CS1800
11/3 - Tues.

Admin
- exam 2 Fri

Agenda
1. conditional prob
2. Bayes
3. exam prep

1. Conditional Probability

↳ last time... independent events
- rolling a die
- flipping a coin

Now events are dependent! two drawers with socks

$P(E \cap F) = ?$

- pick 2 socks from drawer A

draw from A
draw from B

Formula $P(E \cap F) = P(E) \cdot P(F)$

Product rule
E, F independent

- Drawer A: 10 pink
  5 orange

	Drawer A- Pink	Drawer A - Orange	Pr(Drawer B)
Drawer B - Pink	$2/3 \cdot 2/3 = 4/9$	$1/3 \cdot 2/3 = 2/9$	$4/9 + 2/9 = 2/3$
Drawer B - Orange	$2/3 \cdot 1/3 = 2/9$	$1/3 \cdot 1/3 = 1/9$	$2/9 + 1/9 = 1/3$
Pr(Drawer A)	$4/9 + 2/9 = 2/3$	$2/9 + 1/9 = 1/3$	

1st pick:
Pr(pink) = 2/3
P(orange) = 1/3

P(exactly one orange) = O,P or P,O
2/9 or 2/9
2/9 + 2/9
= 4/9

→ Formula

Pr(2nd pick is pink) ?
Pr(P2)

- 1st pick impacts 2nd pick

	1st = pink	1st = orange	Pr(second draw)
2nd = pink	$10/15 \cdot 9/14 = 9/21$	$5/15 \cdot 10/14 = 5/21$	
2nd = orange	$10/15 \cdot 5/14 = 5/21$	$5/15 \cdot 4/14 = 4/21$	
Pr(first draw)			

$P(P1 \cap P2) = P(P1) \cdot P(P2|P1)$ $\quad P(O1 \cap P2) = P(O1) \cdot P(P2|O1)$

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- Pr of 2nd pick depends on what happened in first pick

Formula $Pr(E \cap F) = P(E) \cdot P(F|E)$
↳ Pr of F, given that E happened

$Pr(F \cap E) = P(E \cap F) = P(F) \cdot P(E|F)$

revisiting: Pr(2nd pick is pink) = P(p2)

1. answer from table

	1st = pink	1st = orange	Pr(second draw)
2nd = pink	9/21	5/21	9/21 + 5/21 = 14/21
2nd = orange			
Pr(first draw)			

2. Formulas from table

$P(P1 \cap P2) = P(P1) \cdot P(P2|P1)$ $\quad P(O1 \cap P2) = P(O1) \cdot P(P2|O1)$

$Pr(P2) = P(P1 \cap P2) + P(O1 \cap P2)$

$= 9/21 + 5/21$

$= 2/3$

Formula

$P(E)$ but we don't directly have $P(E)$

$P(E) = P(E|F) \cdot P(F) + P(E|\neg F) \cdot P(\neg F)$

2. Bayes Theorem → probability in the real world

already had $P(E \cap F) = P(F) \cdot P(E|F)$

Formula $\quad P(E|F) = \frac{P(E \cap F)}{P(F)}$

Formula (Bayes)

$P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{P(F|E) \cdot P(E)}{P(F)}$

PS advice: write down what you know!

$P(E), P(E|F), P(F|E), P(F), P(\neg F)$ ...

Las Vegas Aces (WNBA)

- star: Aja Wilson

What we know:

- Aja plays 30/44 games
- Aces win 75% of games Aja plays in
- Aces win 40% of games she doesn't play

1. Pr(Aces win)?

2. Pr(Aja | win)?

What do we know?

A = Aja W = win

$P(A) = 30/44 = .682$ $\quad P(W) = ?$

$P(W|A) = 75\%$ $\quad P(W|\neg A) = 40\%$

$P(A|W) = ?$ $\quad P(A|\neg W) = ?$

$P(W) = P(W|A) \cdot P(A) + P(W|\neg A) P(\neg A)$

$= (.75)(.682) + (.4)(.318)$

$= .6387$

(same as Pr(2nd sock is pink))

$\rightarrow P(A|w) = \frac{P(w|A) \cdot P(A)}{P(w)} = \frac{(.75)(.682)}{.6387} = .800$

exam prep

Two's comp

Sets

Counting

probability