Oct 7

Admin

- No class on 12th Oct
- Project meeting sign ups out

Agenda

- More Sets
- Cartesian Product
- Power Set
- More Cardinality.

Revisit Subset

$A \subseteq B$ is True iff $\forall x : x \in A \Rightarrow x \in B$

$A \subset B$ is True iff $\forall x : (x \in A \Rightarrow x \in B) \wedge A \neq B$

↓

"strict subset"

OR "proper subset"

Alternate definition for Set Equality

$A = B \iff A \subseteq B \wedge B \subseteq A$

in numbers x, y

$$x = y \iff x \leq y \wedge y \leq x$$

Example through everything Sets

$$U = \{0, 1, 2, \ldots, 9\} = \{n \in \mathbb{N} \mid n \leq 9\}$$

1. P = set of all prime numbers (in U).

goal: translate to set builder & roster notation (list everything)

$$P = \{n \in U \mid n \text{ has no factors except from } \{1, n\}, n \neq 1\} = \{2, 3, 5, 7\}$$

2. $E = \{n \in U \mid n \text{ is divisible by } 2\}$

goal: translate to English & roster notation

E = set of all even numbers in U $= \{0, 2, 4, 6, 8\}$

3. $S = \{n \in U \mid n^2 \in U\}$

goal: translate to English & roster notation

S = set of numbers whose square is also in U. $= \{0, 1, 2, 3\}$

4. $O = \{1, 3, 5, 7, 9\}$

goal: to Set Builder & English.

$O = \{n \in U \mid n \text{ is not divisible by } 2\}$ = "set of all odd numbers in U."

$= \{n \in U \mid n \notin E\}$

$O = \overline{E}$

Using P, E, S, O $\quad \{1, 3, 5, 7, 9\}$

① $\{1, 9\} = O - P$

② $\{3, 5, 7\} = P \setminus E \quad = O - (O \setminus P)$

$= P \cap O$

③ $\overline{(P \cap S) \setminus O}$

$\overline{P \cap S} = \{2, 4, 6, 8\} \cup \overline{\{2, 4, 6, 8\}} = U \setminus \{2, 4, 6, 8\}$

$= \{0, 1, 3, 4, \ldots, 9\}$ → 9 elements.

