CS1800
10/9 - Fri :)

Admin

- Tue no class 10/13
- PS3 due 10/11
- proposal due 11/1

Agenda

1. Set equality
2. Set cardinality
3. Set Functions

$\mathbb{Z}$ $\mathbb{Z}^+$ $\mathbb{N}$

Ints positive natural

1. Set Equality

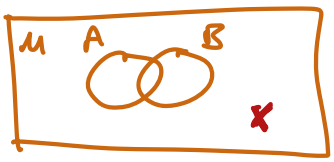
$A = B$ $\quad \forall x \, [(x \in A \Rightarrow x \in B) \wedge (x \in B \Rightarrow x \in A)]$

↳ prove with the laws of set equality (can to laws of logical equivalence)

$\overline{A \cap B} = (A \cup B) - (A \cap B)$?

↳ not always true

- proved with counterexample



$(\overline{A \cap B}) - B = \overline{B}$? → always true!

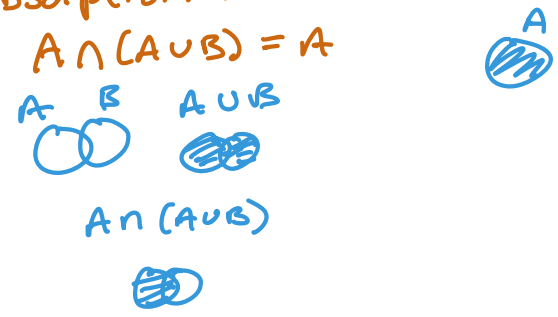
$(\overline{A \cap B}) \cap \bar{B}$ — def of difference

$(\bar{A} \cup \bar{B}) \cap \bar{B}$ — de morg

$\bar{B}$ — absorption!

Absorption law illustrated

$A \cap (A \cup B) = A$



Steps to prove

- look for laws to apply
- try to simplify
- use only $\cup, \cap, -$
- complements and counterparts next to each other
- do one step at a time

2 Set Cardinality

$|S|$ = # of distinct elements in S

$S = \{$snickers, kitkat$\}$ $\quad |S| = 2$

deck of cards

- 52 cards
- 4 suits, each with 13 values
  ↳ A, 2, 3, ..., 10, J, Q, K
  spades, clubs, diamonds, hearts
- half are red: diamonds, hearts
- half are black: spades, clubs
- face cards: J, Q, K

M = whole deck $\quad |M| = 52$

$A = \{x \mid x \text{ is a red card}\}$ $\quad |A| = 26$

$B = \{x \mid x \text{ is face card}\}$ $\quad |B| = 12$

$C = \{x \mid x \text{ is club}\}$ $\quad |C| = 13$

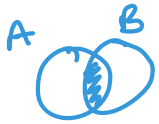
• cardinality • description

$A \cap B$

$|A \cap B| = 6$

red face cards

ex: JD, QH

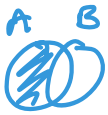


$A - B$

$|A - B| = 20$

red, non-face

ex: 7H, 3D, AD



$B - A$

$|B - A| = 6$

non-red face cards

ex: JC, QS, KC



$B - C$

$|B - C| = 9$

non-AND face cards

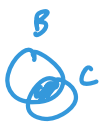
ex: JD, QH, KS

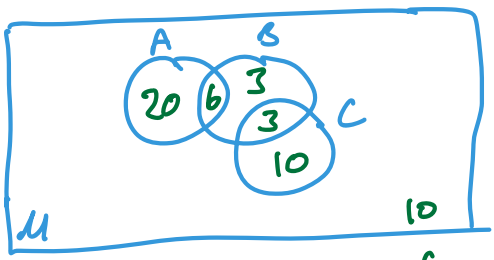


$B \cap C$

$|B \cap C| = 3$

club face





$|U| = 20 + 6 + 3 + 3 + 10 + 10 = 52$

What about $A \cup B$? all cards that are red or face

$|A \cup B| = 32$

why... $|A| + |B| = 26 + 12 = 38$!!

↳ red ↳ face

$|A \cup B| = |A| + |B| - |A \cap B|$

↳ in this example:

$|A \cup B| = 26 + 12 - 6 = 32$

↳ red face cards

How many times do we count...

QS	B	one
8D	A	one
2H	A	one
JH	A, B	two
KH	A, B	two

Belongs to two sets? gets counted twice (red face cards) $(A \cap B)$

↳ Principle of Inclusion/Exclusion (PIE)

3. Set Functions — powerset, cartesian product

2:39

Powerset $\mathcal{P}$ (subsets) $A \subseteq B$ $\forall x \; x \in A \Rightarrow x \in B$



ex) pancake toppings $P = \{c, n, b\}$

do you want ___ topping? Yes/no

- every row in truth table is a subset of P
- 2^n possible subsets

C	N	B	
T	T	T	$\{\{c, n, b\},$
T	T	F	$\{c, n\},$
T	F	T	$\{c, b\},$
T	F	F	$\{c\},$
F	T	T	$\{n, b\},$

- $\mathcal{P}(P)$ = set of all possible subsets of P

F	T	F	$\{n\}$,
F	F	T	$\{b\}$,
F	F	F	$\{\}$ $\}$

$\{c,n,b\} \subseteq \{c,n,b\}$ → every set is a subset of itself

$\{\} \subseteq \{c,n,b\}$ → empty set is a subset of every set

in general... $\mathcal{P}(A) = \{S \mid S \subseteq A\}$

- powerset is a collection
- every element is a set

ex) $A = \{1,2,3\}$

$\mathcal{P}(A) = \{\{1,2,3\}, \{1,2\}, \{2,3\}, \{1,3\}, \{1\}, \{2\}, \{3\}, \{\}\}$

	T/F?
$\{1,2\} \in \mathcal{P}(A)$	T
$\{1,2\} \subseteq \mathcal{P}(A)$	F
$\{\{1\},\{2\}\} \subseteq \mathcal{P}(A)$	T
$\{\} \in \mathcal{P}(A)$	T
$\lvert\mathcal{P}(A)\rvert = 10$	F ($\lvert\mathcal{P}(A)\rvert = 8$)

Cartesian product $A \times B$

↳ input: two sets A, B

output: set of ordered pairs (a,b)

$A \times B = \{(a,b) \mid a \in A \wedge b \in B\}$

$(a,b) \neq (b,a)$

order matters within the pair!

Context: Strangehouse, watching TV

$L = \{$ rookie, matlock, wednesday $\}$

$T = \{$ archer, family guy $\}$

$\lvert L \cap T \rvert = 0$

disjoint

evening: Leary show, then Tom show

- order matters! (Leary shows are longer)!!

How many ways to spend an evening at Strangehouse?

$L \times T = \{(r,a), (r,f), (m,a), (m,f), (w,a), (w,f)\}$

↳ L → T

$\lvert L \times T \rvert = \lvert L \rvert \cdot \lvert T \rvert = 3 \cdot 2 = 6$

$|L \cup T|$ = how many shows do we like? $|L| + |T| - |L \cap T| = 5$

$|L \times T|$ = how many ways to spend the evening? $|L| \cdot |T| = 6$