NAME:

CS1800 Discrete Structures

Exam Practice 1

Strange & Varma *Date: September 27, 2026*

- **Actual vs. Practice:** The primary difference in the actual exam vs. this practice exam is number of subparts per problem. You can expect each of the 7 problems to only have 1 (or maybe 2) subparts instead of the multiple in this exam. The relative proportion of points will be similar in the actual exam, but the total will be different. The instructions below will also be the same in the actual exam.

- **Logistics:** Read all these instructions. Write your name on the top of every page. **Write your ID in the blank space provided above the points table on this page.**
- **Duration, Problems and Points:** This exam lasts for **90 minutes** and has **7** problems totaling **278** points. Make sure to read all problems to decide an order to attempt them that would allow quickest progress.
- **Partial Credit:** Points are determined on the basis of both the correctness and the clarity of your solution. Show your work, as partial credit may be given. Present the best solution/approach you can, even if it doesn't meet all desired properties.
- **Reference sheet:** You are allowed to bring one 8.5x11–inch cheat sheet, with anything written/typed on it, both sides of the sheet can be used. No other materials or devices are permitted. No collaboration of any kind is permitted.
- **Policy on cheating:** Students who violate the above rules on scholastic honesty are subject to disciplinary penalties. Any student caught cheating will *immediately* receive an **F** (failing grade).

  Sign below and confirm that you are strictly abiding by the rules of this test.

Signature: _____

ID: _____

NAME: _____

Question	1	2	3	4	5	6	7	Total
Points	40	36	40	64	20	28	40	268

Best of luck!

Commutative Laws	$p \wedge q \equiv q \wedge p$ $p \vee q \equiv q \vee p$
Associative Laws	$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$ $(p \vee q) \vee r \equiv p \vee (q \vee r)$
Distributive Laws	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$ $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
Identity Laws	$p \wedge T \equiv p$ $p \vee F \equiv p$
Annihiltor/Domination Laws	$p \wedge F \equiv F$ $p \vee T \equiv T$
Idempotence Laws	$p \wedge p \equiv p$ $p \vee p \equiv p$
Complement/Negation Laws	$p \wedge \neg p \equiv F$ $p \vee \neg p \equiv T$
Double Negation Law	$\neg\neg p \equiv p$
Absorption Laws	$p \wedge (p \vee q) \equiv p$ $p \vee (p \wedge q) \equiv p$
De Morgan's Laws	$\neg(p \wedge q) \equiv \neg p \vee \neg q$ $\neg(p \vee q) \equiv \neg p \wedge \neg q$

Table 1: Laws of Boolean Algebra for reference.

Problem 1 (TT $\longrightarrow$ Circuit) — 10 x4

(a) Draw the corresponding circuit for the Truth Table in Table 2. A, B are inputs, the last column is the output and the rest are intermediate outputs of logic gates.

A	B	X	Y	Output
0	0	1	1	1
0	1	0	1	1
1	0	1	0	1
1	1	0	0	1

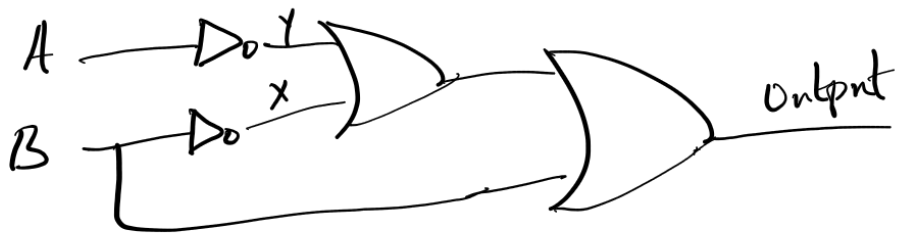
Table 2

$X \to \neg B$

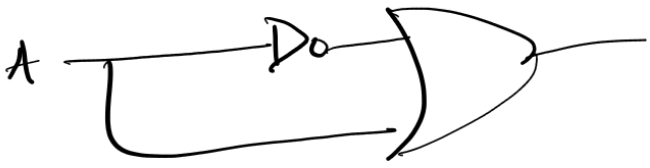
$Y \to \neg A$

$X \vee Y \vee A$

$X \vee Y \vee B$



OR



via complement law $A \vee \neg A = T$

(b) Draw the corresponding circuit for the Truth Table in Table 3. A, B are inputs, the last column is the output and the rest are intermediate outputs of logic gates.

