CS1800
10/30-Fri

Admin

- PS6 due 10/31
- proposal due 11/1
- exam 2 11/6
- Larry's OH this 1:3 4-5pm

Agenda

1. Expected value
2. Indicator Random Vars
3. (in) dependent events

1. Expected value

last time... sample space S, event space E, $E \subseteq S$
all events in E equally likely

today... probability formulas

↳ answering this question: Random Variables

- RV - outcome of particular experiment
- usually X - associated with value
- X has numeric value - we'll compute its average

↳ experiment

- change the question!
- If I run the experiment infinite times, what happens on average?

experiment: roll a 6-sided die
X is random var, associated with outcome
$P(X=2) = 1/6$ (1/6 is prob tied to every value X could be)

expected value of this RV $E[X]$

- every outcome $s_i \in S$ has:
  - $p(s_i)$ probability
  - X_i value

Formula #1

$$E[X] = \sum_{s_i \in S} p(s_i) \cdot X_i$$

experiment: roll a die

outcome $s_i \in S$	$P(s_i)$	X_i
1	1/6	1
2	1/6	2
3	1/6	3
4	1/6	4
5	1/6	5
6	1/6	6

$$E[X] = 1/6 \cdot 1 + 1/6 \cdot 2 + 1/6 \cdot 3 + 1/6 \cdot 4 + 1/6 \cdot 5 + 1/6 \cdot 6$$

$$= 3.5$$

(average of all #s when I roll the die over + over)

experiment: flip a coin ⓗ ⓣ

RV X, assoc. with # tails

$Pr(\text{heads}) = 1/2$, $P(\text{tails}) = 1/2$

S_i	$P(S_i)$	X_i
H	$1/2$	0
T	$1/2$	1

$E[X] = 1/2 \cdot 0 + 1/2 \cdot 1 = \frac{1}{2}$

experiment: flip a coin 3 times

outcomes: order matters HTH ≠ THH

X = RV assoc. with #tails

$E[X] = ?$

S_i	$Pr(S_i)$	X_i
TTT	$1/8$	3
TTH	$1/8$	2
THT	$1/8$	2
THH	$1/8$	1
HTT	$1/8$	2
HTH	$1/8$	1
HHT	$1/8$	1
HHH	$1/8$	0

$$E[X] = \sum_{S_i \in S} P(S_i) \cdot X_i$$

$E[X] = 3 \cdot 1/8 + 2 \cdot 1/8 + 2 \cdot 1/8 + 1 \cdot 1/8 + 2 \cdot 1/8 + 1 \cdot 1/8 + 1 \cdot 1/8 + 0 \cdot 1/8$

$= 3 \cdot \frac{1}{8} + 2 \cdot 3/8 + 1 \cdot 3/8 + 0 \cdot 1/8$ $= 1.5$

X = RV assoc. with # distinct values (TTT = 1, THT = 2, ...)

$E[X] = ?$

S_i	$Pr(S_i)$	X_i
TTT, HHH	$2/8$	1
THH, THT, HTH, HHT, HTT, TTH	$6/8$	2

$E[X] = 2/8 \cdot 1 + 6/8 \cdot 2$

$= 1.75$

2. Indicator Random Variable

↳ $X_i = 1$ happens, 0 doesn't happen

RVs in general X_i = assoc. with outcome of experiment

Formula #2

X_i for $i = 1, 2, \ldots, n$

$$E(X_1 + X_2 + X_3 + \ldots X_n) = E[X_1] + E[X_2] + \ldots + E[X_n]$$

why we like this...

