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Joint bank account by husband and wife; Only assembly statements (not *C*) are assumed atomic. This version models the code at assembly level, and so the “if” statement is no longer atomic. It will assert an error when $total \neq 120$, even though initially, $account = 100$, and

$cash[\text{“husband”}] = cash[\text{“wife”}] = 10.$

In the “Model” sub-window, try initializing the constant “*N*” to 1.

Note that if you remove the labels

w0b, *w0c*, *w1b*, *d0b*, *d0c*, *d1b*, then there will be no assertion error.

EXTENDS *Naturals*, *Sequences*, *TLC* Sequences required for “procedure” stmt

CONSTANT *N* *N* is number of iterations. Assign to it in model overview.

```

--algorithm bank {
  variables account = 100, cash = [i ∈ {“husband”, “wife”} ↦ 10],
            iterations = [i ∈ {“husband”, “wife”} ↦ N];
  Note that we need to define iterations[“husband”] and iterations[“wife”].
  We do not want a single global variable, iterations, that is
  shared between “husband” and “wife”.
  In model, replace “N” (a constant) by value for iterations

  The procedures withdraw and deposit have been translated here
  to pseudo-assembly language
  Note that “register1” and “register2” were declared as local variables
  inside the processes for husband and wife.
  procedure withdraw( amount1 )
    variable register1, register2;
  {
    withdraw_start: register1 := amount1;          lw register1, (amount1)
    w0b:              register2 := account - register1;
                    lw register2, (account) ; sub register2, register2, register1
    w0c:              account := register2;        sw register2, (account)

    w1:              register2 := cash[self] + register1;
                    lw register2, (cash[self]) ; add register2, register2, register1
    w1b:             cash[self] := register2;     sw register2, (cash[self])

    w2:              return ;
  }

  procedure deposit( amount1 )
    variable register1, register2;
  {

```

```

deposit_start: register1 := amount1;          lw register1, (amount1)
d0b:          register2 := account + register1; lw register2, (account)
              add register2, register2, register1
d0c:          account := register2;           sw register2, (account)

d1:          register2 := cash[self] - register1;
              lw register2, (cash[self])
              sub register2, register2, register1
d1b:          cash[self] := register2;        sw register2, (cash[self])

d2:          return ;
}

process ( spouse ∈ { "husband", "wife" } )
  variable total ;
  { start: while ( iterations[self] > 0 ) {
    We hard-wire the max amount below, but this could have been a CONSTANT .
    s1: with ( amount ∈ 1 .. 2 )
        call withdraw(amount);
    s2: with ( amount ∈ 1 .. 2 )
        call deposit(amount);
    s3: iterations[self] := iterations[self] - 1 ;
        total := account + cash["husband"] + cash["wife"];
    } ;
    assert iterations[self] = 0 ;

    if ( iterations["husband"] = 0 ∧ iterations["wife"] = 0 ) {
      total := account + cash["husband"] + cash["wife"];
      print total ;
      assert total = 120 ;
    }
  } end process block
} \* end algorithm
BEGIN TRANSLATION
Procedure variable register1 of procedure withdraw at line 33 col 14 changed to register1_
Procedure variable register2 of procedure withdraw at line 33 col 25 changed to register2_
Parameter amount1 of procedure withdraw at line 32 col 22 changed to amount1_
CONSTANT defaultInitValue
VARIABLES account, cash, iterations, pc, stack, amount1_, register1_,
          register2_, amount1, register1, register2, total

vars ≙ ⟨ account, cash, iterations, pc, stack, amount1_, register1_,
        register2_, amount1, register1, register2, total ⟩

ProcSet ≙ ( { "husband", "wife" } )

Init ≙ Global variables

