Predicates & Quantifiers

CS1800 Fall 2026

A predicate is a generalization of a logic statement

Northeastern University
Khoury College of Computer Sciences

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$P(x)$	$x + 1 = 3$

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$P(x)$	$x + 1 = 3$
$S(x, y)$	$4 + x < 8y$

A predicate is a generalization of a logic statement

$P(x)$	$x + 1 = 3$
$S(x, y)$	$4 + x < 8y$

Why are these not *exactly* logic statements?



Students, write your response!

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Turn a predicate into a logic statement

Turn a predicate into a logic statement

1. Plug in values for the variables
2. Use ✨**quantifiers**✨!

Plug in values for the variables

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$P(x)$	$x + 1 = 3$	predicate

Plug in values for the variables

$P(x)$	$x + 1 = 3$	predicate
$P(14)$	$14 + 1 = 3$	logic statement

Plug in values for the variables

$P(x)$	$x + 1 = 3$	predicate
$P(14)$	$14 + 1 = 3$	logic statement
$P(2)$	$2 + 1 = 3$	logic statement

Plug in values for the variables

$S(x, y)$	$4 + x < 8y$	predicate

Plug in values for the variables

$S(x, y)$	$4 + x < 8y$	predicate
$S(10, 2)$	$4 + 10 < 8 * 2$	logic statement

Turn a predicate into a logic statement

1. Plug in values for the variables

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$\exists$ there exists

$\forall$ for all

Turn a predicate into a logic statement

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$\exists$ there exists ~ at least one thing

$\forall$ for all ~ literally ALL THE THINGS

Turn a predicate into a logic statement

1. Plug in values for the variables
2. Use ✨**quantifiers**✨!

$\exists$ there exists ~ at least one thing

$\forall$ for all ~ literally ALL THE THINGS

(Specify the domain... all WHAT things?)

✨Quantifiers ✨

$P(x)$	$x + 1 = 3$	predicate

✨Quantifiers ✨

$P(x)$	$x + 1 = 3$	predicate

domain = all integers

✨Quantifiers ✨

$P(x)$	$x + 1 = 3$	predicate

domain = all integers

logic statement

$\forall x\, P(x)$	

✨Quantifiers ✨

$P(x)$	$x + 1 = 3$	predicate

domain = all integers

logic statement

$\forall x\ P(x)$	*For all integers x, x+1 = 3*

✨Quantifiers ✨

$P(x)$	$x + 1 = 3$	predicate

domain = all integers

logic statement

$\forall x\, P(x)$	*For all integers x, x+1 = 3*

What's the truth value of this logic statement?



Students choose an option

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✨Quantifiers ✨

$P(x)$	$x + 1 = 3$	predicate

domain = all integers

logic statement

$\forall x\ P(x)$	*For all integers x, x+1 = 3*

False!

✨Quantifiers ✨

$P(x)$	$x + 1 = 3$	predicate

domain = all integers

logic statement

$\forall x\ P(x)$	*For all integers x, x+1 = 3*

How could we add quantifiers to make it True?



Students, write your response!

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✨Quantifiers ✨

$P(x)$	$x + 1 = 3$	predicate

domain = all integers

logic statement

$\forall x\ P(x)$	*For all integers x, x+1 = 3*
$\exists x\ P(x)$	*Some integer x exists such that x + 1 = 3*


STRIKE FIRST · STRIKE HARD · NO MERCY ·
Cobra Kai



Johnny



Johnny

Domain = all characters on the show



Johnny

Domain = all characters on the show

- *karate(x)*... *x* studies karate
- *johnny(x)*... Johnny fights *x*

} predicates

- *karate(x)*... *x* studies karate
- *johnny(x)*... Johnny fights *x*



- *karate(x)*... x studies karate
- *johnny(x)*... Johnny fights *x*



$$\exists x \; karate(x) \wedge johnny(x)$$

what's this in English?

Students, write your response!

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- *karate(x)*... x studies karate
- *johnny(x)*... Johnny fights *x*

$$\exists x \; karate(x) \wedge johnny(x)$$

what's this in English?

Johnny fights a karate student

- *karate(x)*... x studies karate
- *johnny(x)*... Johnny fights *x*

$$\exists x \; karate(x) \wedge johnny(x)$$

what's this in English?

Johnny fights a karate student

Johnny fights at least one person who studies karate

- *karate(x)*... x studies karate
- *johnny(x)*... Johnny fights *x*

$$\exists x \; karate(x) \wedge johnny(x)$$

what's this in English?

