CS 3800, Spring 2010
Homework 1
Assigned: Friday, 15 January 2010
Due: Friday, 22 January 2010

Good students should get at least 50 of the 100 possible points.

1. [5 pts] Write down the formal (5-tuple) description of the second DFA pictured in exercise 1.1 on page 83 of the textbook (the one labelled M_2).

2. [4 pts] Draw the state transition diagram for the DFA whose formal description is

   $$(\{q_1, q_2, q_3, q_4\}, \{a, b, c\}, \delta, q_1, \{q_2, q_3\})$$

   where δ is the function listed within the following table:

	a	b	c
q_1	q_1	q_1	q_2
q_2	q_2	q_2	q_3
q_3	q_4	q_4	q_3
q_4	q_4	q_4	q_4

3. [8 pts] For each of the following languages draw the state transition diagram for a DFA with alphabet $\{0, 1\}$ that recognizes the language.

   (a) $\{0, 1, 10, 11\}$

   (b) $\{w \mid w$ contains at least two 0s$\}$

   (c) $\{w \mid w$ contains at most four 0s$\}$

   (d) $\{w \mid w$ starts with 1 and ends with 1$\}$

   (e) $\{w \mid w$ contains an even number of 0s and an odd number of 1s$\}$

   (f) $\{w \mid w$ is a binary numeral that is divisible by 3$\}$

   (g) $\{w \mid w$ is a binary numeral that is divisible by 7$\}$

   (h) $\{w \mid w$ there exist strings x and y such that $w = x1101y\}$

4. [5 pts] Do exercise 1.12 in the textbook.

5. [5 pts] Prove that the regular languages are closed under intersection.

6. [5 pts] Do exercise 1.14(a) in the textbook.

7. [5 pts] Do both parts of exercise 1.17 in the textbook.

8. [5 pts] Prove the following theorem. If B is a language over an alphabet Σ, and $B = B^*$, then $BBB \subseteq B$.

9. [10 pts] Let $L = \{w \mid w$ is a binary numeral with more 0s than 1s$\}$. Is L a regular language? If L is regular, prove it is regular. If L is not regular, prove it is not regular.

10. [8 pts] For each of the following languages, write down a regular expression whose value is the language.

    (a) $\{01, 11, 101, 111\}$

    (b) $\{w \mid w$ is a binary numeral that contains at least three 1s$\}$

    (c) $\{w \mid w$ is a binary numeral that starts with 11 and ends with 00$\}$

    (d) $\{w \mid w$ is a binary numeral that starts with 10 and ends with 00$\}$

    (e) $\{w \mid w$ is a binary numeral that contains an even number of 0s and an odd number of 1s$\}$

    (f) $\{w \mid w$ is a binary numeral that is divisible by 3$\}$

    (g) $\{w \mid w$ is a binary numeral that is divisible by 5$\}$

    (h) $\{w \mid w$ is a binary numeral and there exist strings x and y such that $w = x101y\}$

11. [10 pts] Do problem 1.31 in the textbook.

12. [5 pts] Do problem 1.32 in the textbook.

13. [5 pts] Do problem 1.33 in the textbook.

14. [5 pts] Do problem 1.34 in the textbook.

15. [5 pts] Do problem 1.35 in the textbook.

16. [10 pts] Let $L = \{w \mid x$ is a binary numeral, y is the reverse of x, and $w = xy\}$. Is L a regular language? If L is regular, prove it is regular. If L is not regular, prove it is not regular.