```

$$\begin{aligned}
& \wedge \text{account} = 100 \\
& \wedge \text{cash} = [i \in \{\text{"husband"}, \text{"wife"}\} \mapsto 10] \\
& \wedge \text{iterations} = [i \in \{\text{"husband"}, \text{"wife"}\} \mapsto N] \\
& \text{Procedure withdraw} \\
& \wedge \text{amount1}_- = [self \in ProcSet \mapsto defaultInitValue] \\
& \wedge \text{register1}_- = [self \in ProcSet \mapsto defaultInitValue] \\
& \wedge \text{register2}_- = [self \in ProcSet \mapsto defaultInitValue] \\
& \text{Procedure deposit} \\
& \wedge \text{amount1} = [self \in ProcSet \mapsto defaultInitValue] \\
& \wedge \text{register1} = [self \in ProcSet \mapsto defaultInitValue] \\
& \wedge \text{register2} = [self \in ProcSet \mapsto defaultInitValue] \\
& \text{Process spouse} \\
& \wedge \text{total} = [self \in \{\text{"husband"}, \text{"wife"}\} \mapsto defaultInitValue] \\
& \wedge \text{stack} = [self \in ProcSet \mapsto \langle \rangle] \\
& \wedge \text{pc} = [self \in ProcSet \mapsto \text{"start"}] \\
\\
\text{withdraw_start}(self) \triangleq & \wedge \text{pc}[self] = \text{"withdraw_start"} \\
& \wedge \text{register1}' = [\text{register1}_- \text{ EXCEPT } ![self] = \text{amount1}_-[self]] \\
& \wedge \text{pc}' = [\text{pc} \text{ EXCEPT } ![self] = \text{"w0b"}] \\
& \wedge \text{UNCHANGED } \langle \text{account}, \text{cash}, \text{iterations}, \text{stack}, \\
& \quad \text{amount1}_-, \text{register2}_-, \text{amount1}, \\
& \quad \text{register1}, \text{register2}, \text{total} \rangle \\
\\
\text{w0b}(self) \triangleq & \wedge \text{pc}[self] = \text{"w0b"} \\
& \wedge \text{register2}' = [\text{register2}_- \text{ EXCEPT } ![self] = \text{account} - \text{register1}_-[self]] \\
& \wedge \text{pc}' = [\text{pc} \text{ EXCEPT } ![self] = \text{"w0c"}] \\
& \wedge \text{UNCHANGED } \langle \text{account}, \text{cash}, \text{iterations}, \text{stack}, \text{amount1}_-, \\
& \quad \text{register1}_-, \text{amount1}, \text{register1}, \text{register2}, \text{total} \rangle \\
\\
\text{w0c}(self) \triangleq & \wedge \text{pc}[self] = \text{"w0c"} \\
& \wedge \text{account}' = \text{register2}_-[self] \\
& \wedge \text{pc}' = [\text{pc} \text{ EXCEPT } ![self] = \text{"w1"}] \\
& \wedge \text{UNCHANGED } \langle \text{cash}, \text{iterations}, \text{stack}, \text{amount1}_-, \text{register1}_-, \\
& \quad \text{register2}_-, \text{amount1}, \text{register1}, \text{register2}, \text{total} \rangle \\
\\
\text{w1}(self) \triangleq & \wedge \text{pc}[self] = \text{"w1"} \\
& \wedge \text{register2}' = [\text{register2}_- \text{ EXCEPT } ![self] = \text{cash}[self] + \text{register1}_-[self]] \\
& \wedge \text{pc}' = [\text{pc} \text{ EXCEPT } ![self] = \text{"w1b"}] \\
& \wedge \text{UNCHANGED } \langle \text{account}, \text{cash}, \text{iterations}, \text{stack}, \text{amount1}_-, \\
& \quad \text{register1}_-, \text{amount1}, \text{register1}, \text{register2}, \text{total} \rangle \\
\\
\text{w1b}(self) \triangleq & \wedge \text{pc}[self] = \text{"w1b"} \\
& \wedge \text{cash}' = [\text{cash} \text{ EXCEPT } ![self] = \text{register2}_-[self]] \\
& \wedge \text{pc}' = [\text{pc} \text{ EXCEPT } ![self] = \text{"w2"}] \\
& \wedge \text{UNCHANGED } \langle \text{account}, \text{iterations}, \text{stack}, \text{amount1}_-, \text{register1}_-, \\
& \quad \text{register2}_-, \text{amount1}, \text{register1}, \text{register2}, \text{total} \rangle
\end{aligned}$$

