Oct 5

Admin Updates

- PS2 grades out; Exam 1 out by tomorrow
- PS3 released. DUE: Oct. 11th, Sunday (+48 hour grace period)

- Gradescope submissions: use a single PDF.
  - make them clear (not too zoomed in/out)
  - Please tag pages to problems.

- Project details

SETS

↳ a collection of things

↳ the component pieces/elements of the set matter

↳ order of things don't matter.

↳ duplicates don't matter.

$$S = \{1, 2, 3\} = \{3, 2, 1\} = \{3, 1, 1, 2, 2, 2\}$$

elements of the set S.

"Set Notation"

Roster Notation

$\mathbb{N} = \{0, 1, 2, 3, 4, \dots\}$, $\mathbb{Z} = \{\dots, -2, -1, 0, 1, 2, \dots\}$

$A = \{a, b, c, \dots, x, y, z\}$, $V = \{a, e, i, o, u\}$

$a \in A$

↓

a "belongs to" A

a "is in the set" A

$2 \notin A$, $1378 \in \mathbb{N}$

$-3 \notin \mathbb{N}$, $1378 \in \mathbb{Z}$

the "Empty set" $= \{\} = \emptyset \neq \{\emptyset\}$

Set builder Notation

$E = \{0, 2, 4, 8, \dots\}$

$= \{n \mid (n \in \mathbb{N}) \wedge (n \text{ is divisible by } 2)\}$

$= \{n \in \mathbb{N} : 2 \text{ divides } n\}$

$\mathbb{Q} = \{ p/q \mid p \in \mathbb{Z}, q \in \mathbb{Z}, q \neq 0 \}$

$S = \{ n^2 \mid n \in \mathbb{N} \} = \{ 0, 1, 4, 9, 16, 25, \dots \}$

$\mathbb{R}$ = the set of real numbers.

$W = \{ x, 0, \text{Akshar}, \{\}, \mathbb{Z} \}$

↳ Note: rational numbers aren't elements of W.
the set of rational numbers is an element of W.

Subsets

$A \subseteq B$ → "A is subset of or equal to B"
"A is subset or equal to B"

↳ is true if every element of A also belongs to B
↳ A = B is possible.

$A \subset B$ → "A is (strict) subset of B"

↳ is true if every element of A also belongs to B

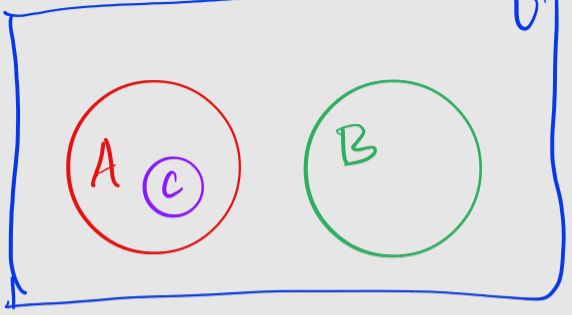
and $A \neq B$

eg: $\mathbb{N} \subseteq \mathbb{Z}$, $\mathbb{N} \subset \mathbb{Z}$

$\mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R} \subset \mathbb{C}$ ↳ complex numbers.

Vowels $\subseteq$ Alphabet

VENN DIAGRAMS



eg: $U = \mathbb{N}$

A = even natural numbers

B = odd square numbers $\{1, 9, 25, 49, 81, \dots\}$

C = powers of 2 (& 0) $\{0, 2, 4, 8, 16, \dots\}$

→ A & B have no elements in common

⇒ A & B are <u>disjoint</u>

→ the set of everything NOT in A is the (but still in the universe)

$A^C = \overline{A}$

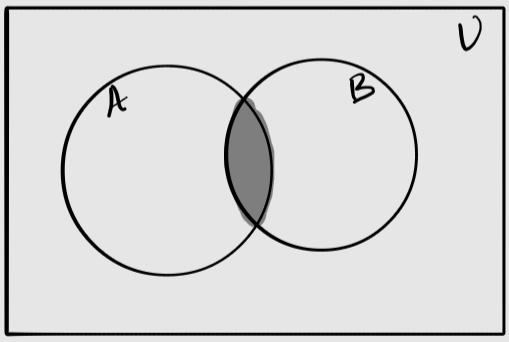
Complement of $A = \overline{A}$

eg. complement of even numbers = odd numbers

↳ Universe is all natural numbers

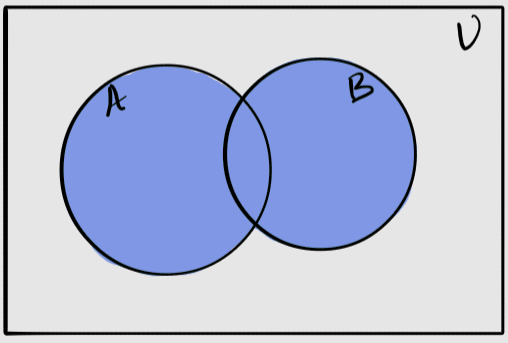
Operations on Sets.

