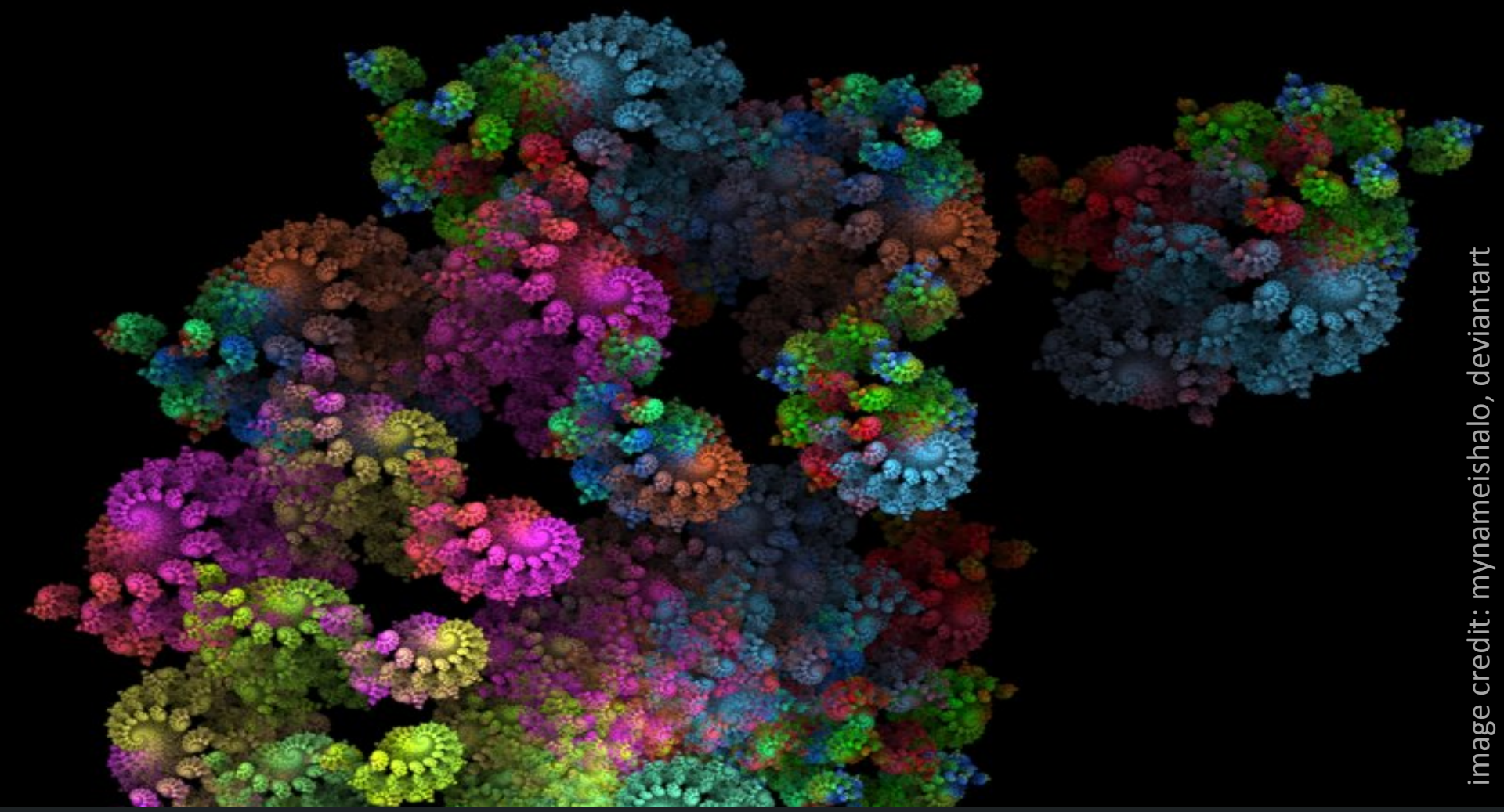


3D Shapes and Transformations

also today: 3D project (initial awareness), Polly

CS 4300/5310

Computer Graphics

ANNOUNCEMENTS

Upcoming Deadlines

- Raytracer
  - February 19th
- 2D Project Team Feedback
  - Today!
- Art Contests
  - ongoing…

and where are we going?

WHERE ARE WE?

2D Graphics

- Image processing
- Image warping

- 2D transformations

3D Graphics

- Raytracing
- Rasterization

- 3D transformations

2D Review

- What does this 2D transformation matrix do?

$$\begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$$

2D Review

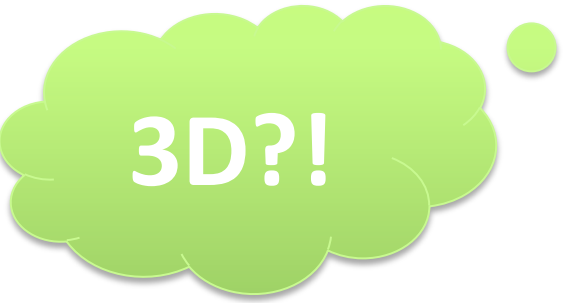
- What does this 2D transformation matrix do?

$$\begin{bmatrix} 2 & 3 \\ 0 & 3 \end{bmatrix}$$

2D Review

- What does this 2D transformation matrix do?

$$\begin{bmatrix} 2 & 0 & 5 \\ 0 & 4 & 10 \\ 0 & 0 & 1 \end{bmatrix}$$



making objects move

3D TRANSFORMS

Scale

Scale

- Same as 2D!

$$\begin{bmatrix} Sx & 0 & 0 \\ 0 & Sy & 0 \\ 0 & 0 & Sz \end{bmatrix}$$

Shear

- 2D: Shear along axis
- 3D: Shear along plane

$$\begin{bmatrix} 1 & d_{xy} & d_{xz} \\ d_{yx} & 1 & d_{yz} \\ d_{zx} & d_{zy} & 1 \end{bmatrix}$$

Rotation

- 2D: Around a point (the origin)
- 3D: Around an *axis*

Rotate a point around the z axis:

$$x' = x \cos \theta - y \sin \theta$$
$$y' = x \sin \theta + y \cos \theta$$
$$z' = z$$

- What is the matrix for this?

