CS1800
10/16-Fri

Admin

- PS4 due 11:59pm
- PS5 out tomorrow
- 11/1 propose
- 11/6 exam #2

Agenda

1. Product rule, sum rule
2. Permutations
3. combinations

↳ combinatorics / counting

0. Review $A = \{1,2,3\}$ $B = \{3,4\}$ $C = \{2,3,5\}$

$|A \cup B|$? 4 $|A \cup B \cup C|$? 5 $\{1,2,3,4,5\}$

$|A| + |B| - |A \cap B| = |A \cup B|$ (PIE)

what is a collection? ↳ set of sets $\{\{-,-\},\{-\}\}$

what is $\mathcal{P}(\{1\})$? $\{\{1\}, \{\}\}$ $\{\{1\}, \emptyset\}$

what is $|\mathcal{P}(\{a,b\})|$ 4

↳ $\mathcal{P}(S)$ is all possible subsets of S

empty set $\{\}$ $\emptyset$

Set containing the empty set $\{\{\}\}$ $\{\emptyset\}$

what is $\{1\} \times \{2,3\}$ set of ordered pairs

$\{(1,2),(1,3)\}$

$= \{(1,3),(1,2)\}$

$\neq \{(3,1),(2,1)\}$ order matters within ordered pair $(a,b) \neq (b,a)$

1. Product rule, sum rule

from last time... $L = \{w, m, r\}$ $T = \{2, t\}$

How many ways are there...?

↳ product rule

Fundamental principle of counting

- 2 tasks
- x ways to do task 1
- y ways to do task 2

xy ways to do task 1 and task 2

$L \times T$ = the ways to spend an evening at Strangehaus

$|L \times T| = 6$

why 6? Because we pick a L shoe and we pick a T shoe

from last time... toppings = {cc, nut, ban} $|\mathcal{P}(\text{toppings})|$?

For each topping, make a decision

cc decision and nut decision and ban decision

2 * 2 * 2

$= 2^3 = 8 = |P(\text{toppings})|$

Sum rule

- 2 tasks
- x ways to do task 1
- y ways to do task 2

$x+y$ ways to do task 1 **or** task 2

(or)
- pick one topping (3)
- pick one syrup (4)

7 ways

~ slides (on website) ~

2. Permutations → like the product rule

How many ways are there...?
- does order matter
- is repetition ok

product rule: tasks, and

repetition ok, or not

ex: 8 walls - Lexy knows diff $3 \cdot 2$
Lexy knows that repeat $3 \cdot 3$

permutations: applies the product rule
- no repetition
- order matters

(ex)
- watch 2 diff Lexy shows $3 \cdot 2$
- 1st, 2nd, 3rd place finishers $535 \cdot 537 \cdot 533$

$\{a, b, c\}$ $n = 3$, $k = 2$

k-permutation: arrangement of k objects from a set

↳ order matters $a, b \neq b, a$

no repetition a, a; b, b; c, c } not valid

2-permutations: a,b; b,a; b,c; c,b; a,c; c,a $\quad \frac{3}{1^{st}} \cdot \frac{2}{2^{nd}}$

2 permutations $= 3 \cdot 2 = 6$

(ex) Halloween $= \{cc, nes, sk, kk, mw\}$

$n = |\text{Halloween}| = 5$

5-permutation $k = 5$ $\quad \frac{5}{1^{st}} \cdot \frac{4}{2^{nd}} \cdot \frac{3}{3^{rd}} \cdot \frac{2}{4^{th}} \cdot \frac{1}{5^{th}} = 120$

$n!$ = n factorial

$(n)(n-1)(n-2) \ldots \cdot 2 \cdot 1$

4-permutation $k = 4$ $\quad 5 \cdot 4 \cdot 3 \cdot 2 = 120$

3-permutation $k = 3$ $\quad 5 \cdot 4 \cdot 3 = 60$

2- permutation $k=2$ $5 \cdot 4 = 20$

1- permutation $k=1$ $5 = 5$

↳ permutation $P(n,k)$

$$P(n,k) = \frac{n!}{(n-k)!}$$

$P(5,5) = 5!$

$P(5,4) = \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{1} = 5 \cdot 4 \cdot 3 \cdot 2$

$P(5,3) = \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1} = 5 \cdot 4 \cdot 3$

$P(5,2) = \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{3 \cdot 2 \cdot 1} = 5 \cdot 4$

$P(5,1) = \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{4 \cdot 3 \cdot 2 \cdot 1} = 5$

(ex) 7 ppl in a room, in line for pic

- all 7? $7! = 5040$
- 5 of them? $\frac{7!}{2!} = 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 = 2520$
- 2 of them? $\frac{7!}{5!} = 7 \cdot 6 = 42$

3. Combinations

How many ways...?
- order doesn't matter
- no repetition

k-combination = choose k objects out of a set of n objects

$\{a,b,c\}$ $n=3$ [reminder 2-permutation = 6 of them]

↳ a,b / b,a / a,c / c,a / c,b / b,c

Combination < permutation

$\{a,b\} = \{b,a\}$

$\{a,c\} = \{c,a\}$

$\{c,b\} = \{b,c\}$

2-combination of $\{a,b,c\}$

$n=3$, $k=2$

a,b
b,c
c,a

$C(n,k)$ $\binom{n}{k}$ "n choose k"

$$\frac{n!}{k!(n-k)!}$$

start with permutation $\frac{n!}{(n-k)!}$

to denominator: $k!$

(ex) 7 ppl in a room say hello to some

order doesn't matter
no repetition

How many ways...?

- 7 of the ppl $\frac{7!}{7! \cdot 1} = 1$
- 5 of them $\frac{7!}{2!5!} = 21$
- 2 of them

[reminder: permutation answers 5040
2520
42]

$\frac{7!}{5!2!} = 21$