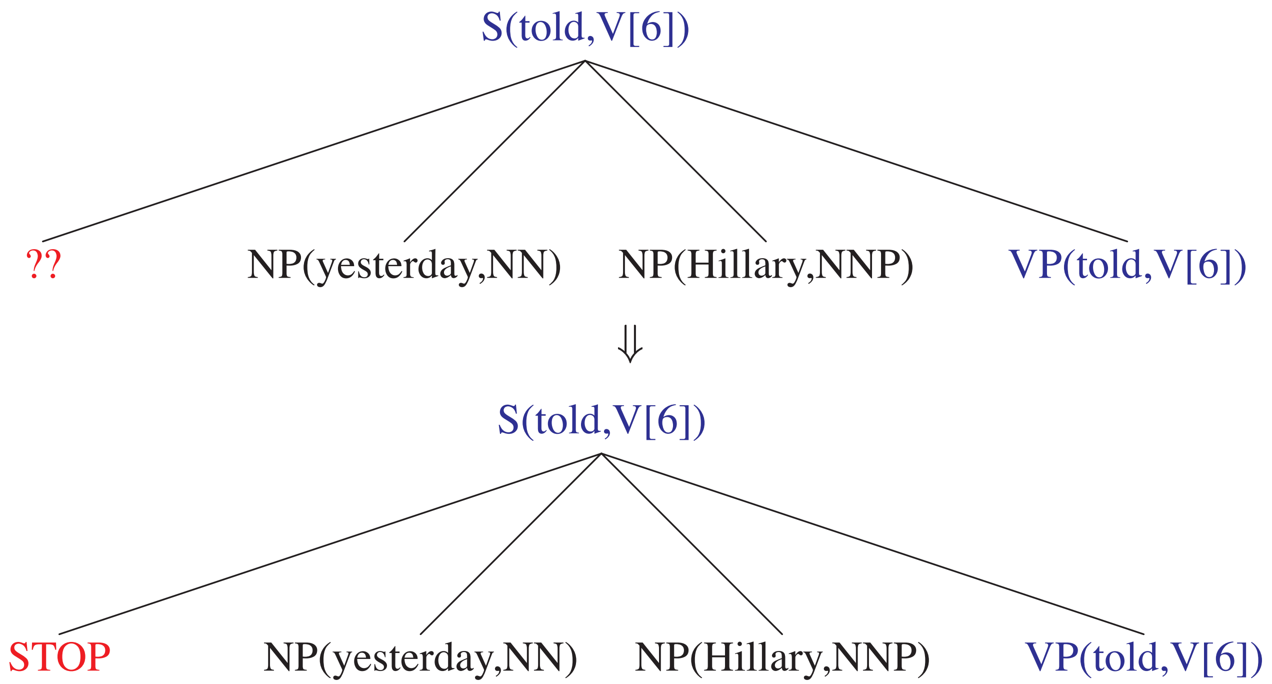
S(told,V[6])

NP(yesterday,NN) NP(Hillary,NNP) VP(told,V[6])

$P_h(\text{VP} \mid \text{S, told, V[6]}) \times P_d(\text{NP(Hillary,NNP)} \mid \text{S,VP,told,V[6],LEFT}) \times$
$P_d(\text{NP(yesterday,NN)} \mid \text{S,VP,told,V[6],LEFT})$

Modeling Rule Productions as Markov Processes

- Step 2: generate left modifiers in a Markov chain



S(told,V[6])

?? NP(yesterday,NN) NP(Hillary,NNP) VP(told,V[6])