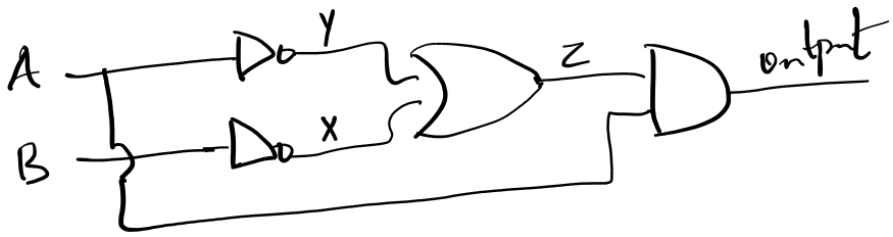
$X \rightarrow \neg B$

$Y \rightarrow \neg A$

$Z \rightarrow X \vee Y$

A	B	X	Y	Z	Output
0	0	1	1	1	0
0	1	0	1	1	0
1	0	1	0	1	1
1	1	0	0	0	0

Table 3



(c) Draw the corresponding circuit for the Truth Table in Table 4. A, B are inputs, the last column is the output and the rest are intermediate outputs of logic gates.

A	B	X	Y	Z	Output
0	0	1	1	0	0
0	1	1	1	1	1
1	0	0	0	1	0
1	1	0	1	1	1

Table 4

(d) Draw the corresponding circuit for the Truth Table in Table 5. A, B are inputs, the last column is the output and the rest are intermediate outputs of logic gates.

A	B	X	Y	Output
0	0	1	0	1
0	1	1	0	1
1	0	0	0	0
1	1	0	1	1

Table 5

Problem 2 (Circuit $\longrightarrow$ Logical Expression) (5+2+2) x4

(a) Convert the circuit in Figure 1 to a logical expression with boolean variables a, b.

Provide a value for a, b such that the output is True and another value for a, b such that the output is False. If no input values work, then say "None".

