CS1800 Discrete Structures

Recitations 5

Strange & Varma *Date: October 22 & 23, 2026*

- This recitation handout contains practice problems. You do *not* need to submit anything.
- We do recommend handwritting your solutions to completion. Do ensure that you are aware of how and why each step in the solution exists.
- For best effects, read through the questions before your recitation section and if possible attempt them on your own. This way you can compare your perspective to the TAs during the recitation.
- If you get stuck on a problem, use your recitation section to get help. Of course, you can also use office hours or post on Campuswire!
- The solutions to this recitation will be posted after Friday. Use those to verify your own attempts and for any clarification.
- If problems are marked $(*)$, those are fundamental and the others are additional practice using concepts from the fundamental practice problems in more complex ways.

Problem 1 (Counting cards)

There are 52 playing cards, 13 of each of the four sutis: Spades, Clubs, Hearts, and Diamonds. The 13 cards in each suit have values 2, 3, 4, ..., 9, 10, Jack (J), Queen (Q), King (K), and Ace (A). The last 4 (J, Q, K, A) are face cards while the others are numeric cards. The cards of Hearts and Diamonds are red in color while Spades and Clubs are black in color. So there are 26 red and 26 black cards.

When we draw cards, we draw them one by one. A hand of n-cards is a set of cards, without a specific ordering among them. Poker hands are a set of five (distinct) cards with certain properties.

(a) How many ways can you make a poker hand such that:

i. **(∗)** There are only red cards in the hand.

ii. There are only even value (numeric) cards in the hand.

iii. There are exactly two face cards in the hand.

iv. **(∗)** All cards in the hand are either cards with prime value or face cards.
So mixed hands are not allowed.

v. There is at least one card from each suit in the hand.

vi. **(∗)** The hand is either a Four of a kind or a Royal Flush.
Four of a kind: there are 4 cards with the same value, one from each suit.
Royal Flush: values: 10, J, Q, K, A, all of the same suit.

vii. **(∗)** There are exactly two cards of the same value. *So other 3 are all distinct.*

viii. **(∗)** There are exactly two cards of the same value and exactly three of a different value.

ix. All cards are consecutive in value. According to the order: A, 2, 3, ..., 9, 10, J, Q, K, A.
Yes, A shows up twice. Further, we are allowed to wrap around. So something like Q, K, A, 2, 3 is a valid ordering.

x. All cards are consecutive in value. According to the order: A, 2, 3, ..., 9, 10, J, Q, K, A.
Yes, A shows up twice. However, this time, we are NOT allowed to wrap around. So something like Q, K, A, 2, 3 is a NOT valid ordering.

(b) We draw 12 (distinct) cards in order. How many ways can we draw these cards such that:

i. **(∗)** Red and black cards alternate. It doesn't matter if red or black cards are at the start.

ii. 3 cards of each suit show up and all cards of the same suit are together.

iii. **(∗)** 3 red cards show up together, and they are not on either end.
So the first and last cards are black.

iv. There are exactly 6 cards of Spades, with none being the first or last card drawn.

v. **(∗)** All face cards show up after all numeric cards.

vi. There are four consecutive aces (A) with their suits alternating red and black.

vii. **(∗)** The pattern of suits is not: SHCD SHCD SHCD.

Problem 2 (Balls in Bins)

There are a 100 identical balls in a bag. These must be placed into 4 bins named A, B, C, D. How many ways can these balls be placed into the bins given the following conditions:

(a) in total, if there are no restrictions, except all balls being placed in some bin.

(b) **(∗)** if each bin must have at least one ball?

(c) **(∗)** if the combined total balls in bins A and B must be exactly equal to the combined total balls in bins C and D?

(d) if there must be more balls in bin A compared to all the other bins?

Problem 3 (Pigeons)

1. **(∗)** What's the minimum number of people required to guarantee that there are two people with the same exam score percentage? *Assume: Percentages are rounded so there are no decimals and the scores are between 0 and 100 (both inclusive).*

2. Twenty-five crates of apples are delivered to a store. The apples are of three different sorts, and all the apples in any single crate are of the same sort. Show that among these crates there are at least nine containing the same sort of apple.

