CS1800
Fall 2026
Sets and Set Equality

Membership

$\in$	Is a member of
$\notin$	Is not a member of. $x \notin A \equiv \neg(x \in A)$
$\subseteq$	Subset. Every set is a subset of itself. $A \subseteq B$ is expressed in logic by $x \in A \Rightarrow x \in B$ $A \subseteq B \wedge B \subseteq A \Rightarrow A = B$
$\subset$	Proper subset. $A \subset B$ is expressed in logic by $(x \in A \Rightarrow x \in B) \wedge A \neq B$
$\|S\|$	Cardinality of set S (number of distinct elements in S)

Common Sets

$\mathbb{N}$	Set of natural numbers. $\{0, 1, 2, 3, \ldots\}$
$\mathbb{Z}$	Set of integers. {..., -2, -1, 0, 1, 2, ... }
$\mathbb{Z}^+$	Set of positive integers. {1, 2, 3, ... }
$\mathbb{R}$	Set of real numbers. $\{x \mid x$ is a real number$\}$
{}, $\emptyset$	Empty set

Set Operations and Logical Definitions

$\cup$	Union. $A \cup B = \{ x \mid x \in A \vee x \in B \}$
$\cap$	Intersection. $A \cap B = \{ x \mid x \in A \wedge x \in B \}$
$\overline{A}$	Also written A^c. $\{ x \mid x \notin A \}$
-	Difference. $A - B = \{x \mid x \in A \wedge x \notin B\}$. Definition of difference: $A - B = A \cap \overline{B}$
$A \Delta B$	Symmetric difference. $A - B = \{x \mid (x \in A \wedge x \notin B) \vee (x \notin A \wedge x \in B)\}$.

Definition of symmetric difference: $A \Delta B = (A \cap \overline{B}) \cup (\overline{A} \cap B)$

$A \times B$ Cartesian Product. $A \times B = \{(a, b) \mid a \in A \wedge b \in B\}$.

$\mathcal{P}(S)$ Powerset of *S*. Collection of all subsets of *S*.

Defining Sets

Set builder notation

$A = \{ z \mid z \in \mathbb{Z} \wedge 6 < z < 10 \}$
$B = \{ n \mid n \in \mathbb{W} \wedge n = n^2 \}$
$C = \{ S \mid S \subseteq \{1, 2, 3\} \wedge |S| = 2 \}$
$D = \{x \mid x \, is \; odd \wedge 1 \leq x \leq 50 \}$
$E = \{x \mid x \, is \; prime \wedge 1 \leq x \leq 20 \}$

Roster notation

$A = \{7, 8, 9\}$
$C = \{\{1, 2\}, \{2, 3\}, \{1, 3\}\}$ // Collection -- a set of sets!
$D = \{1, 3, 5, 7, ..., 49\}$ // Roster notation with pattern

Set Equality Laws & Definitions

Difference $A - B = A \cap \overline{B}$

Identity laws
$A \cap U = A$
$A \cup \{\} = A$

Domination laws
$A \cup U = U$
$A \cap \{\} = \{\}$

Idempotent laws
$A \cup A = A$
$A \cap A = A$

DeMorgan's
$\overline{(A \cap B)} = \overline{A} \cup \overline{B}$

$\overline{(A \cup B)} = \overline{A} \cap \overline{B}$

Absorption laws
$A \cup (A \cap B) = A$
$A \cap (A \cup B) = A$

Commutative laws $A \cup B = B \cup A$

$A \cap B = B \cap A$

Associative laws $A \cup (B \cup C) = (A \cup B) \cup C$
$A \cap (B \cap C) = (A \cap B) \cap C$

Distributive laws $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Double complement $\overline{\overline{A}} = A$

Complement laws $A \cup \overline{A} = U$
$A \cap \overline{A} = \{\}$

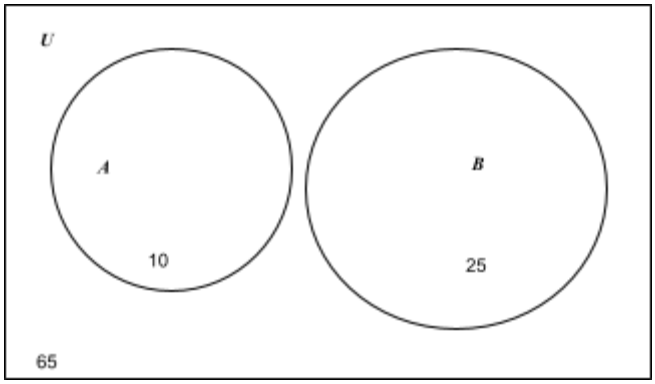
Venn Diagrams

We use a Venn diagram to graphically represent a set or group of sets within a universe. We typically represent the universal set *U* as a box containing one or more circular sets within.

For example, define *U* as the positive integers {1, 2, ..., 100}. Suppose we define two sets within our universe: *A* is the set of all perfect squares between 1 and 100 and *B* is the set of all prime numbers between 1 and 100.

$A = \{1, 4, 9, 16, 25, \ldots, 100\}$
$B = \{x \mid x \text{ is a prime } \wedge\ 1 \leq x \leq 100\}$

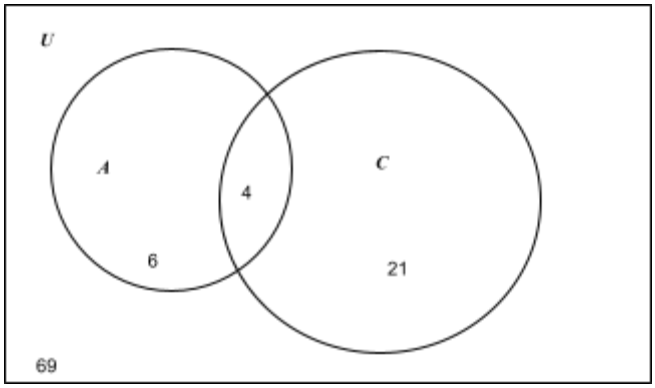
Given these sets, we have $|A| = 10$ and $|B| = 25$. We know that *A* and *B* are disjoint, because a perfect square must have a factor other than 1 and itself (and a prime has **no** factors other than 1 and itself). We can therefore represent our two sets in a Venn diagram, which shows the cardinalities of *A* and *B*. Further, it shows that $|U| = 65 + 10 + 25 = 100$.



Now let's make a change to show how we illustrate the overlap in cardinalities. We'll keep *A* the same, remove *B* from the diagram, and add a third set, *C,* the multiples of 4 between 1 and 100*:*

$C = \{4, 8, 12, 16..., 100\}$

We have that $|C| = 25$. We also know that the intersection of *A* and *C* contains the integers $\{4, 16, 36, 64\}$ and therefore $|A \cap C| = 4$. We represent these overlapping sets in a Venn diagram:



This illustration shows us that $|A| = 6 + 4 = 10, \; and \; |C| = 21 + 4 = 25$.Finally, we have that 69 integers in *U* are not in *A* and are not in *C,* and therefore $|U| = 69 + 6 + 4 + 21 = 100$.