CS1800 Discrete Structures

Solutions to Recitations 7

Strange & Varma *Date: November 12 & 13, 2026*

- This recitation handout contains practice problems. You do *not* need to submit anything.
- We do recommend handwritting your solutions to completion. Do ensure that you are aware of how and why each step in the solution exists.
- For best effects, read through the questions before your recitation section and if possible attempt them on your own. This way you can compare your perspective to the TAs during the recitation.
- If you get stuck on a problem, use your recitation section to get help. Of course, you can also use office hours or post on Campuswire!
- The solutions to this recitation will be posted after Friday. Use those to verify your own attempts and for any clarification.
- If problems are marked $(*)$, those are fundamental and the others are additional practice using concepts from the fundamental practice problems in more complex ways.

Problem 1 (Defective)

There are three machines, X, Y, and Z which produce dice. These dice are supposed to be fair dice but sometimes there are defects which make it biased towards certain outcomes.

- Machine X produces 50% of the dice, of which 3% are defective.
- Machine Y produces 30% of the dice, of which 4% are defective.
- Machine Z produces 20% of the dice, of which 5% are defective.

A dice is chosen randomly from all the dice produced today. Let D be the event that this dice is defective, and X, Y, and Z be the events that this dice was produced by machines X, Y, and Z respectively.

1. Find the probability that the dice is defective, i.e., find $P(D)$.

   Solution:

   $$\begin{aligned} P(D) &= P(X) \cdot P(D|X) + P(Y) \cdot P(D|Y) + P(Z) \cdot P(D|Z) \\ &= 0.5 \cdot 0.03 + 0.3 \cdot 0.04 + 0.2 \cdot 0.05 \\ &= 0.037 \end{aligned}$$

   ■

2. The dice was defective. Which machine was most likely to have produced it?

   Solution:

   We need to find each of $P(X|D)$, $P(Y|D)$, $P(Z|D)$ and find which is the highest. In each of these, we'll reuse $P(D)$ calculated in the previous subpart.

   $$\begin{aligned} P(X|D) &= \frac{P(X) \cdot P(D|X)}{P(D)} \\ &= \frac{0.5 \cdot 0.03}{0.037} \\ &= 0.405 \end{aligned}$$

   $$\begin{aligned} P(Y|D) &= \frac{P(Y) \cdot P(D|Y)}{P(D)} \\ &= \frac{0.3 \cdot 0.04}{0.037} \\ &= 0.324 \end{aligned}$$

   $$\begin{aligned} P(Z|D) &= \frac{P(Z) \cdot P(D|Z)}{P(D)} \\ &= \frac{0.2 \cdot 0.05}{0.037} \\ &= 0.27 \end{aligned}$$

Hence, the most probable machine to have produced the defective dice is Machine X. ■

Problem 2 (Sequences: Terms)

What is the term x_5 for each of these sequences $(x_k)_{k\in\mathbb{Z}^+}$ where x_k is defined as:

1. $x_k = 2^k + 1$

   Solution:

   $$x_5 = 2^5 + 1 = 32 + 1 = 33$$

   ■

2. $x_k = (k+1)^{k+1}$

   Solution:

   $$x_5 = (5+1)^{(5+1)} = 6^6 = 46656$$

   ■

3. $x_k = \lfloor k/2 \rfloor + \lceil k/2 \rceil$

   Solution:

   $$x_5 = \left\lfloor \frac{5}{2} \right\rfloor + \left\lceil \frac{5}{2} \right\rceil = \lfloor 2.5 \rfloor + \lceil 2.5 \rceil = 2 + 3 = 5$$

   In general, x_k will always resolve to k.

   If k is even, then $k/2$ is an integer, let this be m. Both the floor and ceiling of m are equal to m itself, so adding m twice gives us $2m = 2 \cdot k/2 = k$.

   If k is odd, then $k/2$ can be written as $m + 0.5$ for some integer m. The floor of this is m and the ceiling is $m + 1$. Their sum is therefore $2 \cdot m + 1 = 2 \cdot (m + 0.5) = k$. ■

4. $x_k = 8k^2 - 2k + 5$

   Solution:

   $$x_5 = 8 \cdot 5^2 - 2 \cdot 5 + 5 = 195$$

   ■

5. $x_k = 2x_{k-2} - x_{k-1}$ where $x_1 = x_2 = 1$.

Solution:

We'll repeatedly have to use the formula $x_k = 2x_{k-2} - x_{k-1}$ until all subscript values on the right hand side are either 1 or 2. As long as $k-2$ or $k-1$ is ≥ 3, we'll repeat the recursive formula application.

$$\begin{aligned} x_5 &= 2x_3 - x_4 \\ &= 2x_3 - \Big(2x_2 - x_3\Big) \\ &= 3x_3 - 2x_2 \\ &= 3\Big(2x_1 - x_2\Big) - 2x_2 \\ &= 6x_1 - 5x_2 \\ &= 6 \cdot 1 - 5 \cdot 1 \\ &= 1 \end{aligned}$$

Alternatively, we could try producing a few values of x_k for small k to try and find an different, direct/closed-form formula.

$$\begin{aligned} x_3 &= 2x_1 - x_2 = 2 \cdot 1 - 1 = 1 \\ x_4 &= 2x_2 - x_3 = 2 \cdot 1 - 1 = 1 \\ x_5 &= 2x_3 - x_4 = 2 \cdot 1 - 1 = 1 \end{aligned}$$

This seems to tell us that $x_k = 1$ for all k. In the future, we'll use induction to actually prove this fact. ■

Problem 3 (Sequences: Recognition)

For each of the given sequences, determine whether it is an arithmetic sequence, a geometric sequence, a quadratic sequence and determine the formula/function for the value of a_k the k^{th} term of the sequence. If none of these sequence patterns matches the given terms, determine that and give a brief reason. A brief description of each sequence is below.