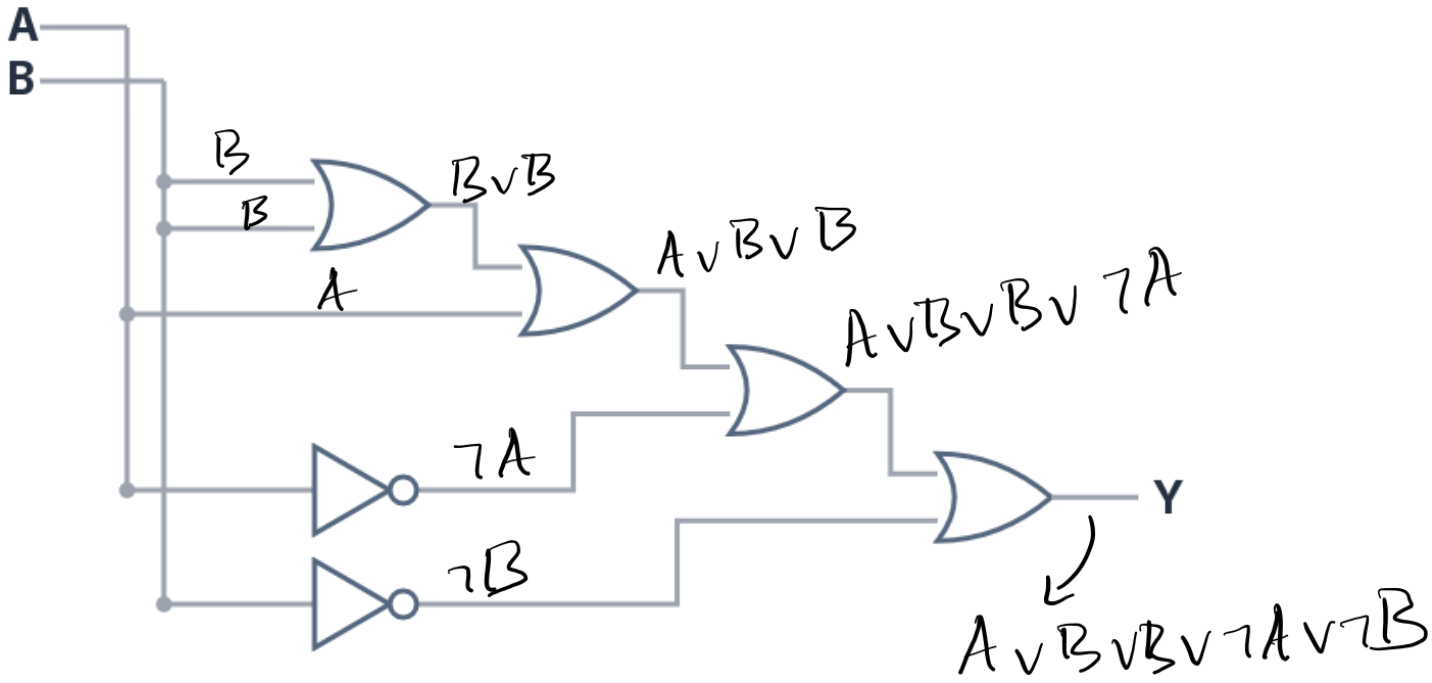


Figure 1

(b) Convert the circuit in Figure 2 to a logical expression with boolean variables a, b.

Provide a value for a, b such that the output is True and another value for a, b such that the output is False. If no input values work, then say "None".

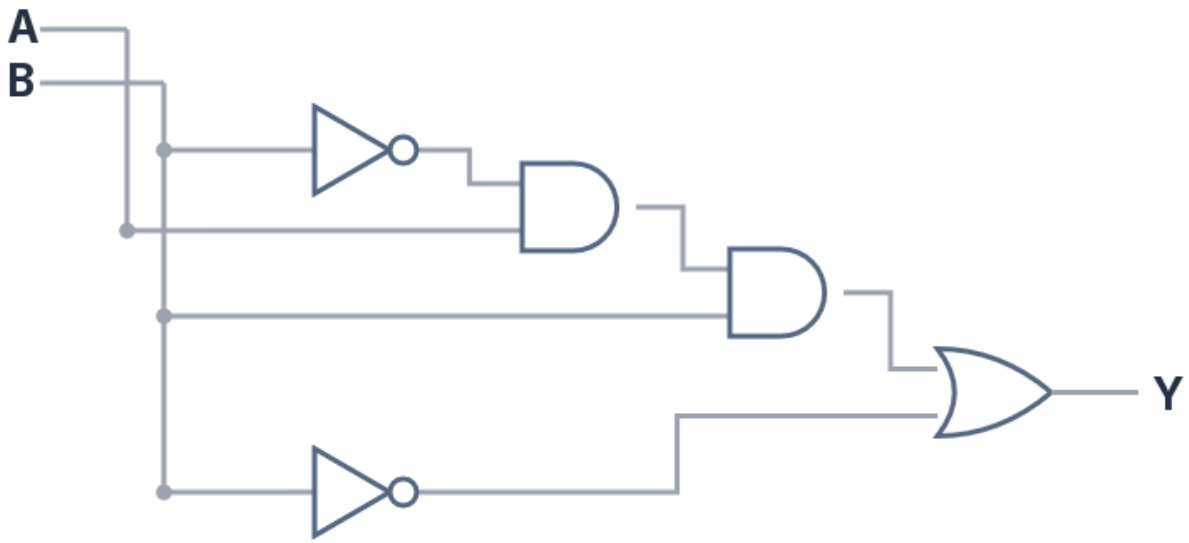


Figure 2

(c) Convert the circuit in Figure 3 to a logical expression with boolean variables a, b.