④ $(E \cup S) - P^c$

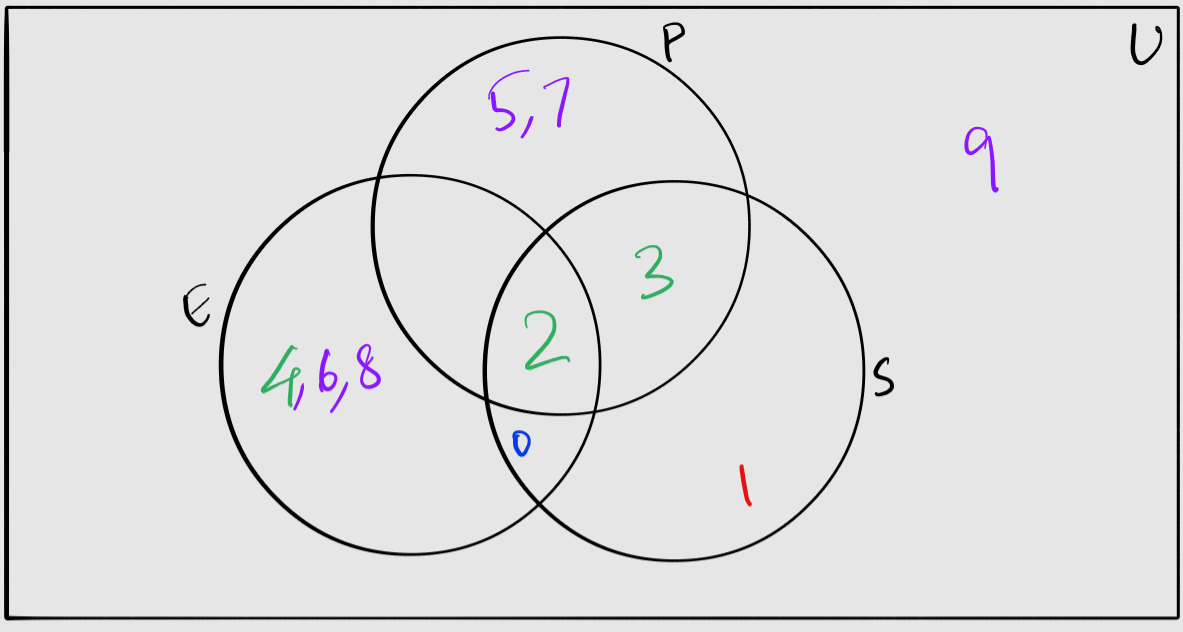
$\{0,2,4,6,8\} \cup \{0,1,2,3\}$

$\{0,1,2,3,4,6,8\}$ \ $\overline{\{2,3,5,7\}}$

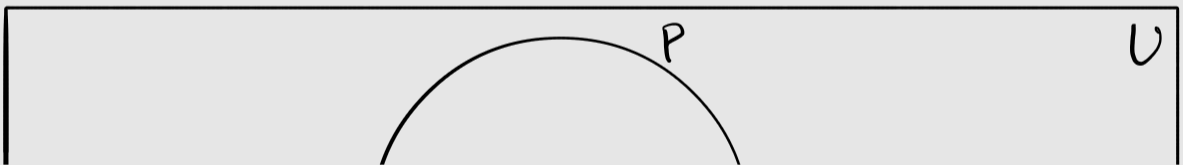
\ $\{0,1,4,6,8,9\}$

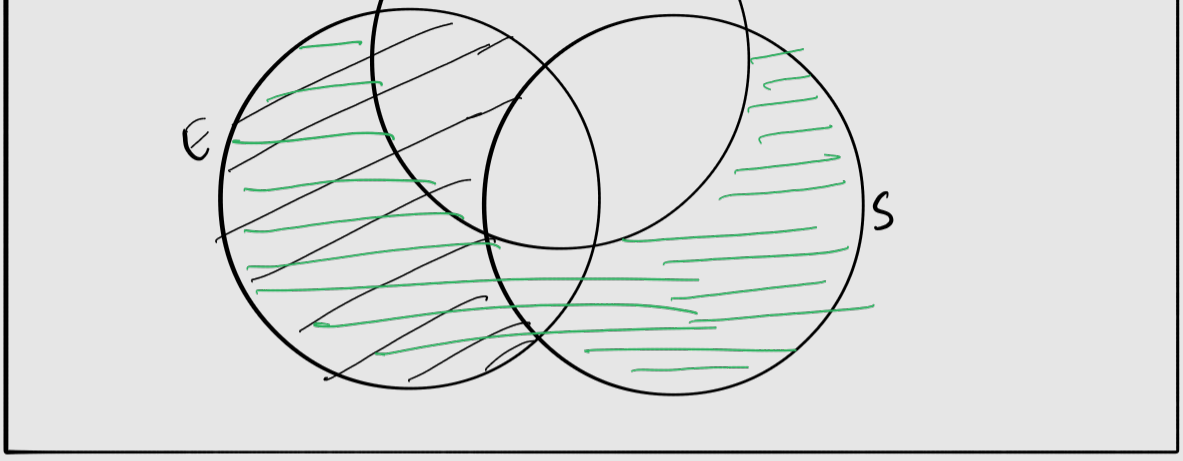
$= \{2,3\}$

Fill Venn Diagram "coorectly"



Shade $E \cap S$ using ▨ & Shade $(E \cup S) - P$ using ▤





Write the expression for the ① combined region (there is any shading)
② region where shading overlaps

Soln

① $(E \cup S) \setminus (P \cap S)$

② $E \setminus (P \cup S)$

Cartesian Product

Given 2 or more sets, their Cartesian Product is

- an ordered pair
- elements in the pair come from the sets (in order)

$\{$

$E \times S = \{(x, y) \mid x \in E \text{ and } y \in S\}$

$E = \{0, 2, 4, 6, 8\}$, $S = \{0, 1, 2, 3\}$

$$E \times S = \{(0,0), (0,1), (0,2), (0,3), (2,0), (2,1), (2,2), (2,3), (4,0), (4,1), (4,2), (4,3), (6,0), (6,1), (6,2), (6,3), (8,0), (8,1), (8,2), (8,3)\}$$

Larger Cartesian Products possible → Cart. Prod. of k-sets will contain k-tuples

$$\{0,1\} \times \{0,1\} \times \{0,1\}$$

$$= \{(0,0,0), (0,0,1), (0,1,0), (0,1,1), (1,0,0), (1,0,1), (1,1,0), (1,1,1)\}$$

$\{0,1\} \times \{\} = \{\}$ because there is no element to

use from the $\{\}$, so no pair can be made. Thus, the result is ϕ.

Power Sets

$P(S) = \mathcal{P}(S)$ = Power Set of S

$= \{A \mid A \subseteq S\}$

$\mathcal{P}(\{0,1\}) = \{\{0,1\}, \{0\}, \{1\}, \{\}\}$

$\{0,1\} \to S$, $\{0\} \to S$, $\{\} \to \phi$

$\mathcal{P}(S)$ always contains S itself & ϕ (the empty set)

$\mathcal{P}(\{\}) = \{\{\}, \phi\} = \{\{\}\}$

$$\mathcal{P}(\{\}) = \{\{\}\}$$

$$\mathcal{P}(\{a\}) = \{\{\}, \{a\}\}$$

If $|S| = 1$, then S is called a singleton set.

$$\mathcal{P}(S) = \{A \mid A \subseteq S, |A| = 0\}$$
$$\cup \{A \mid A \subseteq S, |A| = 1\}$$
$$\cup \{A \mid A \subseteq S, |A| = 2\}$$
$$\vdots$$
$$\cup \{A \mid A \subseteq S, |A| = |S|\}$$

Cardinality

Principle of Inclusion-Exclusion (PIE)

two sets A & B

$$|A \cup B| = |A| + |B| - |A \cap B|$$

$A = \{2, 3, 5, 7\}$, $S = \{0, 1, 2, 3\}$

Eg. $P = \{$

$|P \cup S| = |\{0, 1, 2, 3, 5, 7\}| = 6$ ✓

$= |P| + |S| = 4 + 4 - |\{2, 3\}| = 8 - 2 = 6$