**CS1800**
**Fall 2026**
**Handout: Counting Intro**

**Product, Sum, Subtraction Rules**

*Product Rule*
Let's say that we have **two tasks** we need to do. The product rule states:

- If you have $m$ ways to do Task One, and
- $n$ ways to do Task Two, then
- There are $n \cdot m$ ways to do Task One AND Task Two

That should sound kind-of familiar, because it's a lot like Cartesian product! Remember from last week? If have two sets with $L = \{Ozark,\ Cobra\ Kai,\ Resident\}$ and $S = \{Archer,\ Family\ Guy\}$then $|A \times B| = |A| \cdot |B| = 6$.

That was already counting, so now we're just generalizing what we already knew. There are 3 ways to watch a Laney show, 2 ways to watch a Tom show -- so there are 6 ways to watch a Laney show AND a Tom show.

Whenever you see that AND, think product rule!

> ***Example - Product Rule***
> NEU has 250 majors and 180 minors. **How many ways are there** to choose a major and a minor?
>
> We see that word AND, so we know product rule applies here. The first task is choosing a major (250 ways), and the second is choosing the minor (180). Altogether, we get $250 \cdot 180 = 45000$.

*Sum Rule*
Let's say that we have **two tasks** we need to do. The sum rule states:

- If you have $m$ ways to do Task One, and
- $n$ ways to do Task Two, then
- There are $n + m$ ways to do Task One OR Task Two

Whenever you see that word OR, think sum rule!

> ***Example - Sum Rule***
> NEU has 250 majors and 150 combined majors. But you can't do both! **How many ways are there** to choose a major or a combined-major?

We see that word OR, so we think sum rule. The first task is choosing a major (250 ways), and the second is choosing a combined major (150). Altogether, we get $250 + 150 = 400$.

*Subtraction Rule*
Sometimes you have a counting problem, and you think, oh, I should break this into cases! This often applies in situation where case 1 could happen OR case 2 OR case 3, etc. You set up your counting solution into cases, and add them all together, because of the sum rule.

If you have a lot of cases, though, it may be easier to count all *possible* choices and then subtract the *invalid* choices. This is similar to how we count the size of the complement of a set.

**Permutations and Combinations**

How many ways are there to choose $k$ things from a set of $n$ elements?

| | Repetition not allowed | Repetition allowed |
|---|---|---|
| Order matters | $P(n, k)$ | ??? |
| Order does not matter | $\binom{n}{k}$ | ??? |

Order matters but repetitions are not allowed: **Permutations**

We apply permutations when **order matters.** The permutation formula for choosing $k$objects out of a set of $n$things is:

- $P_k^n = \frac{n!}{(n-k)!}$
- Special case: $P_n^n = n!$

The permutation formula is a close personal friend of the product rule, because we end up multiplying stuff together. We like the permutation formula when we start out with a set of $n$ things, we pull $k$ things out of the set, and arrange them in a certain order.

***Example - Permutation***
10 people are running in the Olympic marathon. **How many ways are there** to have a gold medalist, silver medalist, and bronze medalist?

We have a set of 10 things, that's the runners. Order definitely matters, because first is very different than second, which is very different than third.

There are 10 ways to pick the first-place runner, AND 9 ways to pick second-place, AND 8 ways to pick third place -- so the product rule applies here, but so does the permutation formula! So we have: $P_3^{10} = \frac{10!}{(10-3)!} = \frac{10!}{7!} = 10 \cdot 9 \cdot 8 = 720.$

Order does not matter and repetitions not allowed: **Combinations**

We apply combinations when **order doesn't matter.** The combination formula for choosing $k$objects out of a set of $n$things is:

- $C_k^n = \frac{n!}{k!(n-k)!}$

The combination formula is a close personal friend of sets, because order doesn't matter in our outcomes. We like the combination formula when we start out with a set of $n$ things, we pull $k$ things out of the set, and put them into a new set. Because our outcome is also a set, we ignore ordering.

***Example - Combination***

10 people are running in the Olympic marathon *trials*. **How many ways are there** to field a team of 3 to send to Tokyo?

We have a set of 10 things, that's the runners. Order doesn't matter, because we just send the top-3 people to the Olympics. We are selecting 3 things from our set of 10, so we have:
$C_3^{10} = \frac{10!}{3!(10-3)!} = \frac{10!}{3!\cdot 7!} = 720/6 = 120.$