CS 2810 Day 23 April 15

Admin:
10 mins for TRACE
Quiz3_02: problem 2.iv & 3.iv

Bayes Rule
- binary problems
- parametric likelihoods
- bayes rule & independence

TRACE

TRACE feedback helps me be a better teacher for you all.

TRACE feedback helps NU identify strong / weak teachers.

Please take a few minutes to give feedback about what worked and what didn't.

To zoom students just joining, we'll get started in 6-7 minutes. Please take this time to fill out the trace survey on the course. Thank you!

CONDITIONAL PROB

(INTUITION: "ZOOM INTO CONTEXT B")
SEE LAST ICA DAY 22

PROB A, B HAPPEN TOGETHER

$$P(A|B) = \frac{P(A\,B)}{P(B)}$$

PROB A HAPPENS GIVEN CONDITION B

PROB B HAPPENS

CONDITIONAL PROB (USEFUL ALGEBRAIC MANIPULATION)

PROB A HAPPENS GIVEN CONDITION B

PROB A,B HAPPEN TOGETHER

PROB B HAPPENS

$$P(A|B) = \frac{P(AB)}{P(B)}$$

$$P(A|B)P(B) = P(AB)$$

Takeaway: Multiplying a conditional probability by the probability of condition yields a "full joint" probability of all variables happening together.
(we'll see it works for more than just two vars A, B too!)

Bayes Rule (Glorified Conditional Probability)

See previous "Takeaway"

$$P(A|B)P(B) = P(AB) = P(B|A)P(A)$$

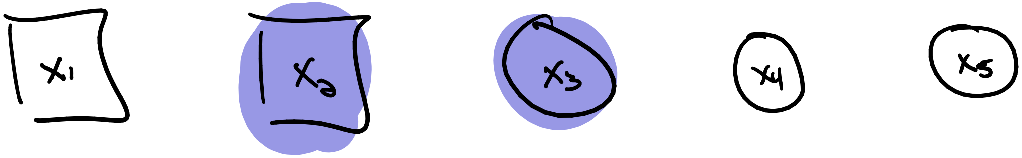
$$\Rightarrow \quad \boxed{P(A|B) = \frac{P(B|A)P(A)}{P(B)}}$$

Notice: this formula "swaps" the order of the conditioning: P(A|B) on left P(B|A) on right
Its typical in a Bayes question to be given variables in one order while question asks for other.

Marginalizing (Remember this?)

$B=1$ Shape is blue

$C=1$ Shape is circle



	$B=0$	$B=1$
$C=0$	x_1 $1/5$	x_3 $1/5$
$C=1$	x_4 x_5 $2/5$	x_2 $1/5$

$$P(B=1) = P(B=1, C=0) + P(B=1, C=1)$$

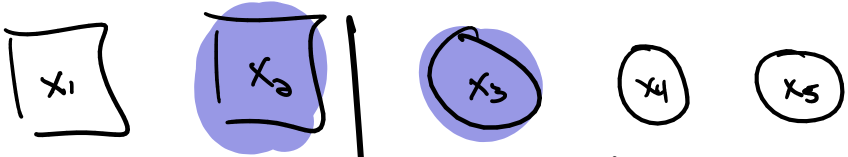
$$= 1/5 + 1/5$$

Remember: To compute P(B) we can sum P(B, A) for all outcomes in sample space of A

$$P(B=b) = \sum_a P(B=b, A=a)$$

Marginalizing (with a conditional)