Johnny fights a karate student

Johnny fights at least one person who studies karate

There's someone on the show who studies karate and fights Johnny

- *karate(x)*... x studies karate
- *johnny(x)*... Johnny fights *x*

$$\forall x \; karate(x) \Rightarrow johnny(x)$$

what's this in English?

Students, write your response!

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- *karate(x)*... x studies karate
- *johnny(x)*... Johnny fights *x*

$$\forall x \; karate(x) \Rightarrow johnny(x)$$

what's this in English?

Johnny fights everyone who studies karate

- *karate(x)*... x studies karate
- *johnny(x)*... Johnny fights *x*

$$\forall x \; karate(x) \Rightarrow johnny(x)$$

what's this in English?

Johnny fights everyone who studies karate

If you study karate, then Johnny fights you

- *karate(x)*... x studies karate
- *johnny(x)*... Johnny fights *x*

$$\forall x \; karate(x) \Rightarrow johnny(x)$$

same meaning?

$$\forall x \; karate(x) \wedge johnny(x)$$



Students choose an option

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- *karate(x)*... x studies karate
- *johnny(x)*... Johnny fights *x*

$$\forall x \; karate(x) \Rightarrow johnny(x)$$

same meaning?

$$\forall x \; karate(x) \wedge johnny(x)$$

Nope!

1. Johnny fights everyone who studies karate
2. EVERYONE in this universe studies karate and fights Johnny

Negating Predicates



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Similar to Logic Negation

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- Simplify $\Longrightarrow$ first
- Go left to right
- Distribute negation across all terms in a statement
- Apply DeMorgan's law where needed

Similar to Logic Negation

- Simplify $\Longrightarrow$ first
- Go left to right
- Distribute negation across all terms in a statement
- Apply DeMorgan's law where needed
- Flip quantifiers!

Negation Example

$\forall x\ karate(x) \Rightarrow johnny(x)$

Negation Example

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Johnny fights everyone who studies karate

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$\forall x\ karate(x) \Rightarrow johnny(x)$

Johnny fights everyone who studies karate

$\neg(\forall x\ karate(x) \Rightarrow johnny(x))$

Negation Example

$\forall x\ karate(x) \Rightarrow johnny(x)$

Johnny fights everyone who studies karate

$\neg(\forall x\ karate(x) \Rightarrow johnny(x))$

$\exists x\ \neg(karate(x) \Rightarrow johnny(x))$

flip quantifier

Negation Example

$\forall x\ karate(x) \Rightarrow johnny(x)$

Johnny fights everyone who studies karate

$\neg(\forall x\ karate(x) \Rightarrow johnny(x))$

$\exists x\ \neg(karate(x) \Rightarrow johnny(x))$ — *flip quantifier*

$\exists x\ \neg(\neg karate(x) \vee johnny(x))$ — *implication*

Negation Example

$\forall x\ karate(x) \Rightarrow johnny(x)$

Johnny fights everyone who studies karate

$\neg(\forall x\ karate(x) \Rightarrow johnny(x))$

$\exists x\ \neg(karate(x) \Rightarrow johnny(x))$ — *flip quantifier*

$\exists x\ \neg(\neg karate(x) \vee johnny(x))$ — *implication*

$\exists x\ (\neg\neg karate(x) \wedge \neg johnny(x))$ — *demorgan*

Negation Example

$\forall x\ karate(x) \Rightarrow johnny(x)$

Johnny fights everyone who studies karate

$\neg(\forall x\ karate(x) \Rightarrow johnny(x))$

$\exists x\ \neg(karate(x) \Rightarrow johnny(x))$ — *flip quantifier*

$\exists x\ \neg(\neg karate(x) \vee johnny(x))$ — *implication*

$\exists x\ (\neg\neg karate(x) \wedge \neg johnny(x))$ — *demorgan*

$\exists x\ (karate(x) \wedge \neg johnny(x))$ — *double neg*

Negation Example

$\forall x\ karate(x) \Rightarrow johnny(x)$

Johnny fights everyone who studies karate

$\exists x\ (karate(x) \wedge \neg johnny(x))$

What's the negation in English?



Negation Example

$\forall x\ karate(x) \Rightarrow johnny(x)$

Johnny fights everyone who studies karate

$\exists x\ (karate(x) \wedge \neg johnny(x))$

What's the negation in English?

There is a karate student who Johnny doesn't fight!