CS1800 Discrete Structures

Recitations 7

Strange & Varma *Date: November 12 & 13, 2026*

- This recitation handout contains practice problems. You do *not* need to submit anything.
- We do recommend handwritting your solutions to completion. Do ensure that you are aware of how and why each step in the solution exists.
- For best effects, read through the questions before your recitation section and if possible attempt them on your own. This way you can compare your perspective to the TAs during the recitation.
- If you get stuck on a problem, use your recitation section to get help. Of course, you can also use office hours or post on Campuswire!
- The solutions to this recitation will be posted after Friday. Use those to verify your own attempts and for any clarification.
- If problems are marked $(*)$, those are fundamental and the others are additional practice using concepts from the fundamental practice problems in more complex ways.

Problem 1 (Defective)

There are three machines, X, Y, and Z which produce dice. These dice are supposed to be fair dice but sometimes there are defects which make it biased towards certain outcomes.

- Machine X produces 50% of the dice, of which 3% are defective.
- Machine Y produces 30% of the dice, of which 4% are defective.
- Machine Z produces 20% of the dice, of which 5% are defective.

A dice is chosen randomly from all the dice produced today. Let D be the event that this dice is defective, and X, Y, and Z be the events that this dice was produced by machines X, Y, and Z respectively.

1. Find the probability that the dice is defective, i.e., find $P(D)$.
2. The dice was defective. Which machine was most likely to have produced it?

Problem 2 (Sequences: Terms)

What is the term x_5 for each of these sequences $(x_k)_{k\in\mathbb{Z}^+}$ where x_k is defined as:

1. $x_k = 2^k + 1$
2. $x_k = (k+1)^{k+1}$
3. $x_k = \lfloor k/2 \rfloor + \lceil k/2 \rceil$
4. $x_k = 8k^2 - 2k + 5$
5. $x_k = 2x_{k-2} - x_{k-1}$ where $x_1 = x_2 = 1$.

Problem 3 (Sequences: Recognition)

For each of the given sequences, determine whether it is an arithmetic sequence, a geometric sequence, a quadratic sequence and determine the formula/function for the value of a_k the k^{th} term of the sequence. If none of these sequence patterns matches the given terms, determine that and give a brief reason. A brief description of each sequence is below.

Arithmetic Sequences: The difference between consecutive terms is always the same. The general term is $a_k = a + (k-1)d$ where a is the initial value (first term) and d is the common difference between consecutive terms.

Geometric Sequences: The ratio of consecutive terms is always the same. The general term is $a_k = a \cdot r^{(k-1)}$ where a is the initial value (first term) and r is the common difference between consecutive terms.

Quadratic Sequences: The difference between consecutive terms is itself an arithmetic sequence. So the second level difference is always the same. The general term is $a_k = ak^2 + bk + c$ for some coeefficients a, b, c. We can find these coefficients by solving equations $a_1 = a \cdot 1^2 + b \cdot 1 + c$, $a_2 = \ldots$, and $a_3 = 9a + 3b + c$. Usually, this involves subtracting manipulating equations to isolate variables.

(a) -28, -22, -16, -10, ...

(b) 5, -7, -12, -19, ...

(c) 5, -7, -19, -31, ...

(d) 4, 20, 100, 500, ...

(e) 6, 11, 18, 27, 38, 51, ...