Provide a value for a, b such that the output is True and another value for a, b such that the output is False. If no input values work, then say "None".

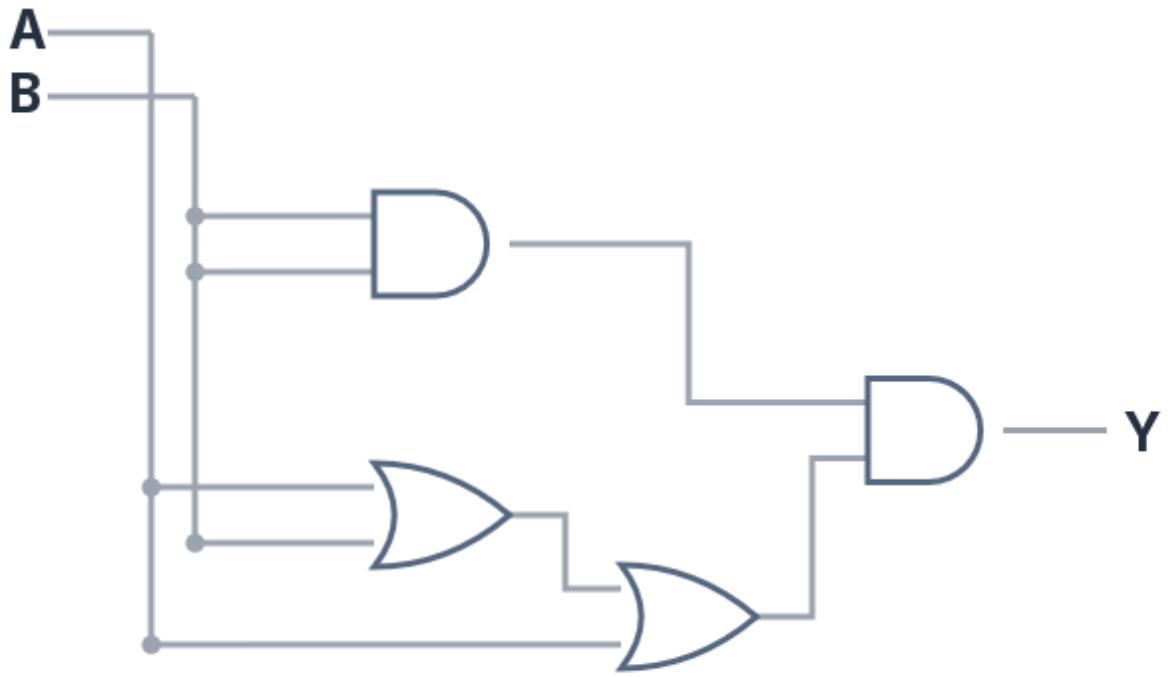


Figure 3

(d) Convert the circuit in Figure 4 to a logical expression with boolean variables a, b.

Provide a value for a, b such that the output is True and another value for a, b such that the output is False. If no input values work, then say "None".