3. **(∗)** The combined amount earned by 5 wizard apprentices is 1500 mana coins each week. The new spell book costs 320. Will all of them be able buy this new spell book?

4. You have 6 machines, each has a 6-core processor which can run 6 processes in parallel. When client jobs come to you at the start of the day, you need to assign them to one of these machines: 1 job uses exactly one core. If a job isn't able to start immediately, then the job doesn't finish by end of day and you lose that client because their work didn't get done. If any of the machines has less than 5 jobs running, then it is considered a waste of electricity since your profits won't make up for the cost of keeping these machines running. For each of these days, determine whether jobs are dropped (client lost), things run fine, or if machines are left idle (electricity wasted). You can assume the most optimal assignment of jobs to machines is used each day.

Day 1	Day 2	Day 3	Day 4	Day 5
30	39	25	32	36

Table 1: Table mapping each day to number of jobs.

5. You have been chosen to play a game against Aliens, the winners keep control over Earth. The challenge/wager is a mathematical game with the following rules:

   - Each box in a 3×3 arrangement of boxes is filled with one of the numbers $-1, 0, 1$ in a totally random way by the Universe (neither you nor the Aliens control it).
   - The sum of numbers in the boxes along the 3 rows and the 3 columns, and the diagonals are considered.
   - If any two sums are equal, you win and Aliens leave forever. If no two sums are equal, Aliens win and you lose Earth forever.

They explain that depending on what "the diagonals" means, the wager is different. If you only ask for the sum one of the two diagonals to be considered (your choice which), then the wager is only that all resources will belong to the Aliens and humans aren't replaced. If you ask for the sums along both diagonals to be considered, the wager will additionally include humans leaving Earth and renting a barebones settlement on the moon. Further, all humans will have to work for the Aliens in perpetuity, helping with their mining of Earth.

They claim that a sufficiently advanced species will predict the Universe correctly and make the right choice and hence win the wager. Thankfully for you, you seem to know more discrete mathematics than they do. How will you guarantee a win and avoid losing Earth?[1]

Problem 4 (A strong password)

Password strength can (very, very) loosely calculated by computing the logarithm to the base 2 of the number of possible passwords someone has to try to guess a specific password. For example, if all possible passwords are bitstrings of length 3, then there are 8 possibilities and $\log_2 8 = 3$ gives a "strength score" of 3. Compare to all 4-digit card PINs, which are 10^4 in number we get a "strength score" of $\log_2 10^4 = 4 \log_2 10 > 3$. So 4-digit PINs are stronger.

Note 1: This is a very crude measure. Note 2: You should use a calculator to do the logarithm calculation; focus on finding the number of possible passwords of a particular type.

Find the "strength score" for each of these types of passwords. Which type of password is the strongest according to this score.

(a) **(∗)** Length 10. Only lowercase letters.

(b) Length 9. Only letters, but both lowercase and uppercase allowed.

(c) Length 8. Any letter and any digit allowed.

(d) **(∗)** Length 12. Any letter and any digit allowed, as well as any (of 10) punctuations.

(e) **(∗)** Length 12. At least one uppercase (lowercase and digits allowed, no punctuations).

(f) Length 15. At least one uppercase and at least one digit (lowercase allowed, no punctuations).

(g) **(∗)** Length 8–12. Only letters.

(h) **(∗)** Length 20. Only lowercase letters and the space character.

(∗) What length password consisting of only the 10 digits will be stronger than the strongest scoring password type above?

Remark: *It is always easier to increase password strength by increasing its length than by increasing the complexity. It is even better to just use a long, memorable phrase or sentence than a weird, immemorable, alpha-numerical-punctuational jumble. Even better, use a (good) password manager to track passwords, this way you only really need to remember the password to this password manager.*

[1] Clearly, Akshar should only be allowed to make up stories for problems where the time spent reading the elaborate made-up situation won't affect grades.