⇓

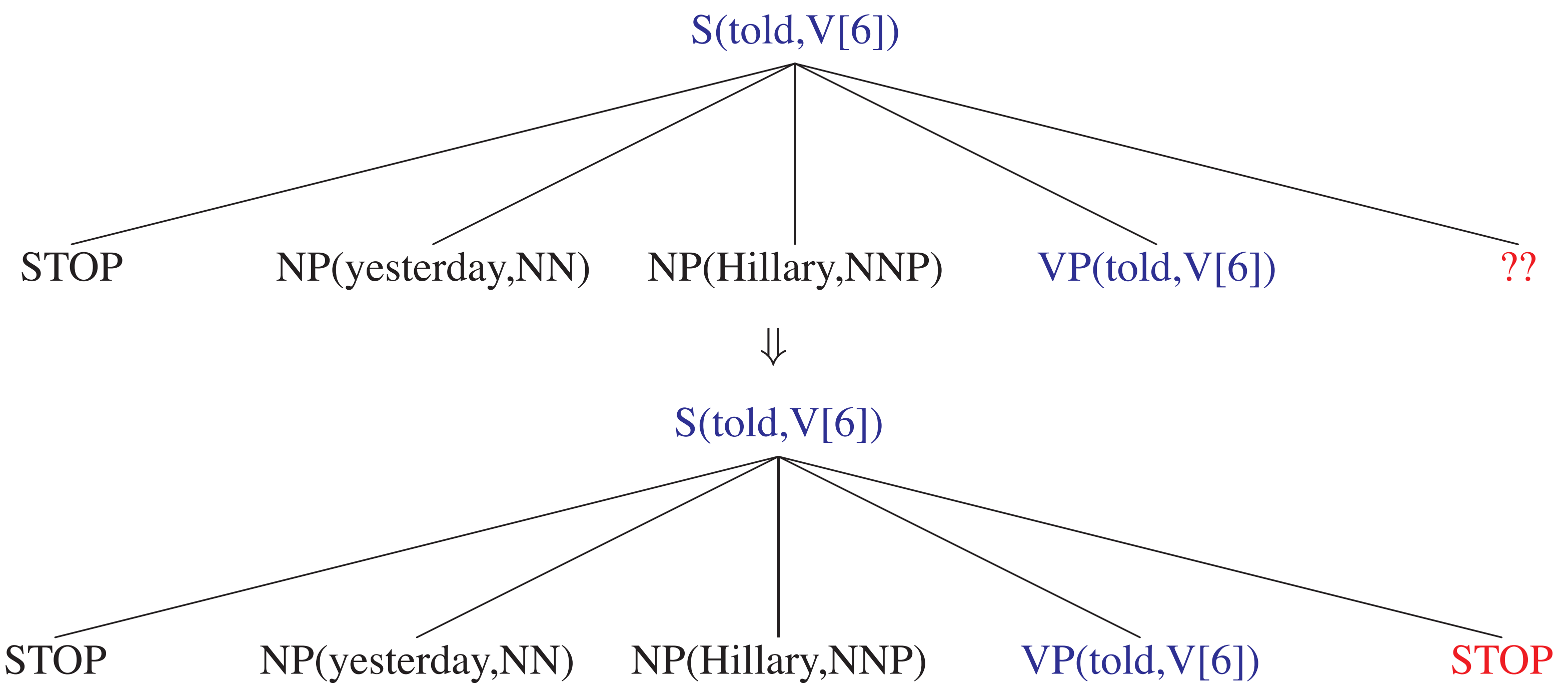
S(told,V[6])

STOP NP(yesterday,NN) NP(Hillary,NNP) VP(told,V[6])

P_h(VP | S, told, V[6]) × P_d(NP(Hillary,NNP) | S,VP,told,V[6],LEFT)×
P_d(NP(yesterday,NN) | S,VP,told,V[6],LEFT) × P_d(STOP | S,VP,told,V[6],LEFT)

Modeling Rule Productions as Markov Processes

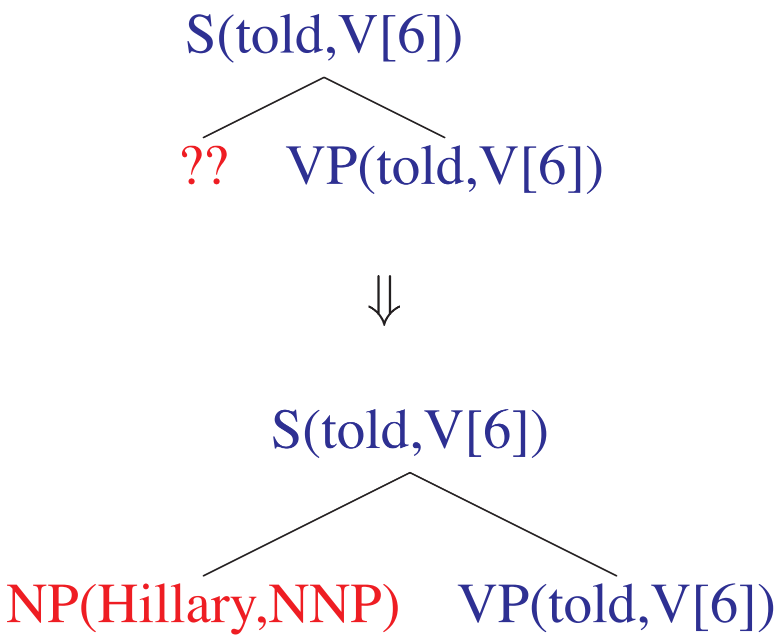
- Step 3: generate right modifiers in a Markov chain



P_h(VP | S, told, V[6]) × P_d(NP(Hillary,NNP) | S,VP,told,V[6],LEFT)×
P_d(NP(yesterday,NN) | S,VP,told,V[6],LEFT) × P_d(STOP | S,VP,told,V[6],LEFT) ×
P_d(STOP | S,VP,told,V[6],RIGHT)

A Refinement: Adding a Distance Variable

- $\Delta = 1$ if position is adjacent to the head.