Rotation

$$\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$$

z axis

y axis

x axis

Rotation: Putting it all Together

- Assume α is angle around x-axis, β around y-axis, γ around z-axis. Compose matrices for general rotation matrix:

$$\begin{bmatrix} \cos\beta\cos\gamma & -\cos\alpha\sin\gamma+\sin\alpha\sin\beta\cos\gamma & \sin\alpha\sin\gamma+\cos\alpha\sin\beta\cos\gamma \\ \cos\beta\sin\gamma & \cos\alpha\cos\gamma+\sin\alpha\sin\beta\sin\gamma & -\sin\alpha\cos\gamma+\cos\alpha\sin\beta\sin\gamma \\ -\sin\beta & \sin\alpha\cos\beta & \cos\alpha\cos\beta \end{bmatrix}$$

Affine Transforms

- What is a linear transformation?

- What is an affine transformation?

Translation: Homogenous Coordinates

- What are homogenous coordinates?

- 3D translation: generalization from 2D

$$\begin{bmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3D: Quick "Quiz"

- What does this 3D transformation matrix do?

$$\begin{bmatrix} 2 & 0 & 0 & 15 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 3 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3D: Quick "Quiz"

- What does this 3D transformation matrix do?

$$\begin{bmatrix} 1 & 0 & 0 & 25 \\ 0 & 1 & 0 & 0 \\ 1.5 & 1.5 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3D: Quick "Quiz"

- What does this 3D transformation matrix do?

$$\begin{bmatrix} \cos(30) & -\sin(30) & 0 & 25 \\ \sin(30) & \cos(30) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Euler Angles

- Pronounced “Oiler”
- Angles describing rotation around x, y, and z axes
  - Or any orthonormal basis vectors…
- Pitch, roll, yaw

The Problem with Euler Angles

- Gimbal Lock!
  - Rotate one of your axes 90 degrees so it perfectly overlaps another
  - Lose one degree of freedom
- Interpolation
  - Euler angles make it look really bad ☹

Avoiding Gimbal Lock with Quaternions

- What is a quaternion?
  - A real number, q
  - A vector, $\mathbf{q}$
- Rotations use *unit quaternions*
  - $q^2 + ||\mathbf{q}||^2 = 1$
  - Magnitude of $\mathbf{q}$ is $\sin(\theta/2)$
  - q is $\cos(\theta/2)$
  - $\mathbf{q}$ is the axis around which the object will rotate
  - θ is the amount of the rotation

Multiplying Quaternions

- Q = (q, $\mathbf{q}$) R = (r, $\mathbf{r}$)

- QR = ($qr - \mathbf{q} \cdot \mathbf{r}$, $\mathbf{q} \times \mathbf{r} + r\mathbf{q} + q\mathbf{r}$)
  - This corresponds to applying R first, then Q

- Rotating a point, **s**, by quaternion R:
  - Extend **s** to quaternion S: [0, s_x, s_y, s_z]
  - Multiply!

Spherical Linear Interpolation (SLERP)

- Smoothly rotate between two quaternion representations Q and R
- Compute $\varphi = \arccos(qr + \mathbf{q}_x\mathbf{r}_x + \mathbf{q}_y\mathbf{r}_y + \mathbf{q}_z\mathbf{r}_z)$
- Interpolation parameter, t ($0 \le t \le 1$)
  - if $t = 0$ or $\varphi = 0$, return Q
  - if $t = 1$ or $\varphi = \pi$, return R
  - else return $(1/\sin\varphi)(\sin((1 - t)\varphi)Q + \sin(t\varphi)R)$

looking ahead…

3D PROJECTS

Project Structure

- 2-3 person groups
  - Prefer not mixing grad/undergrad
- Proposal: March 19th
- Checkpoints: In class March 28th, April 4th
- Presentations: In class April 9th, 11th, 16th
- Report: April 16th

Suggested Topics

- Extension to Raytracer
  - More complex geometry
  - Experiment with different camera types
  - Texturing and/or bump mapping
- Interactive Applications
  - Make a game
  - 3D Data Visualization
  - Interactive Art

Project Goals

- More focused exploration of 3D topics
  - Longer term project than 2D
  - Time for deeper exploration and reflection
- Technical writing and writing critique
- Project critique and peer code review

Writing Requirements

- Project proposal: 3-5 pages
  - What are you going to do?
  - How are you going to do it?
  - Why is it interesting? What is the problem you are trying to solve?
  - What are some related projects?
- Project report: 6-8 pages
  - What did you do?
  - How was it related to what other people have done?
  - What were the results?

What can you do to get started?

- Assignments form a “sampler” of 3D graphics
  - Raytracing
  - Interactive 3D
  - Shaders

- What do you find interesting?
- What do you wish your assignment could do that it can’t?
- What do you wish you knew more about?

building little prototypes

POLLY