CS1800
10/27-Tues.

Admin

- midterm exam
- exam 11/6
  - ↳ practice out by 10/31
- proposal 11/1

Agenda

1. probability overview
2. prob + counting
3. prob + cases

1. Probability Overview

- discrete probability
  - separate outcomes
- last time: pigeonhole principle
  - ↳ outcomes that are 100% guaranteed
- today: outcomes that may or may not happen

experiment - can be repeated
- has well-defined set of outcomes ⇝ ex- rolling a die
  - [singular: die, plural: dice]

sample space - set of all possible outcomes from an experiment ⇝ ex: $\{1,2,3,4,5,6\}$
- ↳ all outcomes are equally likely (today!)

event space - any (non-empty) subset of sample space
- ↳ ex: $E_1 = \{4\}$
  $E_2 = \{1,3,5\}$
  $E_3 = \{3,4,5,6\}$

sample space S
event space E $\quad E \subseteq S$

$\sum$ = add!

prob of an event E, $pr(E)$, $Pr(E)$, $P(E)$ $= \sum_{s \in E} pr(s)$

when probs are equally likely (today...)

$pr(E) = \frac{|E|}{|S|}$

$E_1 = \{4\}$
$Pr(E_1) = \frac{|E_1|}{|S|} = \frac{1}{6}$

$E_2 = \{1,3,5\}$ (sum rule! 1 or 3 or 5)

$Pr(E_2) = \sum_{s \in E_2} pr(s) = Pr(1) + Pr(3) + Pr(5)$
$= \frac{1}{6} + \frac{1}{6} + \frac{1}{6}$
$= \frac{1}{2}$

$E_3 = \{3,4,5,6\}$

↳ $Pr(E_3) = \frac{|E_3|}{|S|} = \frac{4}{6} = \frac{2}{3}$

rolling dice one die $S = \{1,2,3,4,5,6\}$ $|S| = 6$

roll one die twice $\quad Pr(3 \text{ and then a } 4) = \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36}$

↳ product rule: $Pr(3) \cdot Pr(4)$

$\Pr(\text{anything other than 3 then 4}) = \frac{35}{36}$

roll two dice

die #1 1, 2, 3, 4, 5, 6
die #2 1, 2, 3, 4, 5, 6

what is $|S|$? 36
what is S?
↳ die #1 × die #2

$S = \{(1,1), (1,2), (1,3) \ldots, (1,6)$
$(2,1), (2,2), (2,3) \ldots (2,6)$
$\vdots$
$(6,1), (6,2), (6,3) \ldots (6,6)\}$

$\Pr(3,4) = \frac{1}{36}$ → one is (3,4)
→ 36 outcomes

(ex) game of craps - rigged to lose?
were we right to be annoyed by the rules?
↳ roll two dice

win — add to 7 or 11
lose — add to 2, 3, 12
(pass)

$\Pr(\text{lose})$

(1,1) 2
(1,2) 3
(2,1)
(6,6) 12

→ E_1

$\Pr(E_1) = \frac{|E_1|}{|S|} = \frac{4}{36}$

$\Pr(\text{win})$

(1,6) (3,4)
(6,1) (4,3)
(2,5) (5,6)
(5,2) (6,5)

→ E_2 $\Pr(E_2) = \frac{|E_2|}{|S|} = \frac{8}{36}$

$\boxed{\Pr(\text{lose}) = \frac{4}{36}}$ $\boxed{\Pr(\text{win}) = \frac{8}{36}}$ $\boxed{\Pr(\text{win or lose}) = \frac{4}{36} + \frac{8}{36} = \frac{12}{36}}$ ← same rule!

$\boxed{\Pr(\neg\text{win}) = \Pr(\text{lose}) + \Pr(\text{move on})}$ ← same rule!
$= \frac{4}{36} + \frac{24}{36}$
$= \frac{28}{36}$

$\boxed{\Pr(\text{anything else}) = \frac{36}{36} - \frac{12}{36} = \frac{24}{36}}$
↳ subtraction rule

2. Probability + Counting

S = sample space
$E \subseteq S$ = event space

$\Pr(E) = \frac{|E|}{|S|}$

- Computing the cardinality of S, E → counting question

(ex) deck of cards — 52 cards
4 suits, 13 values of each suit
26 red, 26 black

Both: draw cards from a deck
draw 2 cards
Pr(2 hearts)?

order matters → permutation

$|S| = P(52,2) = 52 \cdot 51$

$|E| = P(13,2) = 13 \cdot 12$

$Pr(E) = \frac{|E|}{|S|} = \frac{13 \cdot 12}{52 \cdot 51} = .0588$

$= 5.88\%$



order doesn't matter → combo

$|S| = \binom{52}{2} = 1326$

$|E| = \binom{13}{2} = 78$

$Pr(E) = \frac{|E|}{|S|} = \frac{78}{1326} = .0588$

$= 5.88\%$



3. Probability + cases → ex w/ birthdays
- assume all months equally likely
- assume no knowledge of anyone's bday



Pr(T was born in Oct) = 1/12

Pr(T was born not in Oct) = 11/12

Pr(¬ born in Oct) = 11/12

10 ppl
T has 9 friends

Pr(all 10 born in Oct)

Pr(P1 Oct) · Pr(P2 Oct) · Pr(P3 Oct)...

$\left(\frac{1}{12}\right)^{10} = \frac{1}{12^{10}} = 1.615e^{-11}$

Pr(none of the 10 born in Oct)

Pr(P1 not Oct) · Pr(P2 not Oct) ·

$\left(\frac{11}{12}\right)^{10} = .4189$

Pr(at least one born in Oct)

Correct → Pr(exactly one) + Pr(exactly two) + Pr(exactly 3)....

less annoying: subtraction rule
1 − not "at least one" 1 − .4189 = .58
1 − Pr(none born in Oct)



what is prob that Larry and T were born in same month?

$\Pr(\text{Larry} = \text{March}) = 1$

$\Pr(T = \text{March}) = 1/12$

same month?

$\Pr(\text{Larry month and } T \text{ month})$

$1 \cdot 1/12 = 1/12$