CS1800
9/22-Tues.

Admin

- add/drop today!
- PS2 due Fri 11:59pm
- Exam #1 on 10/2 → Fri!
  - 90 min
  - PS1, PS2, this week
  - 8.5x11 inch cheat, one side only
  - no notes, phones, calcs, etc phones, etc.
  - DAS accommodation? see Sam :)

Agenda

1. normal forms
2. implications/ negations
3. converse, inverse, contrapositive

0. Proof of Logical Equivalence

↳ 2 logic statements
same truth values for all inputs

$p \Rightarrow (p \wedge q) \equiv (\neg p \vee q)$

P	Q	P∧Q	P⇒(P∧Q)	¬P	¬P∨Q
T	T	T	T	F	T
T	F	F	F	F	F
F	T	F	T	T	T
F	F	F	T	T	T

$P \Rightarrow (P \wedge Q)$

$\neg p \vee (P \wedge Q)$ def of implication

$(\neg p \vee P) \wedge (\neg p \vee Q)$ distributive law

$T \wedge (\neg p \vee Q)$ negation

$(\neg p \vee Q)$ identity law

1. truth table
2. laws of logical equivalence
   ↳ one step at a time
   label each law

$P \Rightarrow (P \wedge Q) \equiv (\neg P \vee Q)$

Definitions	$p \Rightarrow q \equiv \neg p \vee q$
Identity laws	$p \wedge T \equiv p$ $p \vee F \equiv p$
Domination laws	$p \wedge F \equiv F$ $p \vee T \equiv T$
Idempotent laws	$p \vee p \equiv p$ $p \wedge p \equiv p$
Commutative laws	$p \vee q \equiv q \vee p$ $p \wedge q \equiv q \wedge p$
Associative laws	$(p \vee q) \vee r \equiv p \vee (q \vee r)$ $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$
Distributive laws	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$ $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
DeMorgan's	$\neg(p \wedge q) \equiv \neg p \vee \neg q$ $\neg(p \vee q) \equiv \neg p \wedge \neg q$
Negation laws	$p \vee \neg p \equiv T$ $p \wedge \neg p \equiv F$
Double negation	$\neg(\neg p) \equiv p$
Absorption	$p \wedge (p \vee q) \equiv p$ $p \vee (p \wedge q) \equiv p$

1. Normal Forms → express logic statements in standard format



last time... ⊕ XOR

P	Q	P⊕Q
T	T	F
T	F	T
F	T	T
F	F	F

$(P \wedge \neg Q) \vee (\neg P \wedge Q)$

$(P \vee Q) \wedge (\neg P \vee \neg Q)$

= OR
= AND

Are they equivalent?

$(P \vee Q) \wedge (\neg P \vee \neg Q)$ FOIL

$(P \wedge \neg P) \vee (P \wedge \neg Q) \vee (Q \wedge \neg P) \vee (Q \wedge \neg Q)$ distr. law

$F \vee (P \wedge \neg Q) \vee (Q \wedge \neg P) \vee F$ negation

$(P \wedge \neg Q) \vee (Q \wedge \neg P)$ identity! done!

Disjunctive Normal Form (DNF) - an OR of ANDs

Conjunctive Normal Form (CNF) - an AND of ORs

$(P \wedge \neg Q) \vee (Q \wedge \neg P)$ [DNF] $(P \vee Q) \wedge (\neg P \vee \neg Q)$ [CNF]

P⊕Q - ex - from truth table

P	Q	P⊕Q
T	T	F
T	F	T
F	T	T
F	F	F

DNF version
- 2 rows where output is true
- either row could happen to generate a true

CNF version
- use or to put together columns

P∨Q	¬P∨¬Q	∧
T	F	F
T	T	T
T	T	T
F	T	F

ex - is that normal form?

maybe not, if:
- a negation (¬) outside a clause, instead of a variable ¬P ✓ ¬(P∨Q) ✗
- mixed signs (P∧Q)∨(R∧S) ✓ (P∧Q)∧(R∨S) ✗

CNF/DNF/neither?

$(A \vee B) \wedge (\neg C \vee D)$	CNF
$(A \wedge B) \vee (C \vee D)$	neither!
$(A \wedge B) \vee (\neg C \wedge D)$	DNF
$\neg(A \wedge B) \vee (C \wedge D)$	neither!

ex - neither → goal!

↳ laws of logical equivalence

→ $\neg(A \wedge B) \vee (C \wedge D)$

→ $(A \wedge B) \vee (C \vee D)$

$(A \wedge B) \vee C \vee D$ DNF! associative

$(\neg A \vee \neg B) \vee (C \wedge D)$ demorgan's

$\neg A \vee \neg B \vee (C \wedge D)$ associative DNF

2:45

2. Implications/Negations

$P \Rightarrow Q$ If P, then Q
Q if P
P implies Q

English ⇝ Logic ⇝ English

P = It's raining
Q = Larry doesn't run today

If it's raining, then Larry doesn't run

P	Q	$P \Rightarrow Q$	
T	T	T	rain, no run
T	F	F	rain, run
F	T	T	no rain, no run
F	F	T	no rain, run

negate the implication!

Logic $P \Rightarrow Q$

$\neg(P \Rightarrow Q)$

$\neg(\neg P \vee Q)$ def of implication

$\neg\neg P \wedge \neg Q$ demorgan's

$P \wedge \neg Q$ double negation

P	Q	$\neg(P \Rightarrow Q)$
T	T	F
T	F	T
F	T	F
F	F	F

English? I'm running and it's raining
It's raining, but I'm running

3. inverse, converse, contrapositive

implication $p \Rightarrow q$
converse $q \Rightarrow p$
inverse $\neg p \Rightarrow \neg q$
contra $\neg q \Rightarrow \neg p$

p	q	$p \Rightarrow q$	$q \Rightarrow p$	$\neg p$	$\neg q$	$\neg p \Rightarrow \neg q$	$\neg q \Rightarrow \neg p$
T	T	T	T	F	F	T	T
T	F	F	T	F	T	T	F
F	T	T	F	T	F	F	T
F	F	T	T	T	T	T	T

What relationships?

$p \Rightarrow q \equiv \neg q \Rightarrow \neg p$

imp $\equiv$ contra

$q \Rightarrow p \equiv \neg p \Rightarrow \neg q$

conv $\equiv$ inverse

Logic → English

If I win the lottery, I will get a flying car (implication)

- **Premise**: If I win the lottery, I will get a flying car.
- I do not have a flying car.
- Therefore, I didn't win the lottery.

follows!

contrapositive

- **Premise:** If I win the lottery, I will get a flying car.
- I did not win the lottery.
- Therefore, I do not have a flying car.

does not follow!

inverse

- **Premise**: If I win the lottery, I will get a flying car.
- I got a flying car.
- Therefore, I won the lottery.

converse

Do these logically follow?

$(A \wedge B) \vee (C \wedge D)$

$(A \vee B) \vee (C \wedge D)$

$A \vee B \vee (C \wedge B)$

$A \wedge B$