$$\begin{aligned}
w2(self) &\triangleq \wedge pc[self] = \text{"w2"} \\
&\wedge pc' = [pc \text{ EXCEPT } ![self] = \text{Head}(\text{stack}[self]).pc] \\
&\wedge register1_ ' = [register1_ \text{ EXCEPT } ![self] = \text{Head}(\text{stack}[self]).register1_] \\
&\wedge register2_ ' = [register2_ \text{ EXCEPT } ![self] = \text{Head}(\text{stack}[self]).register2_] \\
&\wedge amount1_ ' = [amount1_ \text{ EXCEPT } ![self] = \text{Head}(\text{stack}[self]).amount1_] \\
&\wedge stack' = [stack \text{ EXCEPT } ![self] = \text{Tail}(\text{stack}[self])] \\
&\wedge \text{UNCHANGED } \langle \text{account}, \text{cash}, \text{iterations}, \text{amount1}, \text{register1}, \\
&\quad \text{register2}, \text{total} \rangle \\
\\
withdraw(self) &\triangleq \text{withdraw_start}(self) \vee w0b(self) \vee w0c(self) \\
&\quad \vee w1(self) \vee w1b(self) \vee w2(self) \\
\\
deposit_start(self) &\triangleq \wedge pc[self] = \text{"deposit_start"} \\
&\wedge register1' = [register1 \text{ EXCEPT } ![self] = \text{amount1}[self]] \\
&\wedge pc' = [pc \text{ EXCEPT } ![self] = \text{"d0b"}] \\
&\wedge \text{UNCHANGED } \langle \text{account}, \text{cash}, \text{iterations}, \text{stack}, \\
&\quad \text{amount1_}, \text{register1_}, \text{register2_}, \\
&\quad \text{amount1}, \text{register2}, \text{total} \rangle \\
\\
d0b(self) &\triangleq \wedge pc[self] = \text{"d0b"} \\
&\wedge register2' = [register2 \text{ EXCEPT } ![self] = \text{account} + \text{register1}[self]] \\
&\wedge pc' = [pc \text{ EXCEPT } ![self] = \text{"d0c"}] \\
&\wedge \text{UNCHANGED } \langle \text{account}, \text{cash}, \text{iterations}, \text{stack}, \text{amount1_}, \\
&\quad \text{register1_}, \text{register2_}, \text{amount1}, \text{register1}, \text{total} \rangle \\
\\
d0c(self) &\triangleq \wedge pc[self] = \text{"d0c"} \\
&\wedge \text{account}' = \text{register2}[self] \\
&\wedge pc' = [pc \text{ EXCEPT } ![self] = \text{"d1"}] \\
&\wedge \text{UNCHANGED } \langle \text{cash}, \text{iterations}, \text{stack}, \text{amount1_}, \text{register1_}, \\
&\quad \text{register2_}, \text{amount1}, \text{register1}, \text{register2}, \text{total} \rangle \\
\\
d1(self) &\triangleq \wedge pc[self] = \text{"d1"} \\
&\wedge register2' = [register2 \text{ EXCEPT } ![self] = \text{cash}[self] - \text{register1}[self]] \\
&\wedge pc' = [pc \text{ EXCEPT } ![self] = \text{"d1b"}] \\
&\wedge \text{UNCHANGED } \langle \text{account}, \text{cash}, \text{iterations}, \text{stack}, \text{amount1_}, \\
&\quad \text{register1_}, \text{register2_}, \text{amount1}, \text{register1}, \text{total} \rangle \\
\\
d1b(self) &\triangleq \wedge pc[self] = \text{"d1b"} \\
&\wedge \text{cash}' = [\text{cash} \text{ EXCEPT } ![self] = \text{register2}[self]] \\
&\wedge pc' = [pc \text{ EXCEPT } ![self] = \text{"d2"}] \\
&\wedge \text{UNCHANGED } \langle \text{account}, \text{iterations}, \text{stack}, \text{amount1_}, \text{register1_}, \\
&\quad \text{register2_}, \text{amount1}, \text{register1}, \text{register2}, \text{total} \rangle \\
\\
d2(self) &\triangleq \wedge pc[self] = \text{"d2"} \\
&\wedge pc' = [pc \text{ EXCEPT } ![self] = \text{Head}(\text{stack}[self]).pc] \\
&\wedge register1' = [register1 \text{ EXCEPT } ![self] = \text{Head}(\text{stack}[self]).register1] \\
&\wedge register2' = [register2 \text{ EXCEPT } ![self] = \text{Head}(\text{stack}[self]).register2]
\end{aligned}$$

