Oct 14

Admin

- Office Hours ?
- Questions ?
- Final: Dec 16th, 8-10AM
- PS4, Prob 1, part x. "Hint" ↳ Think Contrapositive!

Counting

- Product Rule
- Sum Rule

Product Rule

Recall: $|A \times B| = |A| \times |B|$

"Proof": "1st" element of A is in $|B|$ pairs.

2nd $|B|$ - - -

⋮

$|A|^{th}$ element of A - - - $|B|$ pairs

$|B| + |B| + \dots + |B|$

$|A|$ terms

Can be extended $|A \times B \times C| = |A| \times |B| \times |C|$

In General $|A_1 \times A_2 \times \dots \times A_n| = |A_1| \times |A_2| \times |A_3| \times \dots \times |A_n|$

Product Rule whenever "this AND this AND ..."

Examples of × Rule in practice

① How many NUIDs exist?

↳ total 9 digits

↳ each digit can be anything in $\{0,1,2,\ldots,9\}$

↳ A

NUID: $\underbrace{A \times A \times A \cdots \times A}_{9 \text{ times}} = A^9$

# of NUIDs $= |A^9| = |A \times A \times \cdots \times A| \overset{\text{Product Rule}}{=} |A| \times |A| \times \cdots \times |A|$ (9 times)

$= 10 \times 10 \times \cdots \times 10 = 10^9$

Qn: When will NEU run out NUIDs?

Assume: Every year 100,000 join $= 10^5$

to use up 10^9 IDs, we need $10^9 / 10^5 \rightarrow 10^4$ years

10,000 years

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eg: CRNs (Course Reg. Numbers)

5 digits: $\{1,2,\ldots,9\} \times \{0,1,\ldots,9\}^4$

# of CRNs: (P. Rule)

$9 \times 10^4 = \underline{90,000}$

Eg. Power sets of $S \rightsquigarrow |S| = n$

Use bitstring repr of subsets of S.

↳ ① Make up an ordering for elements of S.

② Use n 0/1 s to represent the n choices of element not in/in subset

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e.g. $S = \{a, b, c\}$ ← this is the assumed order.

subset $\{a\}: (1,0,0)$ $\{b\}: (0,1,0)$, $\{c\}: (0,0,1)$

$\{a,c\}: (1,0,1)$ $\{b,c\}: (0,1,1)$, $\{a,b\}: (1,1,0)$

$\{\}: (0,0,0)$, $\{a,b,c\}: (1,1,1)$

$$|P(S)| = \left| \{(0,0,0), (0,1,0), (0,1,1), (0,0,1), \dots\} \right|$$

$$= |\{0,1\} \times \{0,1\} \times \{0,1\}| = 2 \times 2 \times 2 = 8$$

Subset: $\underbrace{\{0,1\} \times \{0,1\} \times \{0,1\} \times \dots \times \{0,1\}}_{n \text{ times}}$ → n-bit string

↳ subset of S where $|S| = n$

Product Rule: $\underbrace{2 \times 2 \times 2 \dots \times 2}_{n \text{ times}} = 2^n$

$$|P(S)| = 2^{|S|}$$

Eg. I know my credit card PIN uses the digits 3, 5, 2, 9

B

# of possible PINs → length 4

$$|B \times B \times B \times B| = |B|^4 = 4^4 = 256$$

But I know, each digit only used once.

$$|B \times C \times D \times E| = |B| \times |C| \times |D| \times |E|$$

$$= 4 \times 3 \times 2 \times 1 = 24$$

Depending on whether re-use allowed.

If re-use / replacement not allowed.

then number of rearrangements / permutations of n element set

$$\underline{n} \cdot \underline{(n-1)} \cdot \underline{(n-2)} \cdot \underline{(n-3)} \cdot __ \cdot \cdots \underline{2} \cdot \underline{1}$$

1st 2nd -- n-1 n^{th} slot.

