Oct 26

Admin

- PS6 is out
  due 31st, Sat
- Project Proposal
  due Nov 1st.
- Tutoring assignment
  out.

Agenda

- Wrapup Pigeonhole principle
- Probability
  - Event, Sample space
  - PDF/CDF
  - ~~Joint Probability~~ → next time

Generalized Pigeonhole

Let $x_1, x_2, \dots, x_n$ (# of balls in bin $i = x_i$)

$\bar{x} = \frac{x_1 + x_2 + \dots + x_n}{n}$ ← "x bar"

→ ① $\exists\, i$ s.t. $x_i \geq \bar{x}$

② $\exists\, j$ s.t. $x_j \leq \bar{x}$

guarantees existance of certain bins.

"min ≤ avg ≤ max"

Probability → chance / likelihood

Running an experiment → things can happen } events

Probability of event E : $Pr(E)$

$Pr[E]$

alternate notations $P[E]$, $P(E)$

Tossing coins

Events → Coin → Heads $Pr(H) = 1/2 = 50\%$

→ Coin → Tails $Pr(T) = 1/2 = 50\%$

$$Pr(E) = \frac{\#\text{ of ways } E \text{ can happen}}{\#\text{ of all possible outcomes}}$$

$$= \frac{|E|}{|S|}$$

$|E|$ → Event space.

$|S|$ → Sample space → all possible outcomes

Example : Tossing coins

$$Pr(\text{Heads}) = \frac{|\{H\}|}{|\{H, T\}|} = 1/2$$

Dice, 6 sides, numbers $S = \{1, 2, 3, \dots, 6\}$ on the faces of the dice.

$$Pr(\text{dice roll} \le 3) = \frac{|\{r \mid r \in S, r \le 3\}|}{|S|} = \frac{|\{1,2,3\}|}{6} = \frac{3}{6} = \frac{1}{2}$$

$|S|$ → sample space

$$Pr(\text{dice roll is a square number}) = \frac{|\{r \in S \mid r \text{ is square}\}|}{6} = \frac{2}{6} = \frac{1}{3}$$

Tossing multiple coins

H & T are equally likely $\leftrightarrow$ fair coin

10 fair coins

HHHHTTTHTH → 1 possible outcome

⋮

} set of all possible outcomes → Sample Space.

Que 1. $|\text{Sample Space}| = 2^{10}$

Que 2: $P_r(\text{more H than T}) = |\underline{6\text{Hs}} \text{ or } 7\text{Hs or } 8\text{Hs or } 9\text{Hs or } 10\text{Hs}|$

$\frac{10!}{6!4!} = \binom{10}{6}$

$= \dfrac{\binom{10}{6} + \binom{10}{7} + \binom{10}{8} + \binom{10}{9} + \binom{10}{10}}{2^{10}}$

2^{10}

Prob Fact: $P_r(\overline{E}) = P_r(S - E) = \dfrac{|S-E|}{|S|}$

$= \dfrac{|S| - |E|}{|S|}$

$= 1 - \dfrac{|E|}{|S|} = 1 - P_r(E)$

$P_r(E) + P_r(\overline{E}) = 1$

Q. if chance of rain = 1/10

chance of no rain = $1 - \frac{1}{10}$

$= 9/10$

Que 3 What is prob of at least 2 consecutive Hs.

$$Pr(\geq 2 \text{ consecutive } H) = \frac{?}{2^{10}} = 1 - Pr(\text{no cons. Hs})$$

↓ stars & bars

HH

HH ---------- → HHTHT HH---

_ HH---- _

----- HH---

Overcounting problem.

Not an easy approach.

$$= 1 - \Big[Pr(\text{no cons Hs}, \#H=0) + Pr(\ldots, \#H=1) + Pr(\ldots, \#H=2) + \vdots + Pr(\ldots, \#H=10) \Big]$$

(0)

$Pr(A \cup B) = Pr(A) + Pr(B)$ if A & B are disjoint

(comes from sum rule or P.I.E.)

$Pr(\text{no cons Hs}, \#H=k)$

↳ Ts are stars

10-k Ts/stars

10-k+1 slots for k Hs.

