CS1800
Fall 2026
Handout: Counting with Repetition

Counting problems tend to be of the form **"How many ways can I...?"** It's incredibly useful to be able to solve problems like this in computer science. We already know that sometimes order matters, or not; and sometimes repetition is ok; or not. We know what to do in three of those situations, and today we're diving deep into the fourth.

If we have *n* things and we care about *k* of them, here's how we break it down

Order matters, repetition not OK	Permutation $P_k^n = \frac{n!}{(n-k)!}$
Order matters, repetition OK	Product rule: n^k
Order doesn't matter, repetition not OK	Combination: $C_k^n = \frac{n!}{k!(n-k)!}$
Order doesn't matter, repetition OK	Stars & Bars: $C_k^n = C_{\#stars}^{\#stars + \#bars}$

Indistinguishable Objects + Order Matters
Sometimes we select or order objects that are *indistinguishable* -- like in the word *DISCRETE*, you can't tell the two *E*s apart. Or imagine a tray of donuts -- we can distinguish a chocolate frosted from a jelly donut, but the chocolate frosteds are all the same as each other.

When selecting indistinguishable objects in a **permutation**, we're in a situation where order matters and repetition is OK. So we start in the second row in the table above (order matters, repetition OK -- product rule), but we need to account for overcounting.

Let's say we want to rearrange the letters in the word *GRIZZ,* and we want to know how many such permutations exist. We start by saying, OK, it's a permutation and there are five letters so we begin:

$$P(5,5) = 5! = 120$$

But this overcounts. Let's add subscripts to the letters to see why: $GRIZ_1Z_2$
The $P(5,5)$ solution counts GZ_2RIZ_1 and GZ_1RIZ_2 as two different results, but because the Z's are indistinguishable, they both are GZRIZ and should be counted only once.

There are 2! ways to arrange the two Z's, and so we correct the overcounting by dividing them out:

$$P(5,5)/2! = 5!/2! = 60$$

Stars and Bars

You'll also see this kind of problem called *balls in bins*. In this kind of situation, order doesn't matter, and we're still selecting objects from a set. But now, we have objects that are indistinguishable from one another. We're selecting some number of objects from some number of **categories of objects**.

This is basically a regular old combination question with $C_k^n = \frac{n!}{k! \cdot (n-k)!}$. We're still selecting *k* things, that part doesn't change. But *n* is what gets trippy. To get the value of *n*, we set up dividers between the categories (bars), and split the objects we're choosing (stars) into their appropriate categories.

The value for *n* then comes from the number of dividers PLUS the number of objects. So what's that altogether? It's $C_k^n$where *n* = # stars + # bars and *k* = # stars. Once we figure out *n* we just calculate the combination like any other.

Order doesn't matter, repetition OK: $C_{\#stars}^{\#stars + \# bars}$

Example Problem

A donut shop has 5 flavors of donut and you wish to select 3 donuts. **How many ways are there** to select 3 donuts?

Let's break this problem down the way we'd think about any counting problem seeing it for the first time.

Order matters?

We know this in this problem order doesn't matter. If I pick a chocolate and a sprinkled and a jelly donut, that's the same as picking sprinkled and jelly and chocolate.

Repetition ok?

Definitely. I might want all three of my donuts to be jelly. Perfectly reasonable order at the shop.

So we're in "order doesn't matter, repetition OK" territory. That means stars and bars!

We already know *k* = 3, because that's the number of things we're selecting -- same as other combination questions. Now let's figure out *n*.

Suppose the donut flavors are chocolate, boston cream, vanilla, sprinkled, and jelly. There are four dividers between the five categories of donuts, like this:

```
C     BC    V     S   J
   |     |     |     |
```

And then every time we pick a donut, it goes into its appropriate category. This would mean we picked two chocolate and one vanilla.

```
C     BC    V     S    J
xx |     | x   |     |
```

Consider that each object we care about is either a donut (the 3 we're choosing) or a divider (4 of them). In total, we have $n = 3 + 4 = 7$. And our *k* is still the number of things we're selecting, which is 3.

Now it's a combination question: $C_{\#stars}^{\#stars + \# bars} = C_3^7 = 35$.