CS1800
10/6 - Tues.

Admin

- PS3 out, due 10/11
- PS2, Exam grades out
- Project details
  - ↳ proposal due 11/1
    - meeting 11/16-11/20
    - presso 12/7-12/11
    - deadline 12/12
- 10/13 no class!

Agenda

1. Intro to Sets
2. Set operations
3. Set equality proofs

1. Intro to Sets

- Set is a discrete structure ↳ contains elements
  1. unordered
  2. no duplicates

$A = \{2, 4, 6\}$

name is A
A has 3 elements

- membership

$x \in A$ "x is an element of A"

$2 \in A$ → logic statement (True)

$x \notin A$ "x is not an element of A"

$8 \notin A$ → logic statements? (T)

Snickers $\notin$ A

- set equality

$A = B$ "set A is equal to set B"

$A = \{4, 6, 2\} = \{2, 4, 6\} = \{6, 2, 4\}$

A and B have all the same elements

↳ in logic: $\forall x \quad x \in A \Rightarrow x \in B \quad \wedge \quad x \in B \Rightarrow x \in A$

$A = \{4, 6, 2\} = \{2, 2, 4, 6, 4, 2\} = \{6, 6, 6, 6, 6, 2, 4\}$

- subsets

$A \subseteq B$ "A is a subset of B"

$A \subset B$ "A is a proper subset of B"

$A = \{4, 6, 2\}$

$B = \{4, 6, 2, 17\}$

$A \subseteq B$

↳ logic $\forall x \; x \in A \Rightarrow x \in B$

$A \subseteq B$ A, B might be equal $\{18,19\} \subseteq \{18,19\}$

$A \subset B$ A, B are not equal $\{18\} \subset \{18,19\}$

every set is a subset of itself!

- Common Sets $\{\}, \emptyset$ empty set
  contains no elements
  is a subset of everything!

  $\mathbb{N}$ natural #s
  $\{0,1,2,3,\ldots\}$

  $\mathbb{Z}$ integers
  $\{\ldots,-2,-1,0,1,2,\ldots\}$

  $\mathbb{Z}^+$ positive integers
  $\{1,2,3,\ldots\}$

- universal set
  ↳ specifies the domain
  clarifies the complement

  $\mathcal{U} = \{1,2,3,4,5\}$
  $A = \{2,4,5\}$

  $\bar{A}$ in $\mathcal{U}$, not in A
  ↳ $\bar{A} = \{1,3\}$

$\bar{A}$ "A complement"

2. Set Operations

$\mathcal{U}$ universal set

$\bar{A}$ $x \in \mathcal{U} \wedge x \notin A$ (negation)

$A \cap B$ intersection (and)
$x \in A \wedge x \in B$

$A \cup B$ union (or)
$x \in A \vee x \in B$

A, B are sets
- perform operations
- result is a set!

and/or/not is everything!

$A - B$ difference
$x \in A \wedge x \notin B$

2:30

$A \Delta B$ symmetric diff $(x \in A \wedge x \notin B) \vee (x \notin A \wedge x \in B)$ exclusive or!

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3. Set equality proofs

two sets are equal (after applying set operations)
- always true? no matter in A, B
- just a coincidence from the example?

$A = \{x \mid x$'s first name D$\}$
$B = \{x \mid x$'s last name S$\}$
$U = \{DP, LS\}$

What is in these sets?

$\overline{A \cap B}$	DP, LS
$(A \cup B) - (A \cap B)$	DP, LS
$\overline{B}$	DP
$(\overline{A \cap B}) - B$	DP

→ always true?

$\overline{A \cap B} = (A \cup B) - (A \cap B)$?

↑ just a coincidence!

need to show is not always true
- counterexample: updated U
- $A = \{x \mid x$'s first name D$\}$
- $B = \{x \mid x$'s last name S$\}$

$U = \{DP, LS, JJ\}$

$(\overline{A \cap B}) - B = \overline{B}$?

next time!

always true!! (set equality laws)

choose U such that

$\overline{A \cap B} \neq (A \cup B) - (A \cap B)$

DP, LS, JJ DP, LS