$B=1$	Shape is	Blue
$C=1$	Shape is	Circle



$P(C=0)=2/5$

$P(B=1|C=0)=1/2$

$P(B=0|C=0)=1/2$

$P(C=1)=3/5$

$P(B=1|C=1)=1/3$

$P(B=0|C=1)=2/3$

$$P(B=1)=\sum_{c} P(B=1\ C=c)$$

$$=P(B=1\ C=1)+P(B=1\ C=0)$$

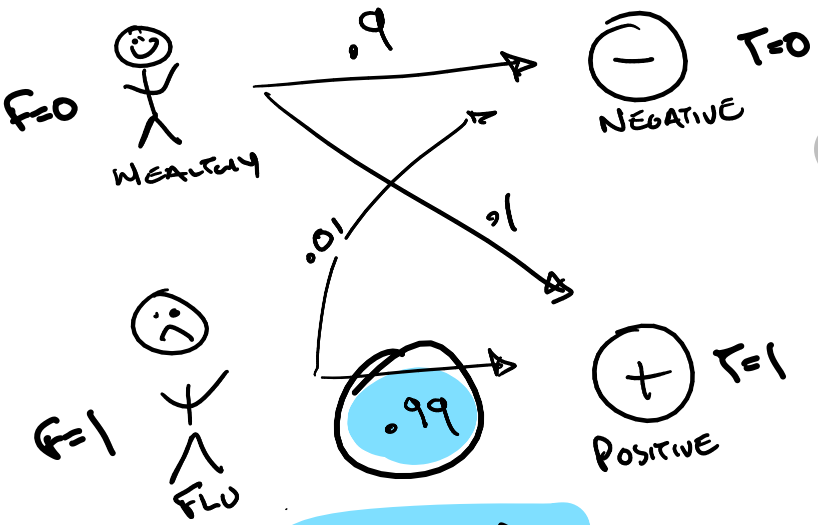
$$=P(B=1|C=1)\,P(C=1)+P(B=1|C=0)\,P(C=0)$$

$$=1/3\cdot 3/5+1/2\cdot 2/5$$

$$=1/5+1/5=2/5$$

Bayes Rule Ex

Given flu occurs in .04 of population, what is the probability one has flu given they test positive?



$$P(F=1) = .04$$

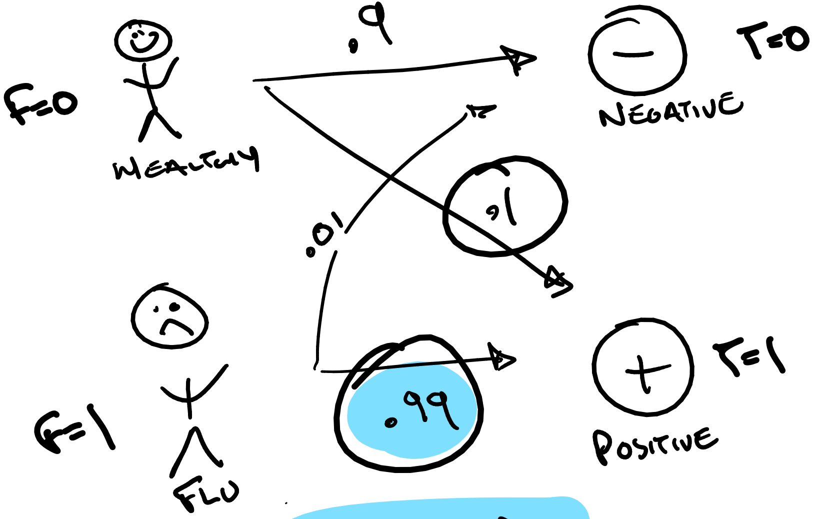
$$P(F=1 \mid T=1) = \frac{P(T=1 \mid F=1)\,P(F=1)}{P(T=1)}$$

$$= \frac{.99 \cdot .04}{.99 \cdot .04 + .1 \cdot .94}$$

$$P(T=1 \mid F=1) = .99$$

Bayes Rule Ex

Given flu occurs in .04 of population, what is the probability one has flu given they test positive?