1) $A \cap B = \{x \mid (x \in A) \wedge (x \in B)\}$



Intersection

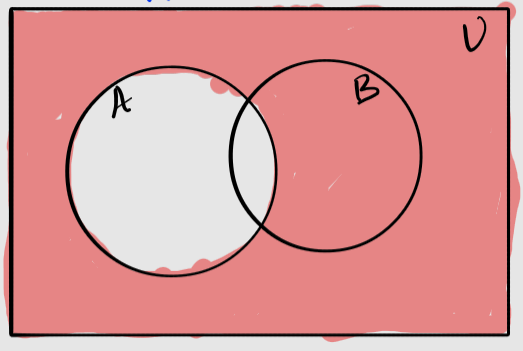
$A \cup B = \{x \mid (x \in A) \vee (x \in B)\}$



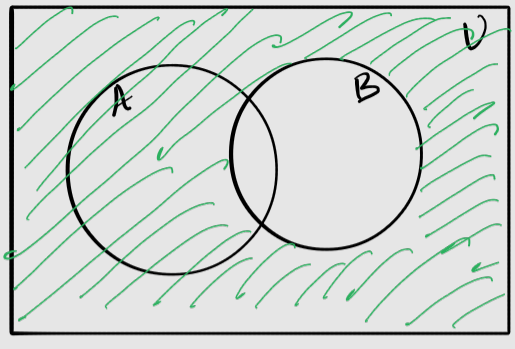
Union

Complement

$\overline{A} = A^c$



$\overline{B} = B^c$



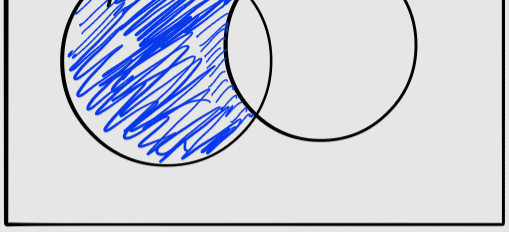
$$\overline{A} = A^c = \{x \mid x \notin A\} = \{x : \neg(x \in A)\}$$

Differences.

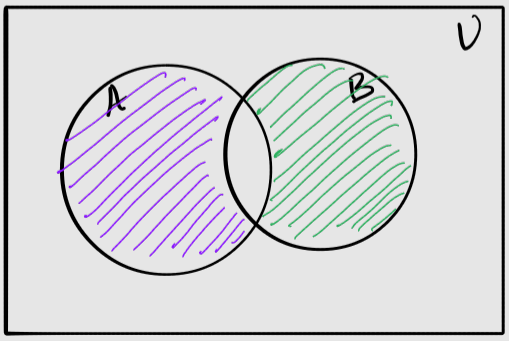
$A - B = A \setminus B$



$= \{x \mid x \in A \wedge x \notin B\}$



Symmetric Difference



$A \Delta B = \{x \mid x \in A \wedge x \notin B\} \cup \{x \mid x \notin A \wedge x \in B\}$

$A \Delta B = (A \setminus B) \cup (B - A)$

$A \Delta B = (A \cup B) \setminus (A \cap B)$

$A \Delta B = \{x \mid (x \in A) \oplus (x \in B)\}$

Examples

$U = \{0, 1, 2, \dots, 9\}$

$A = \{1, 2, 3\}$

$B = \{4, 8, 9\}$

$C = \{3, 4, 5, 6, 7\}$

$D = \{0, 1, 7, 9\}$

i) $(A \cap D) \cup (B \cap C)$

$\{1\} \cup \{4\}$

$= \{1, 4\}$ → Cardinality

$|\{1, 4\}| = 2$

ii) $(A \cup D) \cap B^c$

$\{0, 1, 2, 3, 7, 9\} \cap \{0, 1, 2, 3, 5, 6, 7\}$

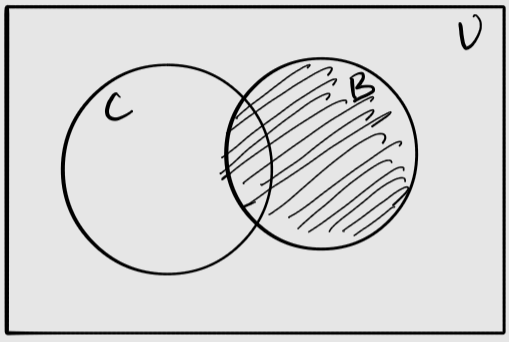
$\{\quad\}$

$= \{0, 1, 2, 3, \ldots\}$

iii) $B \cap (C \cup B)$

↳ Absorption law

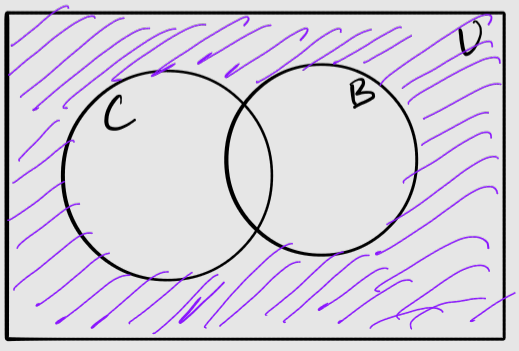
$= B = \{4, 8, 9\}$



iv) $(B \cup C)^c = \overline{B} \cap \overline{C}$

$B = \{4, 8, 9\}$

$C = \{3, 4, 5, 6, 7\}$



$B \cup C = \{3, 4, 5, 6, 7, 8, 9\}$

$\overline{B \cup C} = \{0, 1, 2\}$

$\overline{B} = \{0, 1, 2, 3, 5, 6, 7\} \longrightarrow \overline{B} \cap \overline{C} = \{0, 1, 2\}$

$\overline{C} = \{0, 1, 2, 8, 9\}$

Logic

variables T/F

variable: F

... : T

Sets

$x \in$ or $\notin$?

$x \in \phi$

$x \in U$

$x \notin A$, $x \in \overline{A}$

negation: $\neg$

conjunction: $a \wedge b$

disjunction: $a \vee b$

$x \in A \wedge x \in B, x \in A \cap B$

$x \in A \vee x \in B, x \in A \cup B$