Oct 8

Admin

- No class on Mon, 12th
- Project meeting signups out
  - make groups on your own
  - use the #general chatroom

Agenda

- Revisit terminology
- Example & do everything "sets"
- Cartesian Product
- Power Sets
- Cardinality revisited.

Subsets

$$A \subseteq B \iff \forall x : x \in A \Rightarrow x \in B$$

$$A \subset B \iff \forall x : (x \in A \Rightarrow x \in B) \wedge A \neq B$$

"strict" subset

"proper" subset

Equality of sets

$$A = B \iff \forall x : (x \in A \Rightarrow x \in B) \wedge (x \in B \Rightarrow x \in A)$$

$$A = B \iff A \subseteq B \wedge B \subseteq A$$

in numbers

$$x = y \iff x \leq y \wedge y \leq x$$

Example

"roster notation" "set-builder notation" "English"

$$U = \{0, 1, 2, 3, \dots, 8, 9\} = \{n \in \mathbb{N} \mid n \leq 9\} = \text{first 10 natural numbers (or smallest)}$$

- P = set of prime numbers in U.

Goal: → roster
→ set builder

$$P = \{2, 3, 5, 7\}$$

$$P = \{n \in U \mid n \text{ is not divisible by any number not in } \{1, n\},\ n \neq 1\}$$

$$= \{n \in U \mid n \text{ is prime}\}$$

$$= \{n \in U \mid n \text{ is only divisible by numbers in } \{1, n\} \wedge n \neq 1\}$$

$$= \{n \in U \mid n > 1 \wedge \text{if } m \in U \text{ divides } n \text{ then } m \in \{1, n\}\}$$

- $E = \{n \in U \mid n \text{ is divisible by } 2\}$

Goal: → English
→ roster

E = set of even numbers in U.

$$E = \{0, 2, 4, 6, 8\}$$

$$C = \{n \in U \mid \exists m \in \mathbb{N}: m^2 = n\}$$

Goal: → roster

- $S = \{ n \in U \mid \exists m \in U . \; n = \; \}$ → English

$S = \{ 0, 1, 4, 9 \}$

$S =$ "set of square numbers in U."

- $R = \{ n \in U \mid n^2 \in U \}$

Goal: → roster
→ English

$R = \{ 0, 1, 2, 3 \}$

$R =$ "set numbers in U whose squares are also in U."

- $O = \{ 1, 3, 5, 7, 9 \}$

Goal: → English
→ Set builder

$O =$ "set of odd numbers in U."

$O = \{ n \in U \mid n \notin E \}$

$O = \overline{E} = \{ n \in U \mid n \bmod 2 = 1 \}$

$n \bmod 2$
↳ remainder when n is divided by 2.

Using P, E, S, O

- What is $\{ 1, 9 \} = O \cap S = O \setminus P = O \cap P^c$

$= S - E = \overline{E} \cap \overline{P} = \overline{E \cup P}$

$= \overline{O^c \cup P}$

$= O \cap P^c$

- What is $\{3,5,7\} = P \cap O = P \cap \overline{E}$

$= P \setminus E = O \setminus S = O \setminus \overline{P}$

- What is in $(E \cup S) - \overline{P}$?