$$P(T=1) = P(T=1 \mid F=1)\,P(F=1) + P(T=1 \mid F=0)\,P(F=0)$$

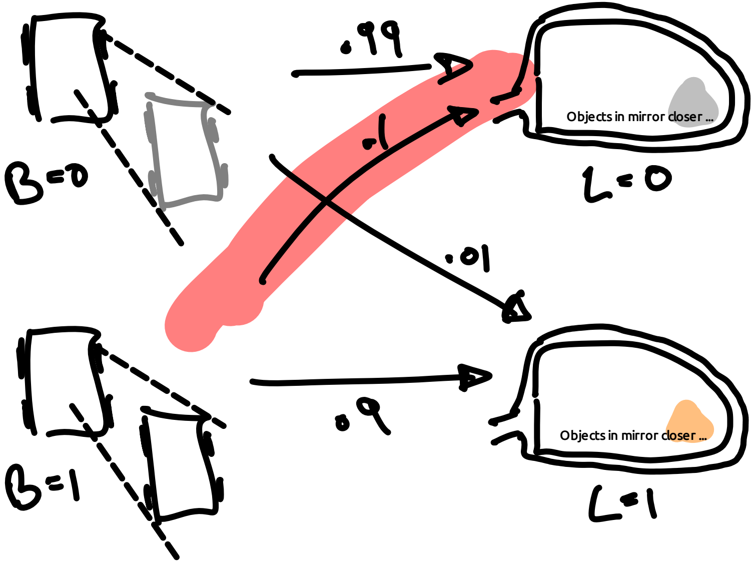
$$= .99 \cdot .04 + .1 \cdot .94$$

$$P(T=1 \mid F=1) = .99$$

In Class Assignment 1

L=1 LIGHT ON   B=1 CAR BLIND SPOT

A blind spot monitor produces a warning light (L=1) when it estimates that a car is in one's blind spot (B=1). Given that the light is off, whats the probability that a car is one's blind spot? (Assume that a car is in your blindspot .02 of the time while driving.)



$$P(B=1 \mid L=0) = \frac{P(L=0 \mid B=1)\,P(B=1)}{P(L=0)}$$

$$= \frac{(.1)(.02)}{(.99)(.98)+(.1)(.02)}$$

L=1 LIGHT ON   B=1 CAR BLIND SPOT

In Class Assignment 1

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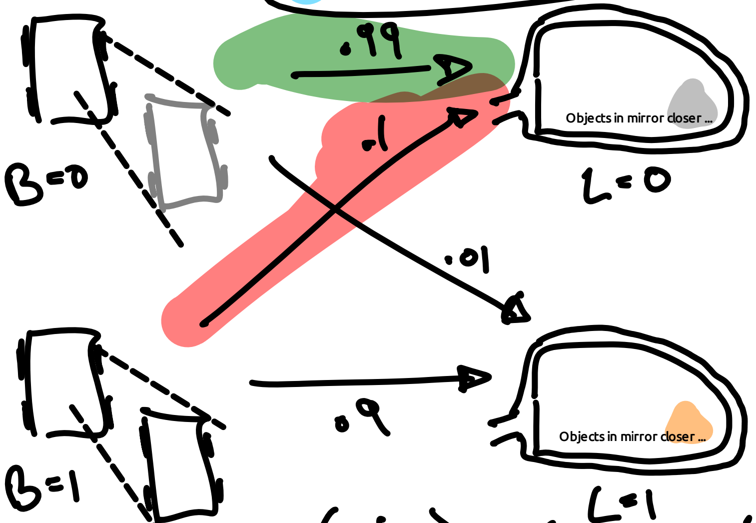