$$\begin{aligned}
& \wedge \text{amount1}' = [\text{amount1 EXCEPT ![self] = Head}(\text{stack}[\text{self}]).\text{amount1}] \\
& \wedge \text{stack}' = [\text{stack EXCEPT ![self] = Tail}(\text{stack}[\text{self}])] \\
& \wedge \text{UNCHANGED} \langle \text{account}, \text{cash}, \text{iterations}, \text{amount1}_-, \text{register1}_-, \\
& \quad \text{register2}_-, \text{total} \rangle \\
\text{deposit}(\text{self}) & \triangleq \text{deposit_start}(\text{self}) \vee \text{d0b}(\text{self}) \vee \text{d0c}(\text{self}) \vee \text{d1}(\text{self}) \\
& \quad \vee \text{d1b}(\text{self}) \vee \text{d2}(\text{self}) \\
\text{start}(\text{self}) & \triangleq \wedge \text{pc}[\text{self}] = \text{"start"} \\
& \quad \wedge \text{IF } \text{iterations}[\text{self}] > 0 \\
& \quad \quad \text{THEN } \wedge \text{pc}' = [\text{pc EXCEPT ![self] = "s1"}] \\
& \quad \quad \quad \wedge \text{total}' = \text{total} \\
& \quad \quad \text{ELSE } \wedge \text{Assert}(\text{iterations}[\text{self}] = 0, \\
& \quad \quad \quad \quad \text{"Failure of assertion at line 74, column 7."}) \\
& \quad \quad \wedge \text{IF } \text{iterations}[\text{"husband"}] = 0 \wedge \text{iterations}[\text{"wife"}] = 0 \\
& \quad \quad \quad \text{THEN } \wedge \text{total}' = [\text{total EXCEPT ![self] = account + cash}[\text{"husband"}] + \text{cash}[\text{"} \\
& \quad \quad \quad \quad \wedge \text{PrintT}(\text{total}'[\text{self}]) \\
& \quad \quad \quad \quad \wedge \text{Assert}(\text{total}'[\text{self}] = 120, \\
& \quad \quad \quad \quad \quad \text{"Failure of assertion at line 79, column 9."}) \\
& \quad \quad \quad \text{ELSE } \wedge \text{TRUE} \\
& \quad \quad \quad \quad \wedge \text{total}' = \text{total} \\
& \quad \quad \wedge \text{pc}' = [\text{pc EXCEPT ![self] = "Done"}] \\
& \quad \wedge \text{UNCHANGED} \langle \text{account}, \text{cash}, \text{iterations}, \text{stack}, \text{amount1}_-, \\
& \quad \quad \text{register1}_-, \text{register2}_-, \text{amount1}, \text{register1}, \\
& \quad \quad \text{register2} \rangle \\
\text{s1}(\text{self}) & \triangleq \wedge \text{pc}[\text{self}] = \text{"s1"} \\
& \quad \wedge \exists \text{amount} \in 1 \dots 2 : \\
& \quad \quad \wedge \wedge \text{amount1}'_ = [\text{amount1}_- \text{ EXCEPT ![self] = amount}] \\
& \quad \quad \quad \wedge \text{stack}' = [\text{stack EXCEPT ![self] = } \langle [\text{procedure} \mapsto \text{"withdraw"}, \\
& \quad \quad \quad \quad \text{pc} \quad \quad \mapsto \text{"s2"}, \\
& \quad \quad \quad \quad \text{register1}_- \mapsto \text{register1}_-[\text{self}], \\
& \quad \quad \quad \quad \text{register2}_- \mapsto \text{register2}_-[\text{self}], \\
& \quad \quad \quad \quad \text{amount1}_- \mapsto \text{amount1}_-[\text{self}]] \rangle \\
& \quad \quad \quad \quad \circ \text{stack}[\text{self}]] \\
& \quad \quad \quad \wedge \text{register1}'_ = [\text{register1}_- \text{ EXCEPT ![self] = defaultInitValue}] \\
& \quad \quad \quad \wedge \text{register2}'_ = [\text{register2}_- \text{ EXCEPT ![self] = defaultInitValue}] \\
& \quad \quad \quad \wedge \text{pc}' = [\text{pc EXCEPT ![self] = "withdraw_start"}] \\
& \quad \wedge \text{UNCHANGED} \langle \text{account}, \text{cash}, \text{iterations}, \text{amount1}, \text{register1}, \\
& \quad \quad \text{register2}, \text{total} \rangle \\
\text{s2}(\text{self}) & \triangleq \wedge \text{pc}[\text{self}] = \text{"s2"} \\
& \quad \wedge \exists \text{amount} \in 1 \dots 2 : \\
& \quad \quad \wedge \wedge \text{amount1}' = [\text{amount1 EXCEPT ![self] = amount}] \\
& \quad \quad \quad \wedge \text{stack}' = [\text{stack EXCEPT ![self] = } \langle [\text{procedure} \mapsto \text{"deposit"}, \\
& \quad \quad \quad \quad \text{pc} \quad \quad \mapsto \text{"s3"}, \\
& \quad \quad \quad \quad \text{register1}_- \mapsto \text{register1}_-[\text{self}], \\
& \quad \quad \quad \quad \text{register2}_- \mapsto \text{register2}_-[\text{self}], \\
& \quad \quad \quad \quad \text{amount1}_- \mapsto \text{amount1}_-[\text{self}]] \rangle \\
& \quad \quad \quad \quad \circ \text{stack}[\text{self}]] \\
& \quad \quad \quad \wedge \text{register1}' = [\text{register1}_- \text{ EXCEPT ![self] = defaultInitValue}] \\
& \quad \quad \quad \wedge \text{register2}' = [\text{register2}_- \text{ EXCEPT ![self] = defaultInitValue}] \\
& \quad \quad \quad \wedge \text{pc}' = [\text{pc EXCEPT ![self] = "deposit_start"}] \\
& \quad \wedge \text{UNCHANGED} \langle \text{account}, \text{cash}, \text{iterations}, \text{amount1}, \text{register1}, \\
& \quad \quad \text{register2}, \text{total} \rangle
\end{aligned}$$