- instead of computing all 2^n possible outcomes (HTT, THT, TTT, HHT, ...)
- isolate one coin flip!

ex - flip a coin 3 times

X is RV assoc. with # tails

above: wrote down all 2^n outcomes, computed probs

now: isolate one flip

X_1 = 1st flip

X_2 = 2nd flip

X_3 = 3rd flip

$$X_1 = \begin{cases} 1 & = \text{flip one is tails} \\ 0 & = \text{flip one is not tails} \end{cases}$$

$$X_1 = \frac{1}{2}\cdot 1 + \frac{1}{2}\cdot 0 = \frac{1}{2}$$

$$X_2 = \begin{cases} 1 & = \text{flip two is tails} \\ 0 & = \text{flip two not tails)} \end{cases}$$

$$X_2 = \frac{1}{2}\cdot 1 + \frac{1}{2}\cdot 0 = \frac{1}{2}$$

$$X_3 = X_2 = X_1$$

flip a coin 11 times?

$E[X] = 11 \cdot \frac{1}{2}$ # tails

$= 6.5$

$$E[X] = E[X_1] + E[X_2] + E[X_3]$$

$$= 1/2 + 1/2 + 1/2$$

$= 1.5$

same as long way!

(ex) draw 4 cards from a deck

52 cards in a deck, 13 spades

X assoc w/ th # spades in my hand

X_i = # spades in card i

$$x_i = \begin{cases} 1 & \text{if spade on card } i \\ 0 & \text{if not spade on card } i \end{cases}$$

$$E[X_i] = 1 \cdot 13/52 + 0 \cdot 39/52$$

$$= 13/52$$

$E[X]$ = expected # spades $= 4 \cdot \frac{13}{52} = 1$

Is surprising? nah! Draw rec 4, don't know what cards are position in deck doesn't impact your probability!

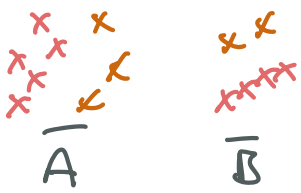
3. (In) dependent events

Two drawers A, B

Two sock colors pink, orange

Pr(sock color)?

- dependent: pick from A, then from A
- independent: pick from A, then from B
  - ↳ Exs today! (A result doesn't change B)



recall: E, S are sets $E \subseteq S$

joint probability: 2 or more events E, F, etc.

E, F where $E \subseteq S$, $F \subseteq S$

$$Pr(E \cap F) = Pr(E) \cdot Pr(F)$$

Formula #3 interested in E and F when E, F independent events

ex drawer A: 10 pink socks, 5 orange

A

$Pr(P) = 2/3$ $Pr(O) = 1/3$

drawer B: 16 pink socks, 8 orange

B

$Pr(P) = 2/3$ $Pr(O) = 1/3$

order matters! [sock from A / sock from B] A = pink, B = orange ≠ B = pink, A = orange

Joint Probability Table

- answer all the related Qs at once!
- Pr(OP) or Pr(PO) or Pr(OO) or Pr(PP)?
- Pr(any pink at all)
- Pr(A is pink)

... so many!

Joint! $P(E \cap F) = P(E) \cdot P(F)$ — independent

	Drawer A- Pink	Drawer A - Orange	Pr(Drawer B)
Drawer B - Pink	$2/3 \cdot 2/3 = 4/9$	$1/3 \cdot 2/3 = 2/9$	$4/9 + 2/9 = 2/3$
Drawer B - Orange	$2/3 \cdot 1/3 = 2/9$	$1/3 \cdot 1/3 = 1/9$	$2/9 + 1/9 = 1/3$
Pr(Drawer A)	$4/9 + 2/9 = 2/3$	$2/9 + 1/9 = 1/3$	

Joint

A pink, B pink = 4/9

A pink, B or = 2/9

A or, B pink = 2/9

A or, B or = 1/9

marginal (or!)

Pr(B pink) = 2/3

Pr(B orange) = 1/3

Pr(A pink) = 2/3

Pr(A orange) = 1/3

B pink, A pink OR B pink, A or

B or, A pink OR B or, A or

A pink, B pink OR B pink, A or

A or, B pink OR A or, B or

other Qs? $Pr(\text{any pink}) = 4/9 + 2/9 + 2/9 = 8/9$

$Pr(\text{no pink}) = 1/9$