Arithmetic Sequences: The difference between consecutive terms is always the same. The general term is $a_k = a + (k-1)d$ where a is the initial value (first term) and d is the common difference between consecutive terms.

Geometric Sequences: The ratio of consecutive terms is always the same. The general term is $a_k = a \cdot r^{(k-1)}$ where a is the initial value (first term) and r is the common difference between consecutive terms.

Quadratic Sequences: The difference between consecutive terms is itself an arithmetic sequence. So the second level difference is always the same. The general term is $a_k = ak^2 + bk + c$ for some coeefficients a, b, c. We can find these coefficients by solving equations $a_1 = a \cdot 1^2 + b \cdot 1 + c$, $a_2 = \ldots$, and $a_3 = 9a + 3b + c$. Usually, this involves subtracting manipulating equations to isolate variables.

(a) -28, -22, -16, -10, ...

Solution:

The difference between consecutive terms is always 6. That means it is an arithmetic sequence starting at initial value $a = -28$ with common difference $d = 6$. The general term is:

$$a_k = a + (k-1)d = -28 + (k-1)6 = 6k - 34$$

Sanity check/smell test/verification:

$$\begin{aligned} a_1 &= 6 - 34 = -28 && \text{Matches} \\ a_2 &= 12 - 34 = -22 && \text{Matches} \\ a_3 &= 18 - 34 = -16 && \text{Matches} \\ a_4 &= 24 - 34 = -10 && \text{Matches} \end{aligned}$$

All known terms match our formula, so we were correct. ■

(b) 5, -7, -12, -19, ...

Solution:

Differences are: -12, -5, -7. These aren't the same, so it isn't an arithmetic sequence.

Second level differences are: 7, 2. These aren't the same either, so it isn't a quadratic sequence either.

The ratios are $^{-7}/_{5}$, $^{-12}/_{-7}$, $^{-19}/_{-12}$. These aren't the same, so it isn't a geometric sequence.

This isn't any of the known sequences. ■

(c) 5, -7, -19, -31, ...

Solution:

The differences are: -12, -12, -12. These are the same, so this is an arithmetic sequence with common difference $d = -12$ and initial value 5. The general term is:

$$a_k = a + (k-1)d = 5 + (k-1)(-12) = 17 - 12k$$

Sanity check/smell test/verification:

$$\begin{aligned} a_1 &= 17 - 12 = 5 && \text{Matches} \\ a_2 &= 17 - 24 = -7 && \text{Matches} \\ a_3 &= 17 - 36 = -19 && \text{Matches} \\ a_4 &= 17 - 48 = -31 && \text{Matches} \end{aligned}$$

■

(d) 4, 20, 100, 500, ...

Solution:

The ratios of successive terms are $^{20}/_{4} = 5$, $^{100}/_{20} = 5$, $^{500}/_{100} = 5$. These are the same, so this is a geometric sequence with common ratio $r = 5$ and initial value 4. The general term is:

$$a_k = a \cdot r^{k-1} = 4 \cdot 5^{k-1}$$

Sanity check/smell test/verification:

$$a_1 = 4 \cdot 5^{1-1} = 4 \cdot 5^0 = 4 \quad \text{Matches}$$
$$a_2 = 4 \cdot 5^{2-1} = 4 \cdot 5^1 = 20 \quad \text{Matches}$$
$$a_3 = 4 \cdot 5^{3-1} = 4 \cdot 5^2 = 100 \quad \text{Matches}$$
$$a_4 = 4 \cdot 5^{4-1} = 4 \cdot 5^3 = 500 \quad \text{Matches}$$

■

(e) 6, 11, 18, 27, 38, 51, ...

Solution:

The differences between terms are: 5, 7, 9, 11, 13. These aren't the same, so we try another level of differences: 2, 2, 2, 2. Therefore it is a quadratic sequence. The general term is:

$$a_k = ak^2 + bk + c$$

We substitute $k = 1$, $k = 2$, and $k = 3$ and then solve for a, b, c.

$$a_1 = a + b + c = 6$$
$$a_2 = 4a + 2b + c = 11$$
$$a_3 = 9a + 3b + c = 18$$

Subtracting first equation from second:

$$\begin{array}{rl} & 4a + 2b + c = 11 \\ - & a + b + c = 6 \\ & 3a + b = 5 \end{array}$$

Subtracting second equation from third:

$$\begin{array}{rl} & 9a + 3b + c = 18 \\ - & 4a + 2b + c = 11 \\ & 5a + b = 7 \end{array}$$

Subtracting the second result from the first, we get:

$$\begin{array}{rl} & 5a + b = 7 \\ - & 3a + b = 5 \\ & 2a = 2 \end{array}$$

Therefore, $a = 1$, plugging that into $3a + b = 5$ gives us $b = 2$. Plugging that into the very first equation gives us $1 + 2 + c = 6 \implies c = 3$.

Sanity check/smell test/verificaiton:

$$
\begin{aligned}
a_1 &= 1 + 2 + 3 = 6 && \text{Matches} \\
a_2 &= 4 \cdot 1 + 2 \cdot 2 + 3 = 11 && \text{Matches} \\
a_3 &= 9 \cdot 1 + 3 \cdot 2 + 3 = 18 && \text{Matches} \\
a_4 &= 16 \cdot 1 + 4 \cdot 2 + 3 = 27 && \text{Matches} \\
a_5 &= 25 \cdot 1 + 5 \cdot 2 + 3 = 38 && \text{Matches} \\
a_6 &= 36 \cdot 1 + 6 \cdot 2 + 3 = 51 && \text{Matches}
\end{aligned}
$$

■