CS1800
10/23 - Fri

Admin

- no Lacey OH today
- tutoring - ducts on CW
- PS5 due 11:59pm
- PS6 at tmw
- 11/1 proposal
- 11/6 exam 2

Agenda

How many ways...?
1. distinq. + combos
2. distinq. + permutations
3. pigeonhole

0. Review

	rep not ok	rep ok
order matters	$P(n,k)$	n^k
order doesn't matter	$C(n,k)$ $\binom{n}{k}$	#stars + #bars #bars

- start with n objects
- want k of them

$$P(n,k) = \frac{n!}{(n-k)!}$$

$$\binom{n}{k} = \frac{n!}{k!\,(n-k)!}$$

ABC
BCA
CAB
CBA
BAC
ACB

{A, C, B}

ceiling $\lceil x \rceil$ next integer up $\lceil\ \rceil$

$\lceil 3.1 \rceil = 4$ $\lceil 3.9 \rceil = 4$ $\lceil 4 \rceil = 4$ $\lceil 4.1 \rceil = 5$

1. Distinguishability & Combinations

last time... order doesn't matter, rep is ok
↳ stars + bars
indist. objects
KK, KK

↳ 4 boxes to help solve counting problems
dist. is another layer

objects - can be dist. or not
↓ cards ↓ KK, dollar bills

boxes - can be dist. or not
↳ where the object goes
↓ labels: box A, box B ↓ no labels: 2 boxes

How many ways...?
- split 10 people into varsity and JV, each of size 5?

↳ order doesn't matter
varsity ABCDE = varsity CDEAB
↳ no repetition
AAAAB not valid!

How many ways...?
- split 10 people

into 2 teams of 5?

Boxes are indistinguishable

↳ order doesn't matter!

ABCDE = EDCAB

↳ no repetition

AAAAB not valid!

before: $\binom{10}{5}\binom{5}{5} = 252$

↳ choose a team ↳ choose another team

two teams: ABCDE, VWXYZ
VWXYZ, ABCDE
same!

252 answer? too big!

overcounts! permutations of teams, nor players

$$\frac{\binom{10}{5}\binom{5}{5}}{2!} = 126$$

$\binom{10}{5} \cdot \binom{5}{5}$

varsity 2nd JV

$\binom{10}{5}\binom{5}{5} = 252$

Boxes are distinguishable
labels = V, JV

varsity ABCDE
JV VWXYZ

≠

varsity VWXYZ
JV ABCDE

How many ways...

- split 6 students into 3 groups of 2?

order matters? no
rep ok? no
boxes distinguishable? no

$$\frac{\binom{6}{2}\binom{4}{2}\binom{2}{2}}{3!} = \frac{90}{6} = 15$$

- split 10 students into a groups (size 6, size 4)?

no
no
yes

$\binom{10}{6} \cdot \binom{4}{4} = 210$

2. Permutations – Distinguishability

How many ways...?

- arrange the letters in NORTH

$n=5 \quad k=5 \quad P(5,5)=5!$

north ≠ thorn ≠ orath

in eastern...

- 2 of the 7 objects are indistinguishable
- (the rest are fine!)
- overcount by all the ways to permute the 2 e's!
- permutation $P(7,7)$ assumes everything is distinguishable!

How many ways...?

- arrange HALLOWEEN ?

$n=9 \quad P(9,9)=9!$

$$\frac{9!}{2! \cdot 2!}$$

$= 90720$

overcounting:

- permute the L's $2!$
- permute the E's $2!$

in distinguishable objects
↳ objects to arrange that we can't tell apart

How many ways...?

- arrange EASTERN
↳ $n=7 \quad k=7$ so $7!$?

eastern ≠ nerstea
↳ $e_2astern$?

permutation counts two diff arrangements.

$$\frac{7!}{2!}$$

- arrange OOFO ?

$n=4 \quad P(4,4)=4!$

overcounting:

- permute the O's $3!$

$$\frac{4!}{3!} = 4$$

OOFO
OOOF
FOOO
OFOO

2:45

3. Pigeonhole Principle

(slides!)

joinpd.com
eok awr