Oct 5

Admin Updates

- PS2 grades out; Exam 1 out by tomorrow
- PS3 released. DUE: Oct. 11th, Sunday (+48 hour grace period)

- Gradescope submissions: use a single PDF.
  - make them clear (not too zoomed in/out)
  - Please tag pages to problems.

- Project details

Sets

↳ a collection of things

↳ only care about presence/absence of object in the set. "element"

↳ order doesn't matter

↳ duplicates don't matter

$$S = \{1, 2, 3\} = \{3, 1, 2\} = \{3, 1, 1, 1, 2, 2, 3\}$$

"Roster Notation"

Natural numbers

Integers

$\mathbb{N} = \{0, 1, 2, 3, 4, \dots\}$, $\mathbb{Z} = \{\dots, -2, -1, 0, 1, 2, \dots\}$

$\mathbb{Z}^+ = \{1, 2, 3, \dots\}$, $\mathbb{R}$ = set of all real numbers

$A = \{a, b, c, \dots, x, y, z\}$, $V = \{a, e, o, i, u\}$

$W = \{x, y, 1, \text{Afshar}, \{e, \pi, \sqrt{2}\}\}$

the empty set $= \{\} = \emptyset \neq \{\emptyset\}$

$x \in A$ → "x belongs to A"
→ "x is in the set A"

opposite $x \notin A = \neg(x \in A)$

eg: $3 \in \mathbb{N}$, $-5 \notin \mathbb{N}$, $\sqrt{2} \notin \mathbb{Z}$, $\pi \in \mathbb{R}$, $\emptyset \notin W$

$b \in A$, $c \in \{a, b, c, \dots, z\}$, $f \notin \{a, e, i, o, u\}$

Set Builder Notation

Even numbers $= E = \{n \mid (n \in \mathbb{Z}) \wedge (n \text{ is divisible by } 2)\}$

"such that"

$$E = \{ n \in \mathbb{Z} : 2 \text{ divides } n \}$$

$$= \{ n \mid n \in \mathbb{Z},\ n \text{ is divisible by } 2 \}$$

Rational Numbers

$$\mathbb{Q} = \{ p/q \mid p \in \mathbb{Z},\ q \in \mathbb{Z},\ q \neq 0 \}$$

eg:

$$Y = \{ y \mid y \in \mathbb{N},\ 2000 \leq y \leq 2026 \}$$

$$B = \{ x+y \mid x \in \mathbb{N},\ y \in \mathbb{N},\ x \leq 3,\ y \leq 1 \}$$

$x \to 0, 1, 2, 3$

$y \to 0, 1$

$x+y \to \{ 0,1,2,3 \atop 1,2,3,4 \}$ $\begin{matrix} x+0 \\ x+1 \end{matrix}$

$$B = \{0, 1, 2, 3, 4\}$$

5 elements.

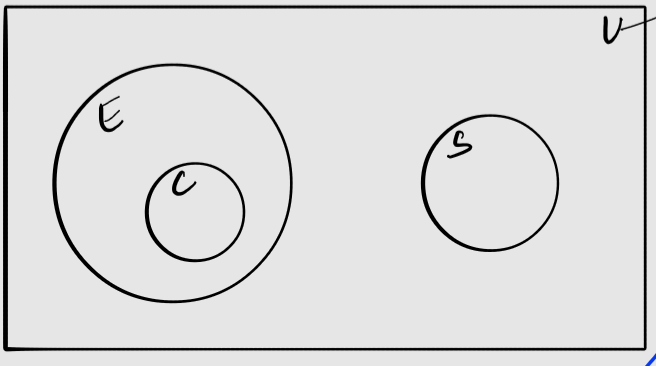
Cardinality of S → number of elements in the set (each only counted once)

- don't count duplicate

↳ $|S| =$ ∞ - count only unique.

$|B| = 5$, $|\{a, e, i, o, u\}| = 5$, $|\{a, b, \dots, y, z\}| = 26$

Venn Diagrams



Universe → all natural numbers

E = even numbers

$S = \{n^2 \mid n \in \mathbb{Z}, n \text{ is odd}\}$

$= \{1, 9, 25, 49, 81, 121, \dots\}$

$C = \{2^n \mid n \in \mathbb{N}, n \geq 1\}$

$= \{2, 4, 8, 16, \dots\}$

Subsets

$C \subseteq E$ → "C is a subset of or equal to E" / "C is subset equal to E"

↳ "every element of C is also an element of E"

↳ also C might be equal to E.

