CS1800
10/20-Tue.

Admin

- no Laney OH 10/23
- PS5 due 10/23
- propose/group 11/1
- exam 2 11/6

Agenda

1. Breakdown
2. Box #4
3. Distinguishability

1. Breakdown

How many ways are there ...? even with same inputs, answer varies depending on the Q

$L = \{\text{Laney shows}\}$ $T = \{\text{Tom shows}\}$

$|L| = 3$ $|T| = 2$

- How many shows do we like? $|L \cup T| = |L| + |T| - |L \cap T| = 5$
- How many ways to watch L, then T? product rule! $|L \times T| = |L| \cdot |T| = 6$
- How many ways to watch L, T, L? ↪ need more info! repetition ok or no?

Key words in counting problems

and	multiply
or	add
"repetition ok"	$n \cdot n$
"no repetition"	$(n)(n-1)$
"at least" / "at most"	? sum rule subtraction rule

Solving this kind of problem...
- think about examples!

ex) 7 ppl, arrange 3 for a pic
- ✗ valid? Chen, Chen, Harper
- ✓ valid? Chen, Nolan, Harper
- same? Nolan, Chen, Harper
  - ↳ no — order matters, no repetition

	repetition not ok	repetition ok
order matters	$P(n, k)$	n^k
order doesn't matter	$C(n, k)$ $\binom{n}{k}$	?

n = # things to start

k = # things we pick

bit string = sequence of 0s, 1s
word, not numeric

How many bit strings exist...?

- of length 7?

- of length 7, that start with 1?

1 _ _ _ _ _ _

↳ leftover: bit string of length 6

$2^6 = 64$

_ _ _ _ _ _ _

0/1 0/1 0/1 .... 0/1 2 choices

2 choices each position

choose 1st and 2nd and 3rd ....

$2^7 = 128$

- of length 7, that have exactly one 0?
  - one zero, 6 1s

valid? 0 1 1 1 1 1 1 ✓

1 0 1 1 1 1 1

1 1 0 1 1 1 1

1 1 1 0 1 1 1

....

these are separate options can't coexist.

first or second or third ...

from the 7 positions, we choose where zero goes

↳ $\binom{7}{1} = 7$

- of length 7, that have exactly 2 zeroes?

valid: 0 0 1 1 1 1 1

1 1 0 0 1 1 1

1 1 1 1 0 0 1

...

0 1 1 1 0 1 1

from the 7 positions, 2 have zeroes

- order doesn't matter
  ↳ pos 1,3 = 3,1
- rep not ok
  ↳ pos 4,4 for the zeroes??

$n = 7$

$k = 2$

Combination!

$$\binom{7}{2} = \frac{7!}{5!\,2!} = \frac{7 \cdot 6}{2} = \frac{42}{2} = 21$$

- length 7, that have at most 2 zeroes?
  ↳ cases!
  - exactly 2 or
  - exactly 1 or
  - none

sum rule!

$21 + 7 + 1 = \boxed{29}$

2:33

2. The 4th Box

	repetition not ok	repetition ok
order matters	$P(n,k)$	n^k
order doesn't matter	$C(n,k)$ $\binom{n}{k}$	?

→ still a combination!

n, k?

order doesn't matter } indistinguishable objects — like 0, 1 in bit string
repetition ok

ex trick or treating : hershey, milky way, kit kat

2 kks are indistinguishable
order doesn't matter, candy in a bucket
repetition is ok

6 pieces of candy
3 treats

outcomes: 3H, 2m, 1kk
6H
1H, 4m, 1kk = 1kk, 4m, 1H

* candies
| separator

Hershey m&w KK

*** | ** | * → 3H, 2m, 1KK

****** | | → 6H

All outcomes:
- length 8
- 6 stars
- 2 bars

* | **** | * → 1H, 4m, 1KK

How many outcomes exist of length 8 with 6 stars?

≡ How many bit strings of length 8 exist with 6 zeroes?

$\binom{8}{2} = \binom{8}{6}$

Can generate... $\binom{n}{k} = \binom{n}{n-k}$

↳ repetition ok
order doesn't matter

$\binom{n}{r}$ #stars + #bars
#F bars

↳ candy: #stars 6, #bars 2 → $\binom{8}{2}$
stars + bars = 8

	repetition not ok	rep ok
order matters	$P(n,k)$	n^k
order doesn't matter	$\binom{n}{k}$	$\binom{\#\text{stars}+\#\text{bars}}{\#\text{bars}}$

(ex) Distr. 10 identical cookies among 4 of us

- order doesn't matter
  - a, b get cookies
  - b, a get cookies
- rep ok
  - Lucy gets all 10 cookies

Stars 10 (cookies)
bars 3 (separators)

$$\binom{13}{3} = \frac{13!}{3!\,10!} = 286$$

3. distinguishability (slides)