NAME:

CS1800 Discrete Structures

Exam Practice 2

Strange & Varma *Date: October 30, 2026*

- **Actual vs. Practice:** The primary difference in the actual exam vs. this practice exam is the length. Each (sub)topic/concept will still show up in the actual exam but the number of times it shows up will be once instead of multiple occurrences as in this practice. The relative proportion of points will be similar in the actual exam, but the total will be different. The instructions below will also be the same in the actual exam.

- **Logistics:** Read all these instructions. Write your name on the top of every page. **Write your ID in the blank space provided above the points table on this page.**
- **Duration, Problems and Points:** This exam lasts for **90 minutes** and has **11** problems totaling **290** points. Make sure to read all problems to decide an order to attempt them that would allow quickest progress.
- **Partial Credit:** Points are determined on the basis of both the correctness and the clarity of your solution. Show your work, as partial credit may be given. Present the best solution/approach you can, even if it doesn't meet all desired properties.
- **Reference sheet:** You are allowed to bring one 8.5x11–inch cheat sheet, with anything written/typed on it, only one side of the sheet can be used. No other materials or devices are permitted. No collaboration of any kind is permitted.
- **Policy on cheating:** Students who violate the above rules on scholastic honesty are subject to disciplinary penalties. Any student caught cheating will *immediately* receive an **F** (failing grade).

  Sign below and confirm that you are strictly abiding by the rules of this test.

Signature: _____

ID: _____

NAME: _____

Question	1	2	3	4	5	6	7	8	9	10	11	Total
Points	15	15	40	41	20	24	20	20	40	35	20	290

Best of luck!

Table 1: Laws of Set Operations. A, B, C are arbitrary sets; U is the universal set; $\emptyset$ is the empty set.

Commutative Laws	$A \cap B \equiv B \cap A$ $A \cup B \equiv B \cup A$
Associative Laws	$(A \cap B) \cap C \equiv A \cap (B \cap C)$ $(A \cup B) \cup C \equiv A \cup (B \cup C)$
Distributive Laws	$A \cap (B \cup C) \equiv (A \cap B) \cup (A \cap C)$ $A \cup (B \cap C) \equiv (A \cup B) \cap (A \cup C)$
Identity Laws	$A \cap U \equiv A$ $A \cup \emptyset \equiv A$
Annihiltor/Domination Laws	$A \cap \emptyset \equiv \emptyset$ $A \cup U \equiv U$
Idempotence Laws	$A \cap A \equiv A$ $A \cup A \equiv A$
Complement/Negation Laws	$A \cap A^C \equiv \emptyset$ $A \cup A^C \equiv U$
Double Negation Law	$\left(A^C\right)^C \equiv A$
Absorption Laws	$A \cap (A \cup B) \equiv A$ $A \cup (A \cap B) \equiv A$
De Morgan's Laws	$\overline{(A \cap B)} \equiv \overline{A} \cup \overline{B}$ $(A \cup B)^C \equiv A^C \cap B^C$

Problem 1 (6-bit Two's Complement) 5+5+5

In this problem, all two's complement is in 6-bit two's complement notation.

(a) Convert -30 into 6-bit two's complement notation.

(b) The numbers $x = 111101$ and $y = 111001$ are in two's complement notation. Add these, verify whether there is overflow or not.

(c) Convert the number $z = 110101$ from two's complement into decimal.

Problem 2 (5-bit Two's Complement) 5+5+5

In this problem, all two's complement is in 5-bit two's complement notation.

(a) Convert -12 into 5-bit two's complement notation.

(b) The numbers $x = 11101$ and $y = 11110$ are in two's complement notation. Add these and verify whether there is overflow or not.

(c) Convert the number $z = 11010$ from two's complement into decimal.

Problem 3 (Laws of Set Operations) 4 x 10

Simplify the following expressions using laws of set operations while labeling the law used in each step. You do not need to specify when the commutative or associative laws are being used.

Note: In the actual exam, the format will be similar to the boolean laws question where you are asked to fill in gaps. The practice is formatted this way to give a slightly more comprehensive practice.

(a) $\overline{(\overline{B} \cup (B \cap \overline{C})) \cap \overline{C} \cap B \cap B \cap \overline{A}}$

(b) $(\overline{B} \cup (C \cap \overline{C} \cap B)) \cap (C \cup C \cup A)$

(c) $C \cup \big(\big((\overline{B} \cap A\big) \cup \overline{C} \cup \overline{A}\big) \cap \big(\overline{B} \cup A \cup A \cup \overline{B}\big)\big)$

(d) $\Big(\big(B \cap A \cap C\big) \cup \big(C \cap B \cap \overline{A}\big)\Big)^{C}$

Problem 4 (Sets) 2+4+2+2+3+5+3+4+5+4+3+4

In this problem, the universe is the 10 digits $U = \{0, 1, 2, \ldots, 9\}$. We also have the sets $A = \{1, 2, 3, 4\}$, $B = \{2, 3, 5, 7\}$, $C = \{0, 2, 4, 8\}$ and $D = \{0, 1, 4, 9\}$.