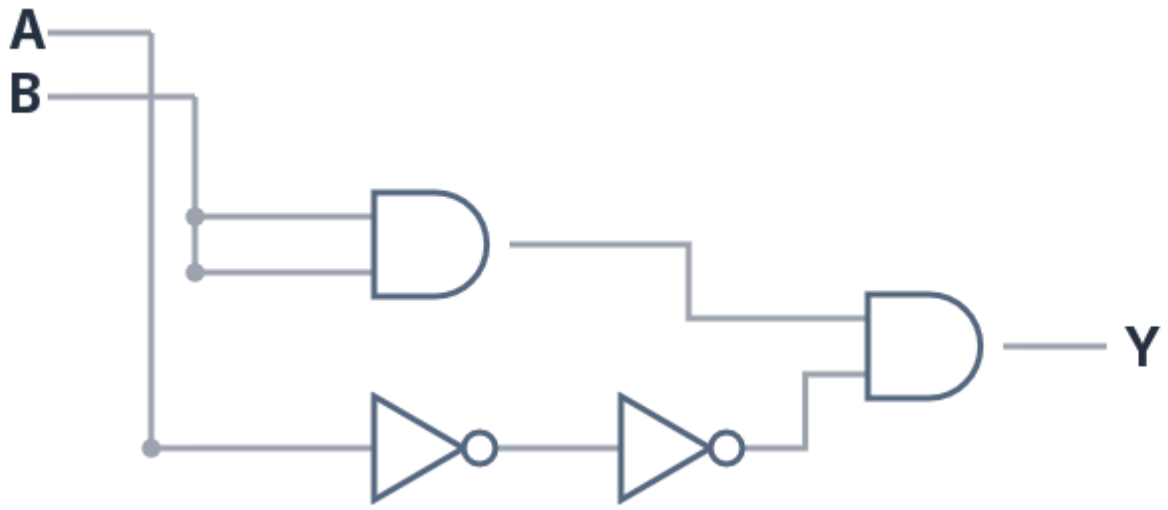


Figure 4

Problem 3 (Logical Expression Equivalence via TT) 10 x4

(a) Are the following statements logically equivalent? If so, show their truth table; if not, show the inputs on which they do not agree.

Statement 1: $(B \vee A) \wedge \neg B \wedge \neg A$ Statement 2: F

(b) Are the following statements logically equivalent? If so, show their truth table; if not, show the inputs on which they do not agree.

Statement 1: $C \vee \neg (C \vee A)$ Statement 2: $C \vee \neg A$

(c) Are the following statements logically equivalent? If so, show their truth table; if not, show the inputs on which they do not agree.

Statement 1: $(A \wedge (B \vee \neg A)) \vee \neg C$ Statement 2: $(A \wedge B) \vee \neg C$

(d) Are the following statements logically equivalent? If so, show their truth table; if not, show the inputs on which they do not agree.

Statement 1: $A \vee (\neg B \vee A)$ Statement 2: $B \vee A$

Problem 4 (Boolean Formula ≡ Boolean Formula) 16 x4

(a) In this simplfication, write beside each step the rule being used in the proof of equivalence or fill in the missing step given the rule. *Note: It is not necessary to perform commutative and associative law applicaiton in separate steps.*

$$C \vee (\neg B \wedge \neg A \wedge (A \vee A \vee B))$$

$$= C \vee (\neg (A \vee B) \wedge (A \vee A \vee B)) \quad \underline{\qquad\qquad} \text{ law}$$

$$= \qquad\qquad \text{Idempotence law}$$

$$= C \vee \mathrm{F} \qquad\qquad \underline{\qquad\qquad} \text{ law}$$

$$= C \qquad\qquad \underline{\qquad\qquad} \text{ law}$$

USE THIS PAGE FOR SCRATCH WORK.
IF CONTINUING AN ANSWER, CLEARLY INDICATE THE QUESTION NUMBER.

(b) In this simplfication, write beside each step the rule being used in the proof of equivalence or fill in the missing step given the rule. *Note: It is not necessary to perform commutative and associative law applicaiton in separate steps.*

$$\neg B \vee \neg A \vee (C \wedge \neg A) \vee \neg B \vee C \vee \neg A$$

$$= \neg B \vee \neg A \vee \neg B \vee C \vee \neg A \qquad \text{Absorption}$$

$$= \neg B \vee \neg A \vee C \vee \neg A \qquad \text{Comm + Idempot}$$

$$= \neg B \vee \neg A \vee C \qquad \text{Comm + Idempotent law}$$

$$= \neg(B \wedge A) \vee C \qquad \text{DeMorgan's law}$$

USE THIS PAGE FOR SCRATCH WORK.
IF CONTINUING AN ANSWER, CLEARLY INDICATE THE QUESTION NUMBER.

