Oct 15

Counting

- Product Rule
- Sum Rule (maybe next time?)

Admin

- final is on Dec 16, Wed, 8-10 AM
- Office Hours!

Product Rule

Recall: $|A \times B| = |A| \times |B|$

"Proof": "first element" from A is in $|B|$ pairs

"second element" — — — — $|B|$ pairs

⋮

"$|A|^{th}$ elem — — — — — $|B|$ pairs

$|A|$ times

$|B| + |B| + \cdots + |B|$

$= |A| \cdot |B|$

Extends to $|A \times B \times C| = |A| \times |B| \times |C|$

In General Product Rule

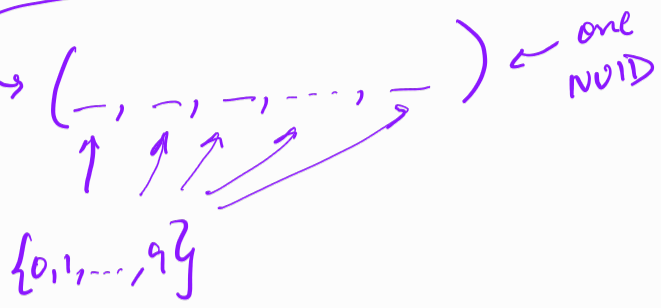
$$|A_1 \times A_2 \times \cdots \times A_n| = |A_1| \times |A_2| \cdots \times |A_n|$$

"pick this AND pick that AND ---"

- ordered list
- element must be from a set → no duplicates

Examples

- How many NUIDs exist? → 9 digits long, uses $\{0, 1, \ldots, 9\}$ ↳ A & $|A| = 10$



$$|NUIDs| = |A| \times |A| \times \cdots \times |A|$$
$$= |A|^9 = 10^9$$

if 100,000 (10^5) new NUIDs used/year

$$\#\text{ of years} = \frac{10^9}{10^5} = 10^4 = 10{,}000$$

- How many CRNs are there? → Course Registration Number
  5 digits, first digit can't be 0.

$$9 \cdot 10 \cdot 10 \cdot 10 \cdot 10 = 9 \cdot 10^4$$
$$= 90000$$

- How many 4-digit PINs exist?
  ↳ if trying one takes 10 seconds, how long to break the PIN by trying all choices

$$\#\text{ of PINs} = 10^4 \times 10 \text{ seconds} = 10^5 \text{ seconds}$$
$$\frac{10^4}{6} \text{ minutes}$$
$$\frac{10^3}{36} \text{ hours} \sim 30 \text{ hours}$$
exact = 1.157 days

Eg: Find $|\mathcal{P}(S)|$ given that $|S| = n$

Goal: count subset

↳ repr. subsets as ordered lists ↳ of the same length.

bitstring representation of subsets.

eg. $R = \{a, b, c\}$ → subsets $\{\}: (0,0,0)$, $\{a\}: (1,0,0)$, $\{b\}$: "010"

bitstring meaning $\{c\}$ = "001", $\{a,b\} = 110$, $\{b,c\} = 011$,

$\{a,c\} = 101$, $\{a,b,c\} = 111$

(↑ , ↑ , ↳)

↑ is a in subset? ↑ is b in subset? ↳ is c in subset?

$|\{(-,-,-)\}| - \in \{0,1\}\}|$

$= |\{0,1\} \times \{0,1\} \times \{0,1\}|$ → Product Rule

$|\{0,1\}| \times |\{0,1\}| \times |\{0,1\}|$

$|\mathcal{P}(R)| = 2 \times 2 \times 2 = 2^3 = 8$

$= |\{0,1\}^3|$

In general (Power set) Set S, $|S| = n$

Set of subsets $= \{(\underbrace{-,-,-,\ldots,-}_{n \text{ terms}}) | - \in \{0,1\}\} = \{0,1\}^n$

$|\mathcal{P}(S)| = |\{0,1\}^n| \xrightarrow{\text{by product rule}} |\{0,1\}|^n = 2^n$

Counting rearrangements of (distinct) things

elements of a set.

- eg. $\{a, b, c\} = A$

rearrangements: abc, bac, bca, cba, acb, cab } 6 things

$3 \times 2 \times 1 = 6$

$|A|$ $|B|$

$|B| = |A| - 1$

in general rearrangements of n elem set

$n \cdot (n-1) \cdot (n-2) \cdots 3 \cdot 2 \cdot 1 = n!$

"n factorial"

q. how many permutations of "COURSE" → 6!

eg. permutations of "AKSHAR" → $\frac{6!}{2!}$

↳ first try "AKA" → AKA, AAK, KAA

$A_1 K A_2 \rightarrow A_1 K A_2, A_2 K A_1, A_1 A_2 K, A_2 A_1 K, K A_1 A_2, K A_2 A_1$

$\frac{6!}{2!}$

using this

Try AKSHAR VARMA

w/o space $\frac{11!}{4!\,2!}$

with space $\frac{12!}{4!\,2!}$