(a) True/False: $2 \in B \cap C$.

(b) Write the set $\{x \in D \mid x > 0\} \Delta \{k^2 \mid k \in B \cap C\}$ in Roster notation.

(c) True/False: $(A \cup D) \subseteq U$.

(d) True/False: $(B \cup \overline{A}) \in \mathcal{P}(U)$.

(e) How many elements will be there in $A \times D$?

(f) Write the set $A \times D \cap \{(n, n^2) \mid n \in U\}$ in set builder notation in as simplified a form as possible.

(g) What is the set: $U \setminus (B \cup D)$ in roster notation?

(h) Translate the following English description into set builder notation: "The set of all pairs where the first element is a subset of the universe and the second element is the cardinality of the first element".

(i) Describe what the following set given in set builder notation contains in English:

$$\{S \subseteq U \mid |S| = 2, \forall a \in S, \forall b \in S : a \neq b \wedge a + b \text{ is even}\}$$

(j) What is the power set of $C \cap D$?

(k) How many elements would there be in $U - C$?

(l) How many elements would there be in the power set of $U - C$?

Problem 5 (To and From Venn) 4 x 5

(a) Shade the Venn diagram in Figure 1 so that exactly the region represented by $A \setminus (B \cup C)$ is shaded.

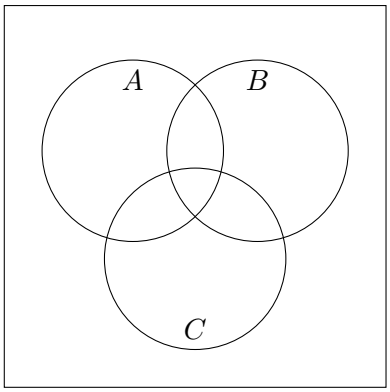


Figure 1: Venn diagram to shade.

(b) Shade the Venn diagram in Figure 2 so that exactly the region represented by $(A \Delta B) \setminus C$ is shaded.

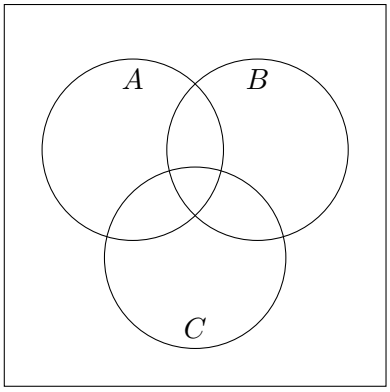


Figure 2: Venn diagram to shade.

(c) Given the shaded Venn diagram in Figure 3, give an expression using set operations that represents the shaded region.

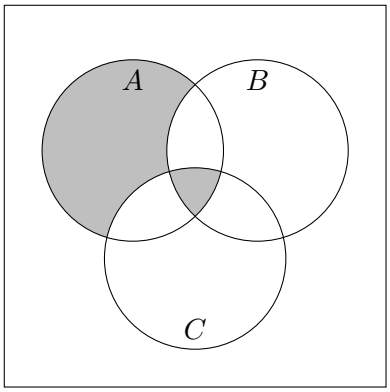


Figure 3: Shaded Venn diagram to find expression for.

Expression:

(d) Given the shaded Venn diagram in Figure 4, give an expression using set operations that represents the shaded region.

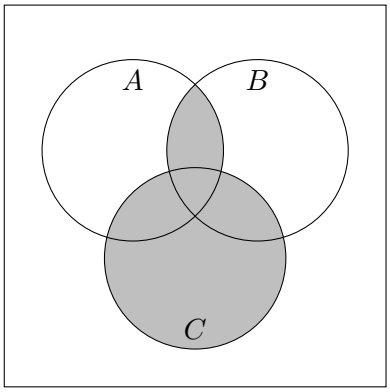


Figure 4: Shaded Venn diagram to find expression for.

Expression:

Problem 6 (Miscellaneous Counting) 4 x 6

(a) 35 students from Northeastern university and 40 students from Boston university visit a museum. The museum functions with limited capacity and allows 20 students from one of the two schools to visit on that day. In how many ways can a team of 20 students visit the museum on that day?

