CS1800
Fall 2026 - TF Sections
Propositional Logic

Basic Logical Operators
Logical Definitions
Logical Equivalence
Logical Equivalence Laws
English to Logic

Basic Logical Operators

$p \wedge q$ Conjunction. p and q. True if p is true and q is true, false otherwise. Same as AND gate.

$p \vee q$ Disjunction. p or q. False if p is false and q is false, true otherwise. Same as OR gate.

$\neg p$ Negation (also written $\sim p$). True if p is false, false if p is true. Same as NOT gate.

Logical Definitions

$p \Rightarrow q$ Implication. By definition, $p \Rightarrow q \equiv \neg p \vee q$. False when p is true and q is false, true otherwise.

$p \oplus q$ Exclusive-or. By definition, $p \oplus q \equiv (\neg p \wedge q) \vee (p \wedge \neg q)$. True when one of p or q is true, false otherwise. Same as XOR gate.

Logical Equivalence

$p \equiv q$ The compound statements p and q are called *logically equivalent* if they have the same truth values for all inputs.

You can demonstrate that two statements are logically equivalent by writing out their truth tables and showing that they have the same values for all inputs. For example, here is how we would show DeMorgan's law, $\neg(p \wedge q) \equiv \neg p \vee \neg q$.

p	q	$p \wedge q$	$\neg(p \wedge q)$	$\neg p$	$\neg q$	$\neg p \vee \neg q$
T	T	T	F	F	F	F
T	F	F	T	F	T	T
F	T	F	T	T	F	T
F	F	F	T	T	T	T

Logical Equivalence Laws

Definitions	$p \Rightarrow q \equiv \neg p \vee q$
Identity laws	$p \wedge T \equiv p$ $p \vee F \equiv p$
Domination laws	$p \wedge F \equiv F$ $p \vee T \equiv T$
Idempotent laws	$p \vee p \equiv p$ $p \wedge p \equiv p$
Commutative laws	$p \vee q \equiv q \vee p$ $p \wedge q \equiv q \wedge p$
Associative laws	$(p \vee q) \vee r \equiv p \vee (q \vee r)$ $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$
Distributive laws	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$ $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
DeMorgan's	$\neg(p \wedge q) \equiv \neg p \vee \neg q$ $\neg(p \vee q) \equiv \neg p \wedge \neg q$
Negation laws	$p \vee \neg p \equiv T$ $p \wedge \neg p \equiv F$
Double negation	$\neg(\neg p) \equiv p$
Absorption	$p \wedge (p \vee q) \equiv p$ $p \vee (p \wedge q) \equiv p$

English to Logic

Sometimes our job in computer science is to translate normal English, possibly-squishy sentences into formal logic, which are easier to reason about and have no room for ambiguity.

English Sentence	**Simple Propositions**	**Compound Proposition**
It's sunny but so cold!	p = It's sunny q = It's so cold	$p \wedge q$
I'll go skiing if there's enough snow	p = I'll go skiing q = There's enough snow	$q \Rightarrow p$
You need a scarf and/or a hat	p = You need a scarf q = You need a hat	$p \vee q$