CS1800
9/18-Fri ☺

Admin

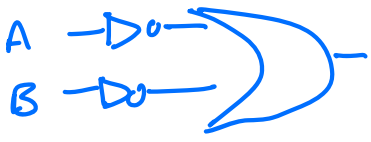
- say your name ☺
- PS1 due 11:59pm
- PS2 out tomorrow!

Agenda

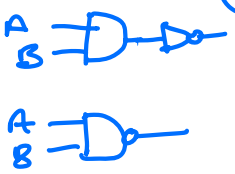
1. Circuits + logic
2. Propositional logic
3. Logical equivalence

0. Review

Circuit



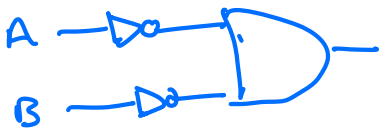
Same thing?



How we know? same output for all inputs

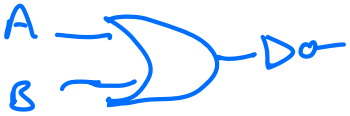
A	B	NOT-A	NOT-B	(NOT A) OR (NOT B)	(A AND B)	NOT (A AND B)
0	0	1	1	1	0	1
0	1	1	0	1	0	1
1	0	0	1	1	0	1
1	1	0	0	0	1	0

↳ Demorgan's law



(NOT A) AND (NOT B)

Something? ✓



NOT(A OR B)

1. Circuits and Logic

↳ so far...



want my computer to do this! (like, adding)



solves problem with logic / circuits

Logic:
- solve something
- prove something
- reason about something

} logic, more general!

Circuit	Logic
input wires A, B, C, ...	propositions P, Q, R, ...
0, 1	False, True

Basics I can do

NOT gate — ¬ negation (~)

everything!

AND gate	$\wedge$ conjunction
OR gate	$\vee$ disjunction
XOR gate	$\oplus$ exclusive or
	$\Rightarrow$ implication

↳ convenience!

output

P	$\neg P$
T	F
F	T

truth value

P	Q	$P \wedge Q$	$P \vee Q$	$\neg(P \wedge Q)$	$P \oplus Q$
T	T	T	T	F	F
T	F	F	T	T	T
F	T	F	T	T	T
F	F	F	F	T	F

$P \oplus Q$ ... with $\wedge, \vee, \neg$

$(P \vee Q) \wedge \neg(P \wedge Q)$

$(\neg P \wedge Q) \vee (P \wedge \neg Q)$ *

$(P \vee Q) \wedge (\neg P \vee \neg Q)$

P	Q	$\neg P$	$\neg Q$	$\neg P \wedge Q$	$P \wedge \neg Q$	$\vee$
T	T	F	F	F	F	F
T	F	F	T	F	T	T
F	T	T	F	T	F	T
F	F	T	T	F	F	F

$(\neg P \wedge Q) \vee (P \wedge \neg Q)$

2. Propositional Logic

Q: • prove
• reason
• compare

saying ⟶ precise

↳ $P, Q, R, \ldots$ inputs?

propositions: meaningful declarative truth value (no zombie's!)

		proposition?	truth value?
P	Prof. Patterson teaches CS2000	Y	T
Q	$1 + 5 = 3$	Y	F
R	Grizz is 5 y/o	Y	T
S	Is Grizz hungry?	N	

T $x + 5 = 3$ N

U For all integers x, $x + 0 = x$ Y T

Programming:

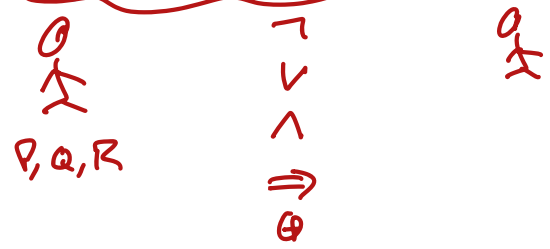
$x = 5$ // does a thing

✓ $x > 4$ // asks y/n

✓ $x + 5 == 3$ // asks y/n

2:38

English → logic → English



1. negation ¬

P = Prof. Patt teaches CS2000

¬P = Prof. Patt does not teach CS2000 ✓

¬P = Prof. Patt teaches CS1800? ∴ not the negation!

truth table respects original!

P	¬P
T	F
F	T

2. conjunction ∧

P = I am wrong

Q = I am loud

P ∧ Q = I'm wrong and loud

P ∧ Q = I'm wrong but I'm loud

P ∧ Q = Q ∧ P = I'm loud and wrong

P	Q	P ∧ Q
T	T	T
T	F	F
F	T	F
F	F	F

3. disjunction ∨

P = x is a prime number

Q = x is an odd number

} not really propositions!

what values can we plug in for x?

P ∨ Q = x is prime or odd

→ 3, 5, 7, 15, 11, , 9 ... prime and/or odd

prime, odd, or both

P	Q	P ∨ Q	P ⊕ Q
T	T	T	F
T	F	T	T
F	T	T	T
F	F	F	F

P ⊕ Q either/or P or Q, not both

→ 2, 15, 9, 21, ...

P and not Q, or Q and not P

4. implication ⇒

$P \Rightarrow Q$

Q if P
P implies Q
if P, then Q
Q if P

ex) P = it's my birthday
Q = we have cake

$P \Rightarrow Q$ If it's my birthday, then we have cake :)

Respect the original statement

P	Q	$P \Rightarrow Q$	
T	T	T	my bday, there's cake
T	F	F	my bday, no cake
F	T	T	not bday, cake
F	F	T	not bday, no cake

where's the False?

3. Logical Equivalence

two circuits can be equivalent
↳ same truth table outputs for all inputs

two logic statements can be equivalent
↳ 1. truth table
2. apply laws of logical equivalence

→ simple one is better!

Definitions	$p \Rightarrow q \equiv \neg p \vee q$
Identity laws	$p \wedge T \equiv p$ $p \vee F \equiv p$
Domination laws	$p \wedge F \equiv F$ $p \vee T \equiv T$
Idempotent laws	$p \vee p \equiv p$ $p \wedge p \equiv p$
Commutative laws	$p \vee q \equiv q \vee p$ $p \wedge q \equiv q \wedge p$
Associative laws	$(p \vee q) \vee r \equiv p \vee (q \vee r)$ $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$
Distributive laws	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$ $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
DeMorgan's	$\neg(p \wedge q) \equiv \neg p \vee \neg q$ $\neg(p \vee q) \equiv \neg p \wedge \neg q$
Negation laws	$p \vee \neg p \equiv T$ $p \wedge \neg p \equiv F$
Double negation	$\neg(\neg p) \equiv p$
Absorption	$p \wedge (p \vee q) \equiv p$ $p \vee (p \wedge q) \equiv p$

$\neg p \vee q$

A AND 1 = A
A OR 0 = A

A AND 0 = 0
A OR 1 = 1



(ex) $\neg(\neg p \vee q) \wedge T \equiv p \wedge \neg q$

$\neg(\neg p \vee q)$	identity
$\neg\neg p \wedge \neg q$	demorgan
$p \wedge \neg q$	double negation
	done!

truth table proof:

p	q	$\neg p$	$\neg p \vee q$	$\neg(\neg p \vee q)$	$\neg(\neg p \vee q) \wedge T$	$\neg q$	$p \wedge \neg q$
T	T	F	T	F	F	F	F
T	F	F	F	T	T	T	T
F	T	T	T	F	F	F	F
F	F	T	T	F	F	T	F