$$\begin{aligned}
& \text{register1} \mapsto \text{register1}[\text{self}], \\
& \text{register2} \mapsto \text{register2}[\text{self}], \\
& \text{amount1} \mapsto \text{amount1}[\text{self}]] \\
& \circ \text{stack}[\text{self}] \\
& \wedge \text{register1}' = [\text{register1} \text{ EXCEPT } ![\text{self}] = \text{defaultInitValue}] \\
& \wedge \text{register2}' = [\text{register2} \text{ EXCEPT } ![\text{self}] = \text{defaultInitValue}] \\
& \wedge \text{pc}' = [\text{pc} \text{ EXCEPT } ![\text{self}] = \text{"deposit_start"}] \\
& \wedge \text{UNCHANGED} \langle \text{account}, \text{cash}, \text{iterations}, \text{amount1}_-, \text{register1}_-, \\
& \quad \text{register2}_-, \text{total} \rangle \\
s3(\text{self}) \triangleq & \wedge \text{pc}[\text{self}] = \text{"s3"} \\
& \wedge \text{iterations}' = [\text{iterations} \text{ EXCEPT } ![\text{self}] = \text{iterations}[\text{self}] - 1] \\
& \wedge \text{total}' = [\text{total} \text{ EXCEPT } ![\text{self}] = \text{account} + \text{cash}[\text{"husband"}] + \text{cash}[\text{"wife"}]] \\
& \wedge \text{pc}' = [\text{pc} \text{ EXCEPT } ![\text{self}] = \text{"start"}] \\
& \wedge \text{UNCHANGED} \langle \text{account}, \text{cash}, \text{stack}, \text{amount1}_-, \text{register1}_-, \\
& \quad \text{register2}_-, \text{amount1}, \text{register1}, \text{register2} \rangle \\
\text{spouse}(\text{self}) \triangleq & \text{start}(\text{self}) \vee s1(\text{self}) \vee s2(\text{self}) \vee s3(\text{self}) \\
\text{Next} \triangleq & (\exists \text{self} \in \text{ProcSet} : \text{withdraw}(\text{self}) \vee \text{deposit}(\text{self})) \\
& \vee (\exists \text{self} \in \{\text{"husband"}, \text{"wife"}\} : \text{spouse}(\text{self})) \\
& \vee \text{Disjunct to prevent deadlock on termination} \\
& ((\forall \text{self} \in \text{ProcSet} : \text{pc}[\text{self}] = \text{"Done"}) \wedge \text{UNCHANGED } \text{vars}) \\
\text{Spec} \triangleq & \text{Init} \wedge \square[\text{Next}]_{\text{vars}} \\
\text{Termination} \triangleq & \diamond(\forall \text{self} \in \text{ProcSet} : \text{pc}[\text{self}] = \text{"Done"})
\end{aligned}$$

END TRANSLATION