$C \subset E$ → "C is a (strict) subset of E"

↳ "everything in C, also in E.

and $C \neq E$."

eg: $\mathbb{N} \subseteq \mathbb{Z}$

$\mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R} \subset \mathbb{C}$

↳ "complex numbers"

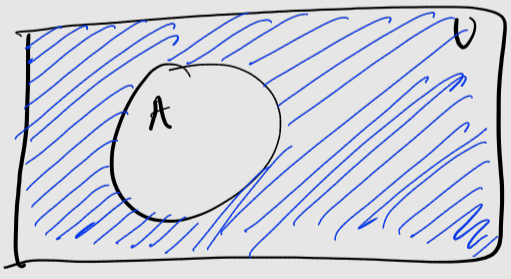


Venn Diagram

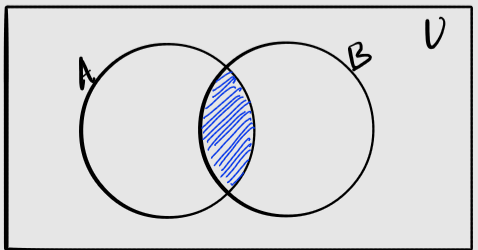
eg $\{a, e, i, o, u\} \subset \{a, b, c, \ldots, z\}$

Set Operations

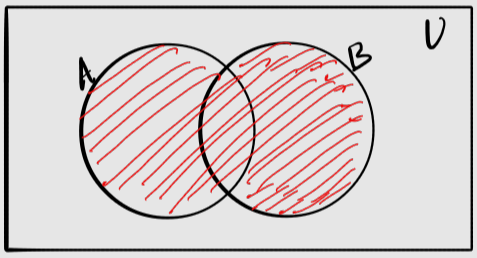
1. Complement: $\bar{A} = A^c = \{x \in U \mid x \notin A\} = \{x \in U \mid \neg(x \in A)\}$



2. Intersection: $A \cap B = \{x \mid (x \in A) \wedge (x \in B)\}$

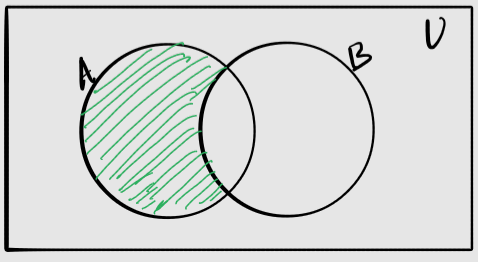


3. Union: $A \cup B = \{x \mid (x \in A) \vee (x \in B)\}$



Differences

$A \setminus B = A - B = \{x \mid x \in A, x \notin B\}$



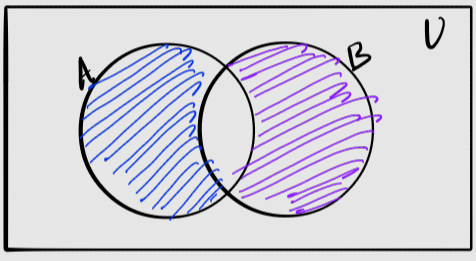
Symmetric Difference

$A \Delta B = \{x \mid x \in A, x \notin B\} \cup \{x \mid x \notin A, x \in B\}$

$= (A \setminus B) \cup (B - A)$

$= \{x \mid (x \in A) \oplus (x \in B)\}$

$= (A \cup B) \setminus (A \cap B)$



Example

$U = \{0, 1, 2, 3, 4, 5, 6\}$

i) $A \cap (B \cup C)^c \rightarrow 1 \cdot 1$

$A = \{1,2,3,4,5\}$

$B = \{1,3,4\}$

$C = \{2,5\}$

$D = \{1,3,5,6\}$

$= A \cap \{1,2,3,4,5\} = A \cap A$

$= \{1,2,3,4,5\} \cap \{0,6\}$

$= \{\}$ ← complement law

ii) $((B \cup C) \setminus D) \Delta A$

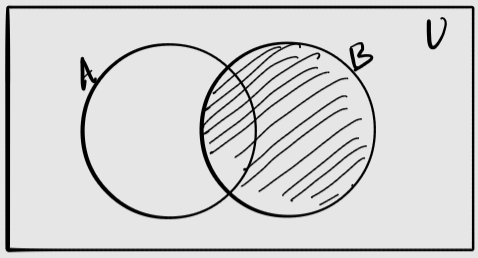
$= (\{1,2,3,4,5\} \setminus \{1,3,5,6\}) \Delta A$

$= \{2,4\} \Delta \{1,2,3,4,5\}$

$= \{1,3,5\}$

iii) $B \cup (C \cap B)$

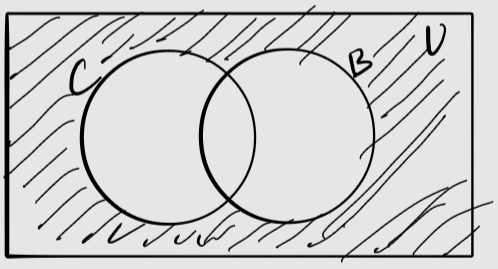
$= B$ ← Absorption Law.



iv) $\overline{(B \cup C)}$

$= \overline{B} \cap \overline{C}$ ← De Morgan's

$= \{0,6\}$ (I think)



Variables: a, b, c

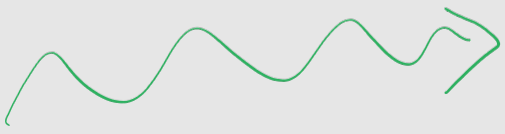
values: $a = T$

$: b = F$

negation: $\neg a$

Conjunction: $a \wedge b$

disjunction: $a \vee b$



sets: A, B, C

belongs to? $x \in A$

$x \notin B$

complement: $x \notin A$

intersection: $x \in A \cap B$

union: $x \in A \cup B$