35 NU 40 BU

order doesn't matter
rep not ok

1 group of 20 from either
case 1: 20 from NU $\binom{35}{20}$
case 2: 20 from BU $\binom{40}{20}$

$$\boxed{\binom{35}{20}+\binom{40}{20}}$$

(b) How many integers are between 0 and 999999 (including both) which don't have any digit repeated side-by-side. For example, 112 wouldn't be counted as the 1 digits are adjacent but 121 is a valid output.

valid: 5, 56, 78, 101, 123
invalid: 55, 77, 110, 11

cases: # digits
1 digit: 10 → 0, 1, ..., 9
2 digits: 9 · 9 → whatever wasn't chosen or 0
↳ can't be zero
3 digits: 9 · 9 · 1 → not in last place
not 0 ↳ not chosen

total: $10 + 9^2 + 9^3 + 9^4 + 9^5 + 9^6$

(c) How many ways can the letters in the word "MISCELLANEOUS" be rearranged?

rearranging letters: order matters
repetition ok? nah

start: all letters distinct
then: save for overcounting

n = # letters
$P(13,13) = 13!$
overcount = L_1, L_2 $2!$
E_1, E_2 $2!$
S_1, S_2 $2!$
$2! \cdot 2! \cdot 2!$

9 (d) There are 11 problems in this exam. How many ways can write this exam if your choices for each problem is to to leave it blank or to attempt something?

each Q: blank, not blank } 2 choices
rep ok
order matters
n^k

2^{11}

Problem 7 (Racing) 6+7+7

A horse race has 12 horses.

(a) How many different ways can the podium be arranged?
(The podium has a spot for only the 1st, 2nd and 3rd place horse)

order matters

no rep

$P(12,3) = 12 \cdot 11 \cdot 10$

(b) How many different ways can the horses finish the race such that neither Grand Valor, Seabiscuit or Secretariat finish in first place?

before restrictions: $P(12,12) = 12!$

Subtract invalid: S wins, everyone else $11!$
GV wins, everyone else $11!$
Sec wins, everyone else $11!$

$12! - 3 \cdot 11!$

(c) How many different ways can the horses finish the race such that one horse (Grand Valor) always beats both Seabiscuit and Secretariat?

Problem 8 (Pigeons at Ruggles) 6+7+7

There are 35 pigeons in a flock that roosts on the top of Ruggles.

(a) What is the maximum number of nests such that there is a nest with a minimum of three pigeons?

(b) If there are 18 nests on top of Ruggles for the 35 identical pigeons, how many different ways can the pigeons be arranged?

(c) Two of the 35 pigeons have paired off and now will share one of the 18 nests with only themselves, eliminating the nest for the other pigeons, how many ways can the remaining identical pigeons be arranged in the remaining nests?

Problem 9 (Cards) **5 x 8**

There are a total of 36 cards. Each card has a number, from 1 to 9 as well as a color from: Cyan, Magenta, Yellow or Black such that every possible number+color combinations exists exactly once. So, the cards can be labelled using the elements of: $\{1, 2, \ldots, 9\} \times \{\text{Cyan, Magenta, Yellow, Black}\}$. How many ways can:

(a) these 36 cards be separated into 6 piles named A, B, C, D, E, F such that each pile has an equal number of cards?

(b) these 36 cards be separated into 6 unlabeled, indistinguishable piles such that each pile has an equal number of cards?

(c) we arrange the odd numbered cards in order such that all same color cards are together?

(d) six cards be chosen such that there are exactly two cards from any three separate colors?

(e) nine cards be chosen such that either, they are all the same color or there are three cards of three different colors?

Problem 10 (What's the chance?) 5+10+10+10

There are three bins named A, B, and C. Bin A contains 3 white and 4 red balls. Bin B contains 5 white and 5 red balls. Bin C contains 6 white and 2 red balls.

(a) What is the probability of picking 3 white balls from bin B, if they are picked one after the other, without replacement (without being put back in before next ball is picked)?

(b) What is the joint probability table of picking one ball from bin A and one from bin B?

	$A =$ White	$A =$ Red
$B =$ White		
$B =$ Red		

Table 2: Joint probability table of picking a single ball from A and B

(c) If one ball is picked from each of the 3 bins, then what is the probability that there are two white and one red ball?

(d) What is the probability that two balls are picked from each of the three bins and all the first balls are white and all the second balls drawn are red?

Problem 11 (That's unlikely) **4 x 5**

You roll a 20-sided dice with faces numbered from 1 to 20. What is the probability of the following events:

(a) The dice roll is 1.

(b) The dice roll is an even number.

(c) The dice roll is greater than 15.

(d) The dice is rolled twice and both rolls are the same.