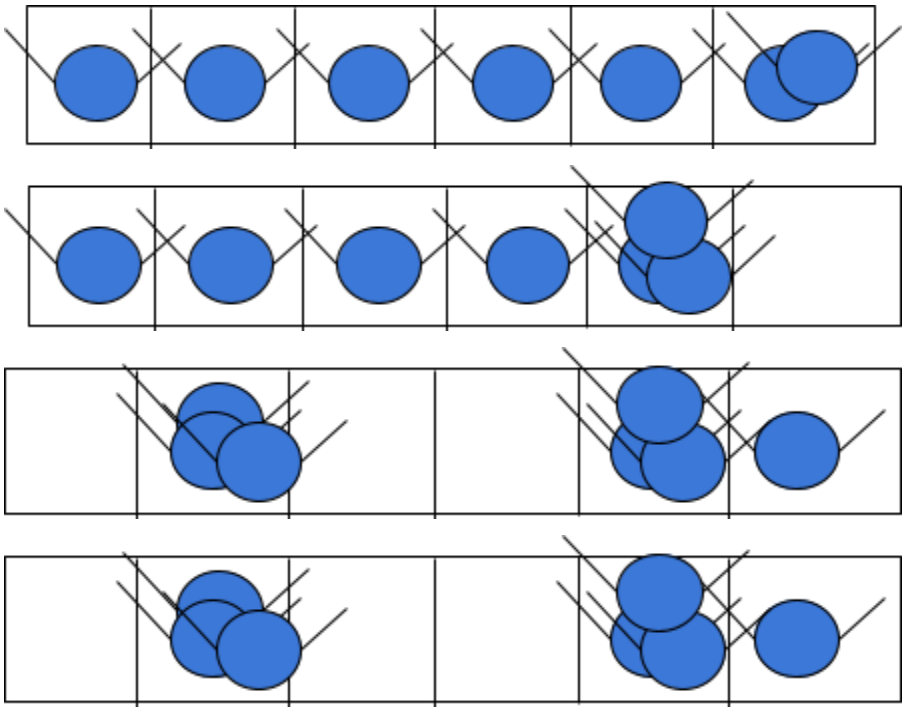
CS1800
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Pigeonhole Principle

The Pigeonhole Principle is applied to counting problems. It makes a **guarantee** about certain counting situations. It's pretty straightforward as counting principles go; its impact really comes in how we apply it, so we have a few examples at the bottom of this page.

This principle starts with the idea that you have a bunch of pigeons, and a bunch of pigeonholes, and every pigeon **must** go into a pigeonhole. If there are more pigeons than pigeonholes, then you are **guaranteed** that at least one pigeonhole contains at least 2 pigeons. This is the closest I could get to drawing pigeons; there are 6 pigeonholes in the drawings below and 7 pigeons.



6 pigeonholes and 7 pigeons. No matter how we distribute the pigeons, we are guaranteed that at least one pigeonhole contains at least 2 pigeons.

In more general terms, the Pigeonhole Principle states that, if you have N objects and $k < N$ boxes, there there must be at least one box with $\lceil N/k \rceil$objects in it.

A quick aside on that $\lceil\rceil$ *notation -- it means "ceiling". The ceiling of a number is the next highest integer.* $\lceil 2.9 \rceil = \lceil 2.5 \rceil = \lceil 2.1 \rceil = \lceil 3 \rceil = 3$

That's all you need to know for pigeonhole. Sometimes we solve for *N*, sometimes for *k*, and sometimes for $\lceil N/k \rceil$, but as long as you can identify those parts you can apply the principle. But as we mention, it's how we apply it that Pigeonhole becomes super interesting, so let's do a few examples.

Example #1 - Drawing Cards from a Deck

In a standard deck of cards, half of the cards are red and half are black. If I'm drawing cards from a shuffled deck and I care only about their colors, then I'd say I have two boxes, one for each color. If I draw three cards, then we have:

- Objects (*N*): cards draw. $N = 3$
- Boxes (*k*): possible colors (red and black). $k = 2$

I am guaranteed that I have at least $\lceil N/k \rceil = \lceil 3/2 \rceil = 2$cards of the same color.

There are four suits in a deck of cards -- clubs, diamonds, hearts, and spades. If I'm drawing cards from a shuffled deck and I care only about their suits, then I'd say I have four boxes, one for each suit.What's the minimum number of cards I need to draw to guarantee that I have at least 3 of the same suit?

- Objects (*N*): unknown
- Boxes (*k*): possible suits. $k = 4$
- Guarantee: $\lceil N/4 \rceil = 3$

In this case, I need to solve for the smallest possible value N such that $\lceil N/4 \rceil = 3$. There are 4 possible values of $N$for which $\lceil N/4 \rceil = 3$. *N* could be 12, 11, 10, or 9. We choose the smallest of these, which is 9.

Example #2 - Birthdays

If you have 367 people in a Zoom meeting, you are guaranteed that at least two of them share the same birthday (month and day). We have:

- Objects (*N*): people in the room. $N = 367$
- Boxes (*k*): possible birthdays. $k = 366$

There are at least $\lceil N/k \rceil = \lceil 367/366 \rceil = 2$people who share the same birthday.

Example #3 - Summing Numbers

We can use the Pigeonhole Principle to show that, if seven distinct numbers are selected from the set $\{1, 2, 3, \ldots, 11\}$, then two of these numbers sum to 12.

We'll construct sets by selecting two numbers in the range 1...11. Each set contains two values that sum to 12: {1, 11}, {2, 10}, {3, 9}, {4, 8}, {5, 7}, {6}.

Notice that the numbers are distinct within each set and across all sets. Each number 1..11 belongs to exactly one set. When a number is selected, place it in the unique set designated for that number. Now we can apply the pigeonhole principle:

- Objects (*N*): numbers selected. There are 7 of these.
- Boxes (*k*): the sets defined above. There are 6 of these.

Since seven numbers are selected and placed in six sets, we have $\lceil N/k \rceil = \lceil 7/6 \rceil = 2$. Therefore, at least one set will contain two numbers. By the way the sets were defined, these two numbers must sum to 12.