(c) In this simplfication, write beside each step the rule being used in the proof of equivalence or fill in the missing step given the rule. *Note: It is not necessary to perform commutative and associative law applicaiton in separate steps.*

$$((B \vee C) \wedge (B \vee \neg A \vee C)) \vee \neg\neg\neg A$$

$=$ Double Negation

$= B \vee (C \wedge (\neg A \vee C)) \vee \neg A$ _____

$=$ _____ $\vee \neg A$ Absorption

USE THIS PAGE FOR SCRATCH WORK.
IF CONTINUING AN ANSWER, CLEARLY INDICATE THE QUESTION NUMBER.

(d) In this simplfication, write beside each step the rule being used in the proof of equivalence or fill in the missing step given the rule. *Note: It is not necessary to perform commutative and associative law applicaiton in separate steps.*

$$\neg\neg\left(A \vee \neg A \vee B\right) \wedge \left(\neg\left(A \wedge C\right) \vee \neg\left(A \vee \neg C\right)\right)$$

$$= \left(A \vee \neg A \vee B\right) \wedge \left(\neg\left(A \wedge C\right) \vee \neg\left(A \vee \neg C\right)\right) \qquad \underline{\qquad\qquad}$$

$$= \left(\mathrm{T} \vee B\right) \wedge \left(\neg\left(A \wedge C\right) \vee \neg\left(A \vee \neg C\right)\right) \qquad \underline{\qquad\qquad}$$

$$= \qquad \text{DeMorgan's}$$

$$= \left(\mathrm{T} \vee B\right) \wedge \neg\left(A \wedge C\right) \qquad \underline{\qquad\qquad}$$

$$= \mathrm{T} \wedge \neg\left(A \wedge C\right) \qquad \underline{\qquad\qquad}$$

$$= \qquad \text{Identity}$$

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Problem 5 (Implications) (6 + 4) x2

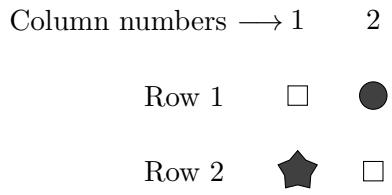


Figure 5: Universe

For each of the following statements, provide its contrapositive, converse, inverse (all in English, no logical operators required). For each of the 4 statements, determine its truth value based on Figure 5.

(a) "If the shape is on the diagonal, then it is a clear square."
[***Hint**: The diagonal has row number equal to column number.*]

(b) "If the shape is shaded then it is not a square."

Problem 6 (Predicates and Quantifiers) (5+2) x4

In the following, you can use the predicates $Square(x)$, $Circle(x)$, $Star(x)$ to denote whether a given shape x is a square, circle or star respectively. Similarly, $Shaded(x)$ and $Clear(x)$ are predicates denoting that a shape has been shaded or is clear (unshaded). $Row(x)$ is the row number (from 1, 2, 3) of a given shape x. $Col(x)$ is the column number (from 1, 2, 3, 4) of a given shape x.