↳ $\binom{10-k+1}{k}$

_★_★_★_

$$Pr(\geq 2 \text{ consecutive } H) = 1 - Pr(\text{no cons. Hs})$$

$$= 1 - \Big[Pr(\text{no cons Hs}, \#H=0) + \ldots + \Big]$$

$$= 1 - \left[\frac{\binom{11}{0}}{2^{10}} + \frac{\binom{10}{1}}{2^{10}} + \frac{\binom{9}{2}}{2^{10}} + \frac{\binom{8}{3}}{2^{10}} + \frac{\binom{7}{4}}{2^{10}} + \frac{\binom{6}{5}}{2^{10}} + \frac{\binom{5}{6}}{2^{10}} + \ldots \right]$$

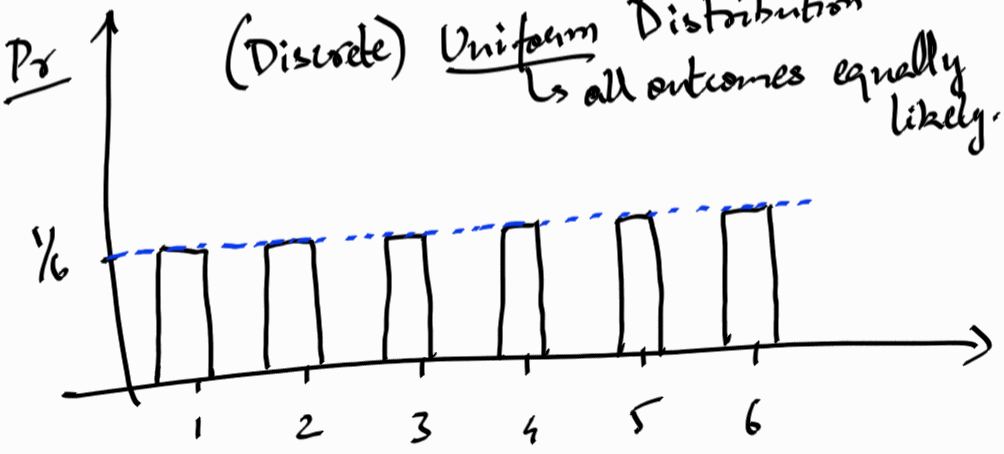
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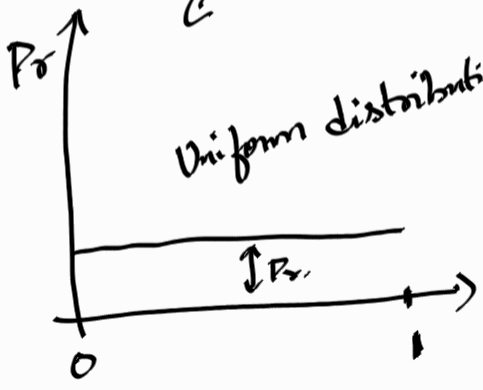
$$Pr(\geq 2 \text{ cons. Hs}) = 1 - \frac{{}^{11}C_0 + {}^{10}C_1 + {}^{9}C_2 + \ldots + {}^{6}C_5}{2^{10}}$$

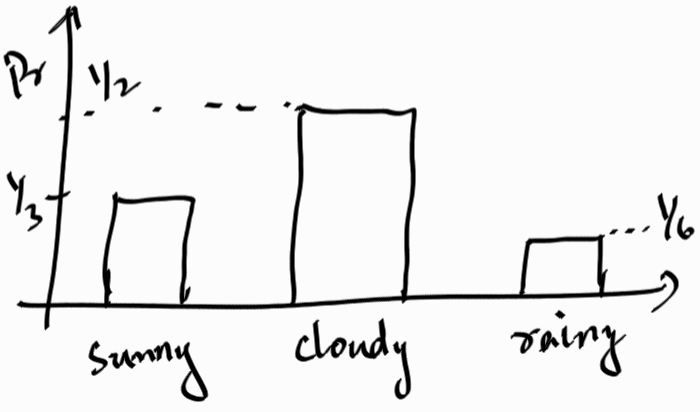
6-sided dice rolls

Probability Mass Function (PMF)

[Prob. Density Fun (PDF) for continuous]







$Pr(\text{rainy}) = 1 - Pr(\text{not rainy})$