$$P(L=0) = P(L=0\ B=0) + P(L=0\ B=1)$$

$$= P(L=0 \mid B=0)P(B=0) + P(L=0 \mid B=1)P(B=1)$$

$$= (.99)(.98) + (.1)(.02)$$

$$P(L \mid B)P(B) = P(LB)$$

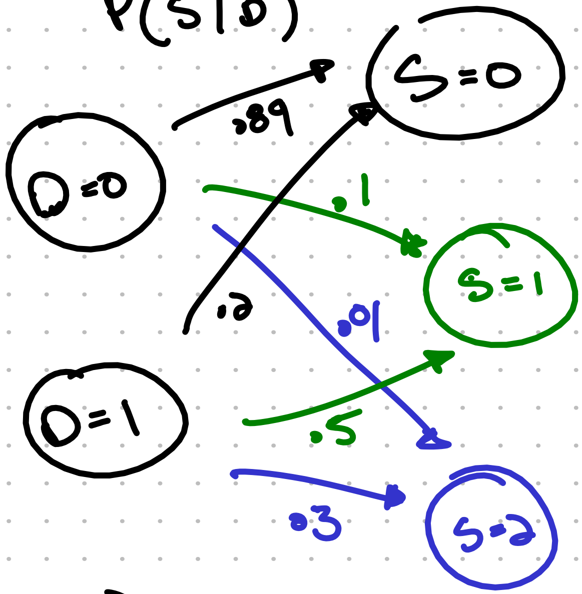
Making Bayes more useful (non-binary A, B variables):

Bayes is applicable in problems where each variable has more than 2 states too!

(see quick example on next page)

Bayes Practice

$P(S|D)$



$P(D=1) = .09$

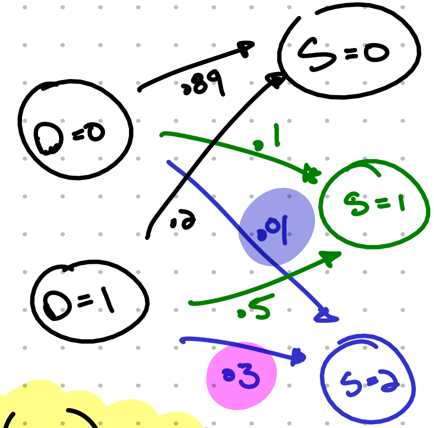
D=0 No Gold Deposit
D=1 Gold Deposit

S=0 No Gold in Stream
S=1 Little Gold in Stream
S=2 Much Gold in Stream

What is Prob of Deposit given Much Gold in Stream?

$$P(D=1 \mid S=2) = \frac{P(S=2 \mid D=1)\, P(D=1)}{P(S=2)}$$

$$= \frac{P(S=2 \mid D=1)\, P(D=1)}{\sum_{d} P(S=2,\ D=d)}$$



$P(D=1) = .09$

$$= \frac{P(S=2 \mid D=1)\, P(D=1)}{P(S=2 \mid D=0)\, P(D=0) + P(S=2 \mid D=1)\, P(D=1)}$$

$$= (.3 \cdot .09) \Big/ \left(.01 \cdot .89 + .3 \cdot .09\right) \approx .752$$

Making Bayes More Useful (parametric likelihoods):

Let's build models / problems where the conditional P(B|A) is parametric
- binomial distribution
- poisson

If a stream is near a gold deposit, one typically finds a gold nugget after an hour of sifting.

$P(X \mid D=1) = \text{Poisson}(\lambda = 7)$

If a stream is not near a gold deposit, one typically finds a gold nuggest after a full day of sifting work (10 hours).

$P(X \mid D=0) = \text{Poisson}(\lambda = .7)$

1% of streams are near gold deposits.

$P(D=1) = .01$

If we find 3 nuggets after 7 hours of sifting a particular stream, whats the probability that this stream is near a gold deposit?