$\{0,2,4,6,8\} \cup \{0,1,4,9\} \rightarrow \{0,1,4,6,8,9\}$

$\{2\}$

- What is $\overline{(P \cap S) \setminus O}$

$\phi \setminus O \rightarrow \overline{\{\}} = U$

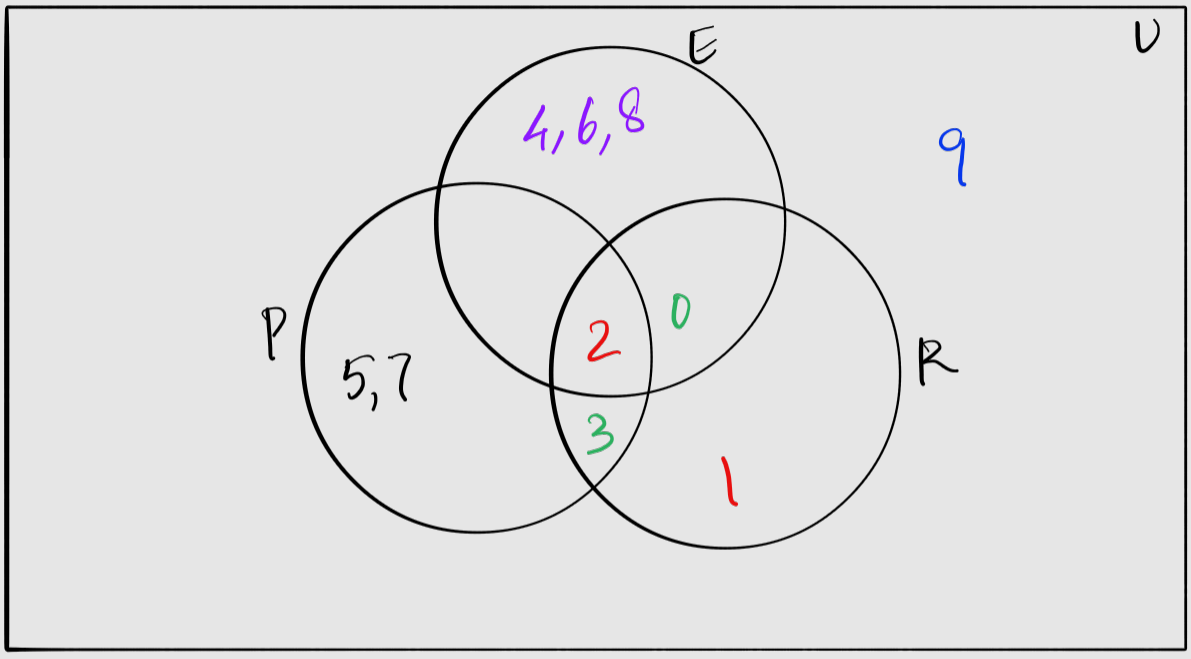
- What is $\overline{(P \cap R) \setminus O}$

$\{2,3,5,7\} \cap \{0,1,2,3\}$

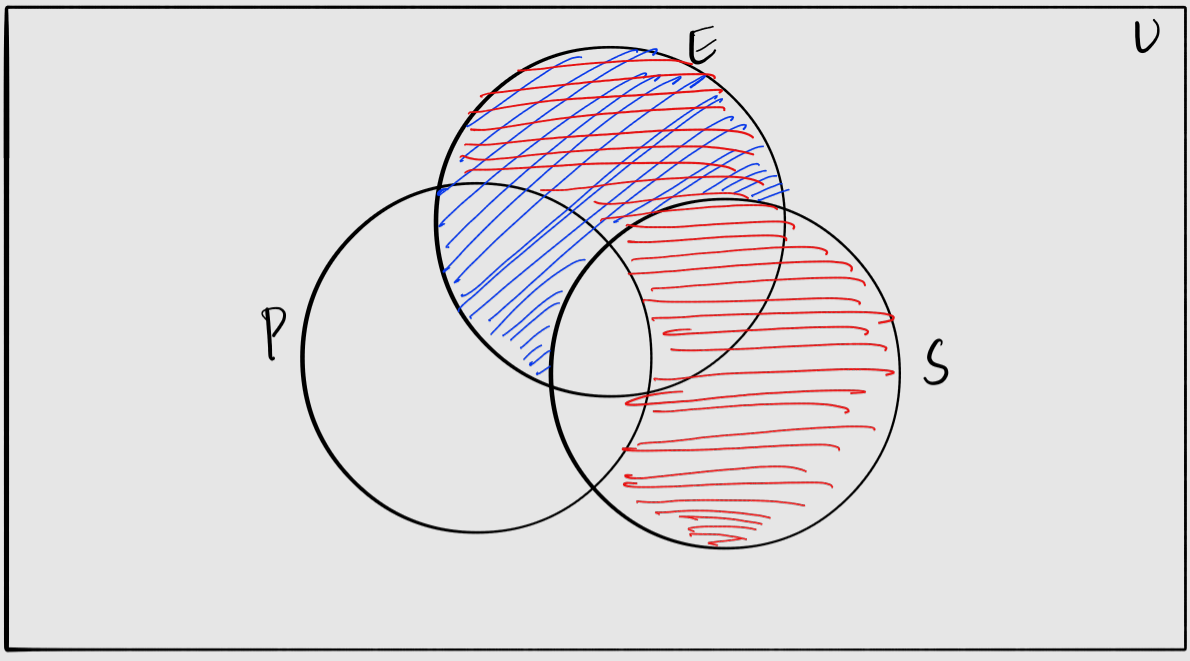
$\{2,3\} \setminus O \rightarrow \overline{\{2\}} = U \setminus \{2\}$

