Oct 26

Admin

- PS6 out
  due Oct 31, Sat
- Project Proposal
  due Nov 1, Sun
  [NO Extra time]
- Tutoring assignments out (rolling)
- Midterm Eval
  ↳ Khoury Admin

Agenda

- Pigeonhole wrapup
- Probability
  ↳ Sample space
  ↳ Event space
- PMF/CMF -ish.
  Uniform distribution.

Pigeonhole: $n+1$ balls into n bins $\Rightarrow$ $\exists$ bin w/ >1 ball

→ guaranteeing existence of certain objects

general form

$x_1, x_2, \dots, x_n$ # balls in bin $i = x_i$

$\bar{x} = \frac{x_1 + x_2 + \dots + x_n}{n}$ "min ≤ avg ≤ max"

$\exists i$ s.t. $\bar{x} \le x_i$

$\exists j$ s.t. $x_j \le \bar{x}$

↓ Probability

deals with uncertainty

chance, likelihood, luck, etc.

Probability

↳ experiment w/ many possible outcome. eg. weather each day.

↳ **Event**: specific outcome ⟶ eg. is it rainy = E
↳ a specific outcome

↳ probability of an event happening.
↳ chance
↳ likelihood

$Pr(E)$ ← Event

Other notation
$Pr[E]$
$P(E)$
$p[E]$

$$Pr(E) = \frac{|E|}{|S|}$$

$|S|$ → Sample space = set of all possible outcomes

e.g. $S = \{\text{sunny, cloudy, rainy}\}$, $E = \{\text{rainy}\} \Rightarrow Pr(E) = \frac{1}{3}$

Example: 6-sided dice. Experiment: roll 6-sided dice, outcome is value of top face.

Sample space $S = \{1,2,3,4,5,6\}$

1) $Pr(\underbrace{\text{roll} < 3}_{\text{low roll}}) = \frac{|\{r \in S \mid r \leq 3\}|}{|S|} = \frac{|\{1,2,3\}|}{6} = \frac{3}{6} = \frac{1}{2} = 50\%$

2) $Pr(\text{roll is a square number}) = \frac{|\{1, 4\}|}{6} = \frac{2}{6} = \frac{1}{3}$

3) $Pr(\underbrace{\text{roll} \geq 4}_{\text{high roll}}) = \frac{|\{4,5,6\}|}{6} = \frac{3}{6} = \frac{1}{2}$

Prob. Fact:

$$Pr(\overline{E}) = Pr(S-E) = \frac{|S-E|}{|S|} = \frac{|S|-|E|}{|S|} = \frac{|S|}{|S|} - \frac{|E|}{|S|}$$

$$= 1 - Pr(E)$$

$$Pr(E) + Pr(\overline{E}) = 1$$

eg above $Pr(\text{roll} \geq 4) = Pr(\text{roll} \not\leq 3) = 1 - Pr(\text{roll} \leq 3) = 1 - 1/2 = 1/2$

↳ complement of E = roll ≤ 3

Success / Failure: Tossing coins → Heads (H) = Success
Tails (T) = Failure

↓

fair coin ⇒ $Pr(H) = Pr(T) = 1/2$

eg. Tossing 10 fair coins → HHHTHTHTTH
HHH ---- HHT
⋮
} Sample space = S

Que 1 What is $|S|$? → 2^{10}

Que 2: $Pr(\text{more H than T}) = Pr(\#\text{ of Hs} \geq 6)$

$$= Pr(\#H=6 \text{ OR } \#H=7 \text{ OR } \dots \text{ OR } \#H=10)$$

$$= \frac{|\quad\quad|}{2^{10}}$$

$$= \frac{|\#H=6| + |\#H=7| + \dots + |\#H=10|}{2^{10}}$$

$$= \frac{{}^{10}C_6 + {}^{10}C_7 + \cdots + {}^{10}C_{10}}{2^{10}}$$

Que 4 $Pr(\text{at least 2 consecutive Hs}) = 1 - Pr(\quad \text{no} \quad \text{consecutive Hs})$

$= 1 - Pr(\text{no consecutive Hs})$

HHHH H T.... } overcounting difficult to compensate for

H HHHH T.... }

$= 1 - \{ Pr(\text{no cons. Hs}, \#H=0)$
$+ Pr(\text{no cons Hs}, \#H=1)$
$+ Pr(_\ _\ \cdots, \#H=2)$
$+ \vdots$
$Pr(\ldots, \#H=10) \}$

if k Hs.
$\Rightarrow$ 10-k Ts.
$\Rightarrow$ 10-k+1 slots to place k Hs.

$\binom{10-k+1}{k}$

T _ T _ T _
pick slots to ensure no cons.