X = # NUGGETS FOUND IN 7 HRS SIFTING

D = 1 IF STREAM NEAR DEPOSIT

If a stream is near a gold deposit, one typically finds a gold nugget after an hour of sifting.

$$P(X \mid D=1) = \text{Poisson}(\lambda = 7)$$

If a stream is not near a gold deposit, one typically finds a gold nuggest after a full day of sifting work (10 hours).

$$P(X \mid D=0) = \text{Poisson}(\lambda = .7)$$

1% of streams are near gold deposits.

$$P(D=1) = .01$$

If we find 3 nuggets after 7 hours of sifting a particular stream, whats the probability that this stream is near a gold deposit?

$$P(D=1 \mid X=3) = \frac{P(X=3 \mid D=1)\,P(D=1)}{P(X=3)}$$

$$P(x=3) = P(x=3\ D=0) + P(x=3\ D=1)$$

$$= P(x=3 \mid D=0)\,P(D=0) + P(x=3 \mid D=1)\,P(D=1)$$

$$P(D=1 \mid x=3) = \frac{P(x=3 \mid D=1)\,P(D=1)}{P(x=3 \mid D=0)\,P(D=0) + P(x=3 \mid D=1)\,P(D=1)}$$

$$\simeq \frac{(.052)(.01)}{(.0283)(.99) + (.052)(.01)} \simeq .0182$$

BAYES RULE TERMS HAVE NAMES

T TARGET VARIABLE OF INTEREST (GOLD DEPOSIT NEAR)

E EVIDENCE (# NUGGETS)

likelihood: probability of evidence under each possible target outcome

"a priori" / prior distribution of target variable before observing any evidence

$$P(T|E) = \frac{P(E|T)\,P(T)}{P(E)}$$

"a posteriori" / posterior the probability of target variable after observing the evidence

If a stream is near a gold deposit, one typically finds a gold nugget after an hour of sifting.

If a stream is not near a gold deposit, one typically finds a gold nuggest after a full day of sifting work (10 hours).

LIKELIHOOD

1% of streams are near gold deposits.

PRIOR

If we find 3 nuggets after 7 hours of sifting a particular stream, whats the probability that this stream is near a gold deposit?

POSTERIOR

X = # NUGGETS | 7 HRS EVIDENCE

G = 1 DEPOSIT TARGET

In Class Assignment

$P(N|C=0) = \text{Binom}(n=5, p=.05)$

In a typical box of chocolates, only 5% of chocolates are coconut flavored.

In a "coconut special", 50% of the chocolates are coconut flavored.

$P(N|C=1) = \text{Binom}(n=5, p=.5)$

If one selects 5 chocolates out of a box and observes that 3 are coconut flavored, whats the probability that the box is a "coconut special" box?

$P(C=1|N=3)$

Assume that coconut special boxes are as common as typical chocolate boxes.

$C=1$ BOX OF CHOCOLATE IS COCONUT SPECIAL

0 IT'S NOT

$N=$ # OF COCONUT CHOCOLATES IN SAMPLE OF 5 CHOCOLATES FROM BOX

$$P(C=1 \mid N=3) = \frac{P(N=3 \mid C=1)\,P(C=1)}{P(N=3)}$$

$$P(N=3) = P(N=3\ C=0) + P(N=3\ C=1)$$

$$= P(N=3 \mid C=0)\,P(C=0) + P(N=3 \mid C=1)\,P(C=1)$$

INDEPENDENCE + CONDITIONAL PROB

INDEPENDENCE

INTUITION:

Random variables x, y are independent if observing any outcome of one doesn't impact our beliefs about the other.

ALGEBRA:

FOR EACH OUTCOME PAIR x, y

$$P(X=x, Y=y) = P(X=x)\,P(Y=y)$$

Bayes Rule shows the equivilence of the algebraic and intuitive definitions above!

INDEPENDENCE + CONDITIONAL PROB

INDEPENDENCE

INTUITION:

Random variables x, y are independent if observing any outcome of one doesn't impact our beliefs about the other.

ALGEBRA:

FOR EACH OUTCOME PAIR x, y

$$P(X=x\, Y=y) = P(X=x)\,P(Y=y)$$

$$P(X|Y) = \frac{P(X\,Y)}{P(Y)} = \frac{P(X)P(Y)}{P(Y)} = P(X)$$

Notice that P(X|Y) = P(X). Observing Y has no impact on the prob of X!