Q. How many rearrangements / permutations of the letters in the word "COURSE" → 6 elements

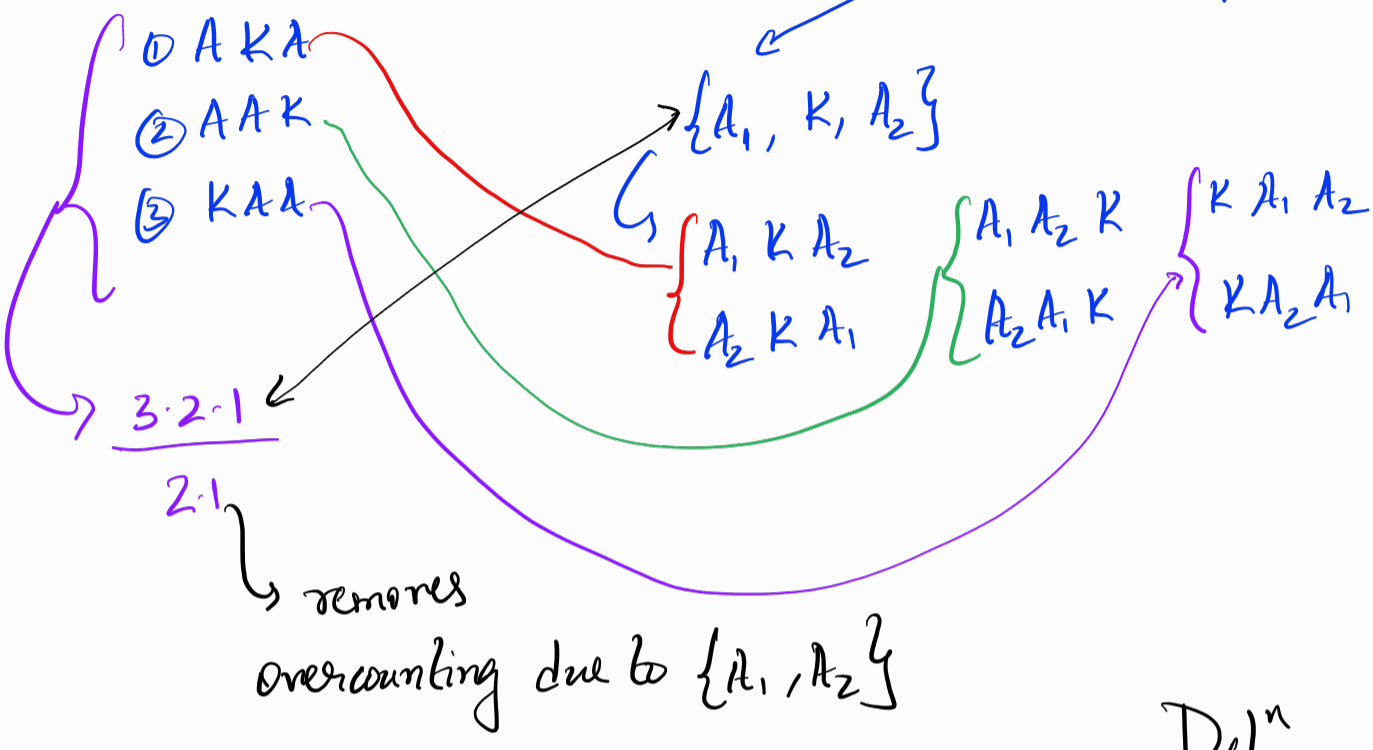
e.g. { COURSE, OUCRSE, ROUCSE, ECOURS, ... }

$$\underline{6} \quad \underline{5} \quad \underline{4} \quad \underline{3} \quad \underline{2} \quad \underline{1} = 720$$

Q. How many rearrangements of "AKSHAR"

↳ first trying rearrangements of "AKA"

↳ compare to 3·2·1



↳ removes overcounting due to $\{A_1, A_2\}$

"AKSHAR" $\longrightarrow \dfrac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1}$

$= \dfrac{6!}{2!}$

Def^n

$n \cdot (n-1)(n-2) \cdots 3 \cdot 2 \cdot 1 = n!$

"n factorial"

Q. "AVAILABLE" $\longrightarrow \dfrac{9!}{2! \cdot 3!}$

Q. AKSHAR VARMA

without space $\dfrac{11!}{4! \cdot 2!}$

with space $\dfrac{12!}{4!\,2!}$

Eg: In Rinendell, License plates

3 distinct letters (↓ a–z) followed by 4 distinct digits 0–9

# of license plates: $\underline{26} \cdot \underline{25} \cdot \underline{24} \cdot \underline{10} \cdot \underline{9} \cdot \underline{8} \cdot \underline{7}$