$= 1 - \left\{ \frac{\binom{11}{0}}{2^{10}} + \frac{\binom{10}{1}}{2^{10}} + \frac{\binom{9}{2}}{2^{10}} + \frac{\binom{8}{3}}{2^{10}} + \cdots \frac{\binom{6}{5}}{2^{10}} + \frac{\binom{5}{6}}{2^{10}} + \frac{\binom{4}{7}}{2^{10}} + \cdots + \frac{\binom{1}{10}}{2^{10}} \right\}$

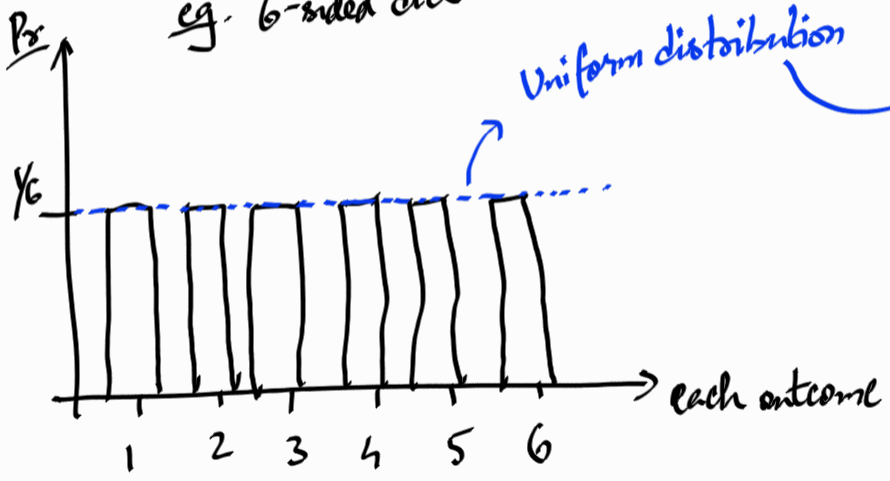
all are 0.

$Pr(\text{at least 2 consecutive Hs})$

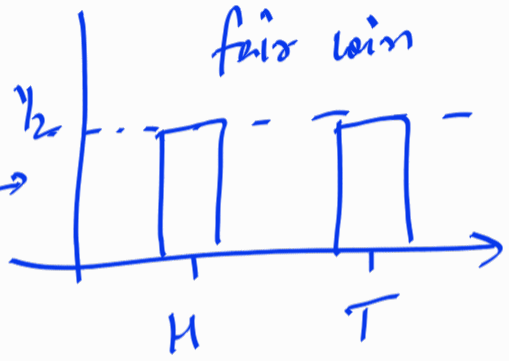
$= 1 - Pr(\text{no cons. Hs}) = 1 - \frac{{}^{11}C_0 + {}^{10}C_1 + {}^{9}C_2 + \cdots + {}^{6}C_5}{2^{10}}$

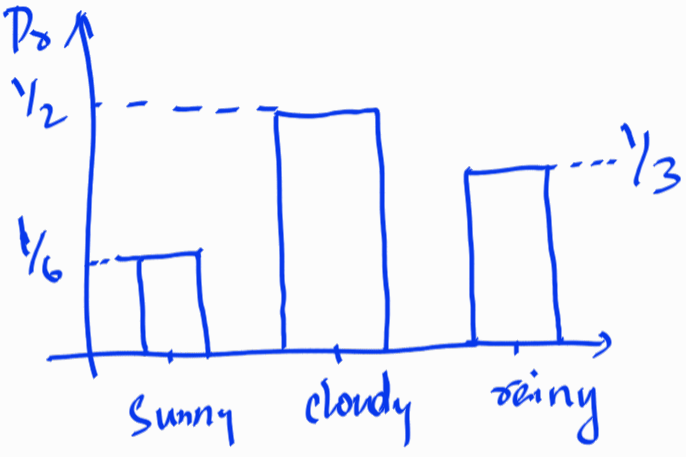
Probability Mass Function (PMF)

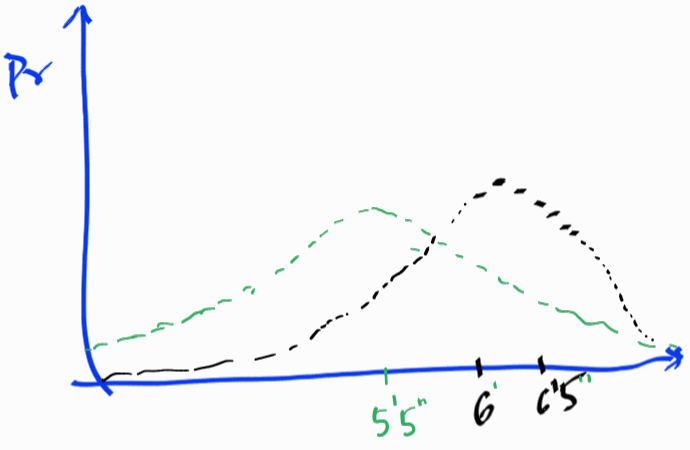
eg. 6-sided dice



Uniform distribution







height of everyone on Earth

height of basketball players