Sep 24

Agenda

- Predicates & Quantifiers
- Number Representations

Recall, Propositions → Statements → TRUE / FALSE

eg: "It is sunny today."

now consider,

x is a real number, $\boxed{x \geq 0}$ → not quite a prop since it depends on value of x.

$2 \geq 0 \rightarrow T$

$5 \geq 0 \rightarrow T$

$-3 \geq 0 \rightarrow F$

Predicates

$P(x) \rightarrow$ 1 → TRUE if x (is an object from some valid domain) has property P.

otherwise 0 → FALSE

a boolean

Examples: (the domain, property)

- (real numbers, is the number ≥ 0) → $P(x)$: $x \geq 0$

(like variable; meaning of P(x))

- (students at NEU, do they have prereqs for CS 3000) → $A(s)$: can s take CS 3000?

more eg.

- (integers, is the number even?) → $Even(n)$ or $E(n)$
  integers: $\{\ldots, -2, -1, 0, 1, 2, \ldots\}$; n

- (shapes, is the shape a square?) → $Sq(x)$
  shapes: $\{\Delta, \bigcirc, \square, \pentagon, \ldots\}$; x

- (pair of integers (d, n), Does d divide n evenly? ↳ without any remainder)
  → $Divides(d, n)$

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eg.

A decent number: $D(n)$

integer
a number n is decent if any of these 2 conditions satisfied.

→ $E(n)$

i) n is even → $E(n)$

ii) the leftmost digit of n is even and n is divisible by 5.

eg: 21, 43, 4, 65, etc.

non-eg: 1, 341, 3, etc.

$L(n)$

$F(n) = \text{Divides}(5, n)$

$$D(n) = E(n) \vee (L(n) \wedge F(n))$$

Side Que: is this in any normal form?
Ans: Yes, DNF.

What is the bool exp to denote that all of these 20 numbers are decent: $a_1, a_2, a_3, \ldots, a_{20}$

think $\{3, 5, 7, 9, 12, \ldots, 48\}$

$D(a_1) \wedge D(a_2) \wedge \ldots \wedge D(a_{20})$ → what to do when so many more?

Quantifiers

$\forall x$: put in front of a predicate $P(x)$ to say that the predicate will be True for all

"for All x"

upgrade of $\wedge$ operator

possible "objects" x.

$$P(x_1) \wedge P(x_2) \wedge \dots$$

eg: $\forall a_i : D(a_i)$

with $i \in 1, 2, \dots, 20$

$\hookrightarrow S(x)$: x is one of $a_1, a_2, \dots, a_{20}$

$\hookrightarrow \forall x : S(x) \wedge D(x)$

$\exists x$: put in front of a predicate $P(x)$ to say that the predicate will be True for at least one of the possible "objects" x.

"there Exists x"

upgrade of $\vee$ operator

$$P(x_1) \vee P(x_2) \vee \dots$$

eg: There is an even prime number.

- $N(n)$: n is a natural number : $\{0, 1, 2, \dots\}$
- $E(n)$: n is an even number

- $P(n)$: the natural number n is prime.

$\exists n : N(n) \wedge P(n) \wedge E(n)$ → evaluates to TRUE if we set $n=2$

Dealing w/ negation

$$\neg\left(\forall x : P(x)\right)$$

by def^n

$$\equiv \neg\left(P(x_1) \wedge P(x_2) \wedge \cdots\right)$$

by De Morgan

$$\equiv \left(\neg P(x_1) \vee \neg P(x_2) \vee \cdots\right)$$

$$\left(Q(x_1) \vee Q(x_2) \vee \cdots\right)$$

$$\equiv \exists x : \neg P(x)$$

$$\neg\, \forall x : P(x) \equiv \exists x : \neg P(x)$$

And

$$\neg\left(\exists x : P(x)\right) \equiv \forall x : \neg P(x)$$

Representations

Number Representation

Computers $\rightarrow$ 0, 1 $\rightarrow$ all available digits

Decimal $\rightarrow$ 0, 1, 2, ..., 9

↳ base 10

$$1541 = 1 \cdot 1000 + 5 \cdot 100 + 4 \cdot 10 + 1 \cdot 1$$

$$= 1 \cdot 10^3 + 5 \cdot 10^2 + 4 \cdot 10^1 + 1 \cdot 10^0$$

$$1100_2 = 1 \cdot 2^3 + 1 \cdot 2^2 + 0 \cdot 2^1 + 0 \cdot 2^0$$

$$= 1 \cdot 8 + 1 \cdot 4 + 0$$

$$= 12_{10}$$

73_{10} $\xrightarrow[\text{binary}]{\text{to}}$ base-2

36	1
18	0
9	0
4	1
2	0
1	0
0	1

1001001_2

2

1

0

0

0

1