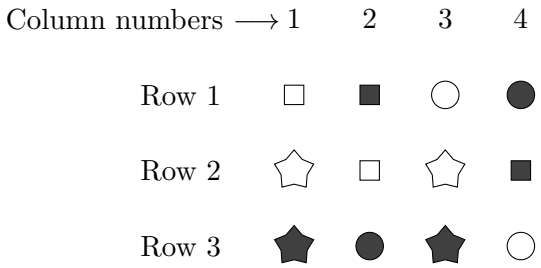


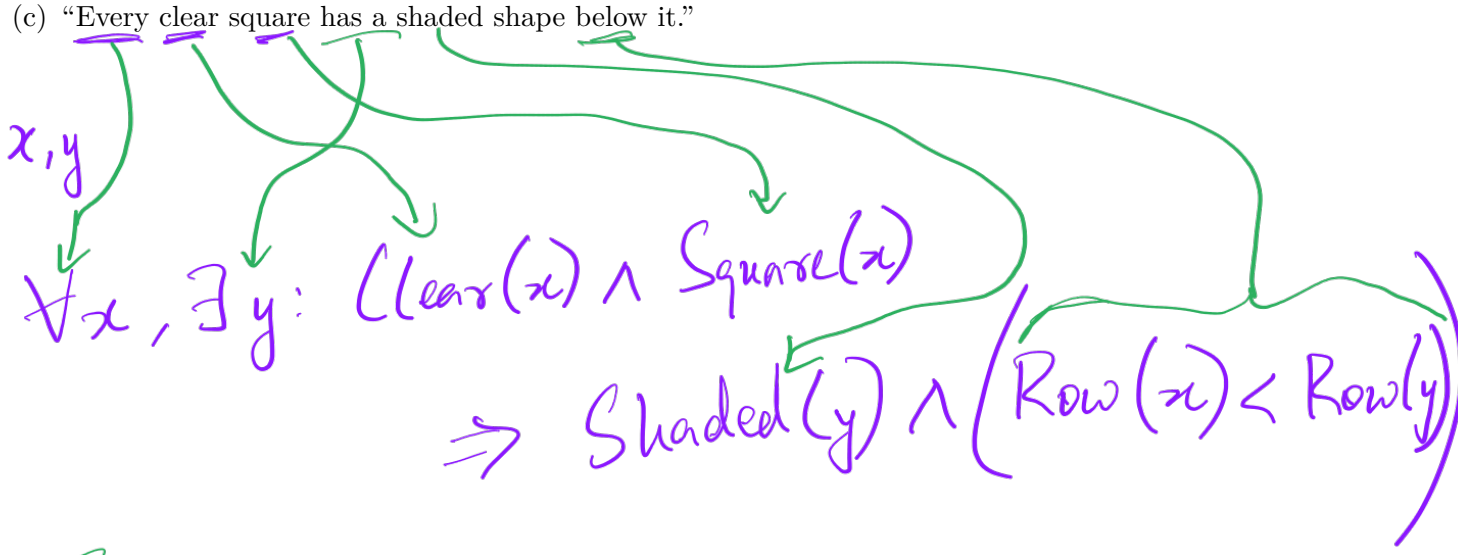
Figure 6: Universe

For the following statements, if they are in English, translate to a Boolean formula and vice versa. For each statement, also mention if it is True/False. Provide a counterexample if the statement is False.

(a) "If the shape is a Star, then it is shaded."

(b) $\forall x : Clear(x) \implies Circle(x)$

(c) "Every clear square has a shaded shape below it."



True Statement: the shaded shapes in last row are below the clear squares.

(d) $\forall x, \exists y : Square(x) \wedge Clear(x) \implies Square(y) \wedge Shaded(y) \wedge \Big(Col(x) < Col(y)\Big)$

Problem 7 (Number Conversions) — 5 x8

(a) `oct2dec`: Convert 1234_8 to decimal; show all steps.

$$1234_8 = 1 \cdot 8^3 + 2 \cdot 8^2 + 3 \cdot 8^1 + 4 \cdot 8^0$$

$$= 1 \cdot 512 + 2 \cdot 64 + 3 \cdot 8 + 4 \cdot 1$$

$$= 512 + 128 + 24 + 4$$

$$\boxed{1234_8 = 668_{10}}$$

(b) `dec2bin`: Convert 1234_{10} to base 2 using the division method; show all steps.

$1234_{10} = 10011010010_2$

$1234 \rightarrow (617, 0)$

$\hookrightarrow (308, 1)$

$\hookrightarrow (154, 0)$

$\hookrightarrow (77, 0)$

$\hookrightarrow (38, 1)$

$\hookrightarrow (19, 0)$

$(9, 1)$

$\hookrightarrow (4, 1)$

$\hookrightarrow (2, 0)$

$\hookrightarrow (1, 0)$

$\hookrightarrow (0, 1)$

(c) `hex2dec`: Convert AB_{16} to decimal; show all steps.

(d) `dec2oct`: Convert 89_{10} to base 8 using the division method; show all steps.

(e) `qua2dec`: Convert 3333_4 to decimal; show all steps.

(f) `dec2qua`: Convert 456_{10} to base 4 using the division method; show all steps.

(g) `bin2dec`: Convert 110101001_2 to decimal; show all steps.

(h) `dec2qua`: Convert 456_{10} to base 2 using the division method; show all steps.