S(told,V[6])

?? VP(told,V[6])

⇓

S(told,V[6])

NP(Hillary,NNP) VP(told,V[6])

$$P_h(\text{VP} \mid \text{S, told, V[6]})\times$$
$$P_d(\text{NP(Hillary,NNP)} \mid \text{S,VP,told,V[6],LEFT,}\Delta = 1)$$

Adding dependency on structure

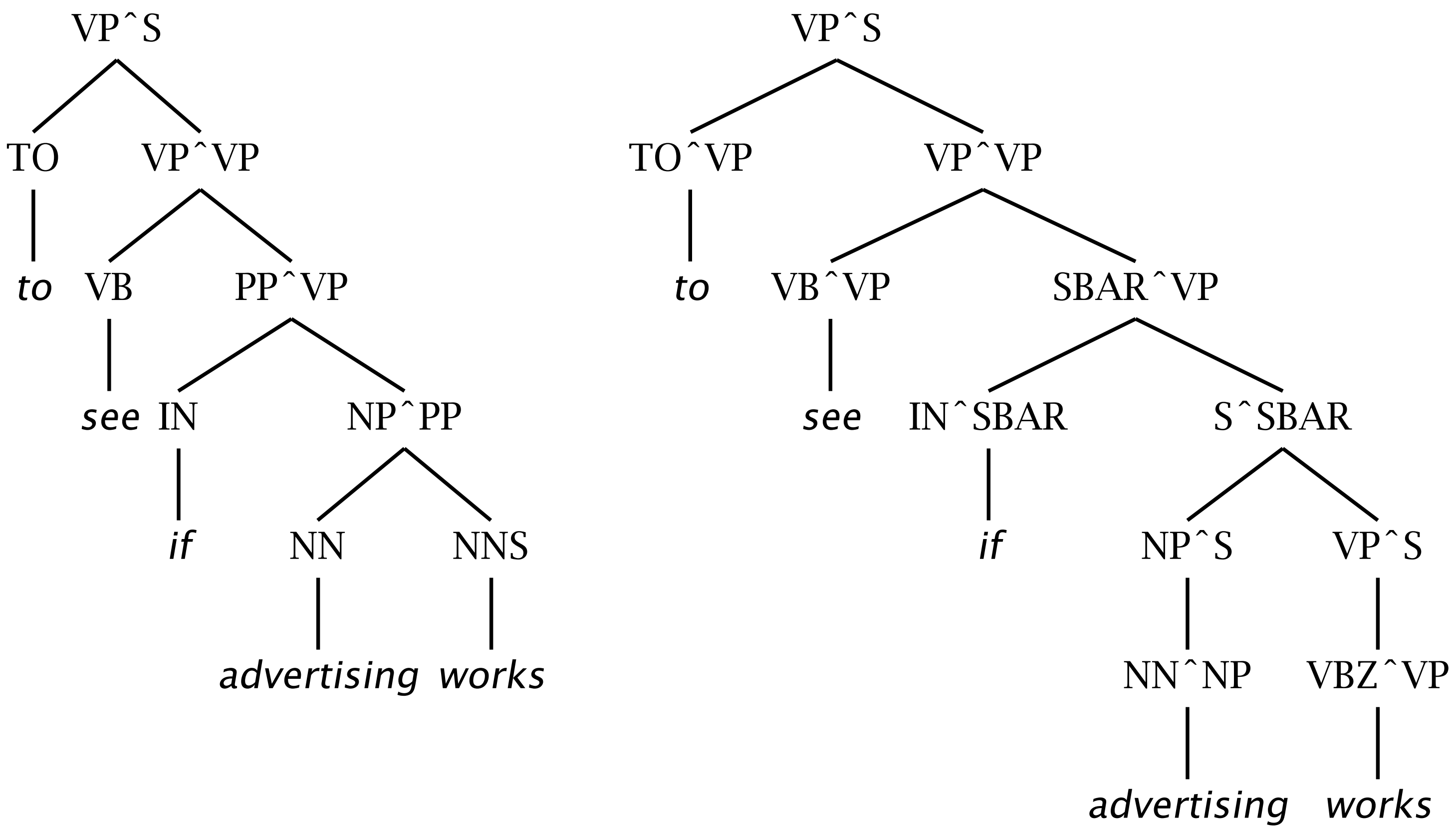
Weakening the independence assumption of PCFG

Probabilities dependent on structural context

- PCFGs are also deficient on purely structural grounds too
- Really context independent?

Expansion	% as Subj	% as Obj
NP → PRP	13.7%	2.1%
NP → NNP	3.5%	0.9%
NP → DT NN	5.6%	4.6%
NP → NN	1.4%	2.8%
NP → NP SBAR	0.5%	2.6%
NP → NP PP	5.6%	14.1%

Weakening the independence assumption of PCFG



(a) (b)