V. Diagram of P, E, R

Venn Diagram



Shade $E \setminus S$ using ▨ & $(E \cup S) - P$ using ▤



Write a simple expression for

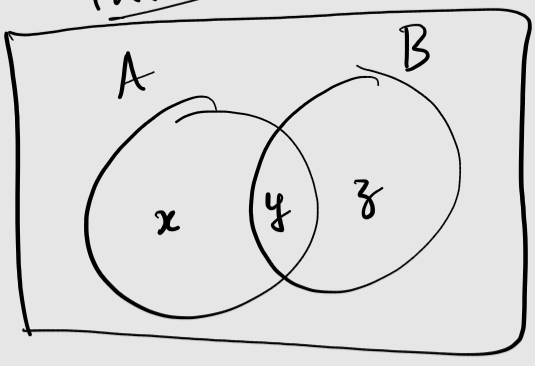
① All shaded region. ↪ $(E \cup S) \setminus (P \cap S)$

② Region for overlapping shading ↪ $(P \cap E) \setminus S$

Cardinality Revisited: Principle of Inclusion-Exclusion (PIE)

$$|O \cup P| = |O| + |P| - |O \cap P|$$
$$= 5 + 4 - 3 = 6 = |\{1,2,3,5,7,9\}|$$

Intuition.



Want: $x + y + z$

$|A| = x + y \longrightarrow |A| + |B| = x + y + y + z$

$|B| = y + z \quad -|A \cap B| \quad -y$

Try yourself

$$|E \cup S| = |E| + |S| - |E \cap S| = 5 + 4 - 2 = 7$$

$|\{0,1,2,4,6,8,9\}|$ ← match.

Cartesian Products

Cartesian

$$A \times B = \{(a,b) \mid a \in A \wedge b \in B\}$$

$$P \times S = \{2,3,5,7\} \times \{0,1,4,9\}$$

$$= \{(2,0),(2,1),(2,4),(2,9),$$
$$(3,0),(3,1),(3,4),(3,9),$$
$$(5,0),(5,1),(5,4),(5,9),$$
$$(7,0),(7,1),(7,4),(7,9)\}$$

In general $|A \times B| = |A| \times |B|$

$$\{0,1\} \times \{0,1\} \times \{a\}$$

$$= \{(0,0,a),(0,1,a),$$
$$(1,0,a),(1,1,a)\}$$

$$|\{0,1\} \times \{0,1\} \times \{a\}| = |\{0,1\}| \times |\{0,1\}| \times |\{a\}| = 2 \times 2 \times 1 = 4 \checkmark$$

$$U \times \{\} = \{\}$$

$$|U \times \{\}| = |U| \times |\{\}| = _ \times 0 = 0$$

Power Sets

$$\mathcal{P}(A) = \{ S \mid S \subseteq A \}$$

$$\mathcal{P}(\{a, b\}) = \{ \{\}, \{a\}, \{b\}, \{a, b\} \}$$

$$\mathcal{P}(\{x\}) = \{ \phi, \{x\} \}$$

Note: if $|S| = 1$ then S is called a singleton set.

$$\mathcal{P}(\{\}) = \{ \phi, \{\} \} = \{ \{\} \} \neq \phi$$

ϕ and $\{\}$ are the same thing

$$\mathcal{P}(\{a, b, c\}) = \{ \{\}, \{a\}, \{b\}, \{c\},$$

$\{a, b\}, \{b, c\}, \{c, a\},$

$\{a, b, c\}$ $\}$