CS1800 Discrete Structures

Recitations 4

Strange & Varma *Date: October 15 & 16, 2026*

- This recitation handout contains practice problems. You do *not* need to submit anything.
- We do recommend handwritting your solutions to completion. Do ensure that you are aware of how and why each step in the solution exists.
- For best effects, read through the questions before your recitation section and if possible attempt them on your own. This way you can compare your perspective to the TAs during the recitation.
- If you get stuck on a problem, use your recitation section to get help. Of course, you can also use office hours or post on Campuswire!
- The solutions to this recitation will be posted after Friday. Use those to verify your own attempts and for any clarification.

Problem 1 (Poker hands)

We'll use set terminology to talk about playing cards. The suits are the set $Suits = \{S, C, H, D\}$ referring to *S*pades, *C*lubs, *H*earts, and *D*iamonds. The card values are from the set $Values = \{2, 3, 4, \ldots, 9, 10, J, Q, K, A\}$ for the cards 2 through 10 and the face cards $Faces = \{J, Q, K, A\}$.

Poker hands are a set of five (distinct) cards with certain properties.

(a) Each playing card is defined as a pair of its suit and its value. Define the set of all playing cards *Cards* using set builder notation.

(b) *Almost* Royal Flush or Face Flush is *almost* the best possible hand[1]; it is described using set builder notation below. Describe, in English what characterizes this hand. Ensure that you understand the role of each term in the set builder notation.

$$FaceFlushes = \left\{ \left\{ (A, s), (K, s), (Q, s), (J, s), (v, s') \right\} \;\middle|\; s \in Suits, (s \neq s') \vee (v \notin Faces) \right\}$$

(c) A very strong poker hand is the "Four of a kind" where you have 4 cards of the same value, one from each suit and any other 5th card. Define a set $4oaks$ using set builder notation containing all possible Four of a kind hands.

(d) Describe in English the characteristics of the Full House hand defined by the set below. Ensure that you understand the role of each term in the set builder notation.

$$FullHouses = \left\{ \left\{ (v, s_1), (v, s_2), (v, s_3), (v', r_1), (v', r_2) \right\} \;\middle|\; \{v, v'\} \subset Values, \{s_1, s_2, s_3\} \subset Suits, \{r_1, r_2\} \in \mathcal{P}(Suits) \right\}$$

(e) Straight is a hand, which contains consecutive values of any suits (a run). The "order" is 2, 3, ..., 10, J, Q, K, A. So the values $7, 8, 9, 10, J$ satisfies the requirement for a run and is a straight hand; similarly $4, 5, 6, 7, 8$ is also a straight. There is special straight: $5, 4, 3, 2, A$ which is also considered a run although here 5 is considered the highest value although A is the highest value in general. Note that this is an exception and $3, 2, A, K, Q$ is not a valid run.

First, assume that there is a set *Runs* of all valid runs. Use that to define the set *Straights* of all straight hands in set builder notation. As a bonus, attempt to write the set *Runs* in set builder notation.

Problem 2 (PIEs!)

(a) Find the number of subsets of $T = \{1, 2, 3, 4, 5, 6, 7, 8\}$ that are subsets of neither $A = \{1, 2, 3, 4, 5\}$ nor $B = \{4, 5, 6, 7, 8\}$. You can use the fact that the number of subsets of a set of size n is 2^n.[2]

(b) In a certain class of 100 students, 70 use a tablet to take notes and 50 use a laptop. How many students solely use a tablet for note taking?

(c) Akshar knows Hindi, Gujarati, and Malayalam apart from English. He's asked around and calculated precise statistics[3] of how many students in this semester's Discrete course know

[1] The basic Royal Flush was too boring, so I modified it.
[2] Another reason to know your powers of 2.
[3] Nope. All made up just for this problem. But I'll be happy to find out actual numbers.

these languages. There are 30 students in the course who know at least one of these three languages but some know two or even all three. There are 20 students who know Hindi, 15 who know Gujarati, and 9 who know Malayalam. There are 8 students who know at least two languages. How many students know all three languages?

Problem 3 (Counting)

Recall: The cardinality of the Cartesian product, $|A \times B| = |A| \times |B|$. In Combinatorics, the general form of this is called the ***Rule of Product or Product Rule****:*

$$|A_1 \times A_2 \times A_3 \times \dots A_n| = |A_1| \times |A_2| \times |A_3| \times \dots |A_n|$$

Similarly, recall the PIE (Principle of Inclusion-Exclusion): $|A_1 \cup A_2| = |A_1| + |A_2| - |A_1 \cap A_2|$ and its more general expressions for more terms. In particular, when all the intersections are empty, that is A_1, A_2, $\dots$, A_n are disjoint sets, then this becomes the ***Sum Rule or Rule of Sum****:*

$$|A_1 \cup A_2 \cup \dots \cup A_n| = |A_1| + |A_2| + \dots + |A_n| \qquad \textbf{iff } |A_i \cap A_j| = 0 \text{ for all } i \neq j$$

(a) You want to dress up (for Halloween?) as a character from one of the TV shows: Game of Thrones, Community, and Sherlock. You have 20 candidates you can dress up as from Game of Thrones, 10 from Community and 8 from Sherlock. How many total different choices do you have for your Halloween dress up.[4]

(b) If each person in your 3 person group for CS1800 decides to dress up as a different character, one from each show from the previous subpart, how many total combinations can there be for your groups character choices?

(c) How many 10 character passwords exist if you can use the digits $\{0, 1, \dots, 9\}$, upper case letters $\{A, B, \dots, Z\}$ and lower case letters $\{a, b, \dots, z\}$?

(d) How many passwords of 8 to 12 characters exist if you only use the digits $\{0, 1, \dots, 9\}$?
[***Hint****: Consider sets $P_8, P_9, \dots, P_{12}$ to be the set of 8, 9, …, 12 character passwords. You need to find $|P_8 \cup P_9 \cup \dots \cup P_{12}|$. Thankfully, any $P_i \cap P_j$ is empty.*]

(e) How many 10 character passwords using the digits $\{0, 1, \dots, 9\}$, upper case letters $\{A, B, \dots, Z\}$ and lower case letters $\{a, b, \dots, z\}$ exist if you must use at least one digit and at least one upper case letter?
[***Hint****: Remove unwanted thing count instead of attempting to count wanted things.*]

[4] Yes, none of these fits horror/Halloween vibes, but I randomly pulled TV shows off the top of my head.

Problem 4 (Why not Venn?)

Is this a true Venn Diagram for four sets? If so, confirm that all possible intersections are representable. Otherwise, point out which intersection is not representatble in the regions of the diagram.

Try labeling each region with the intersection it can be used to represent. For example, $A = A \cap B^C \cap C^C \cap D^C$, $AB = A \cap B \cap C^C \cap D^c$, $ABC = A \cap B \cap C \cap D^C$ and so on.

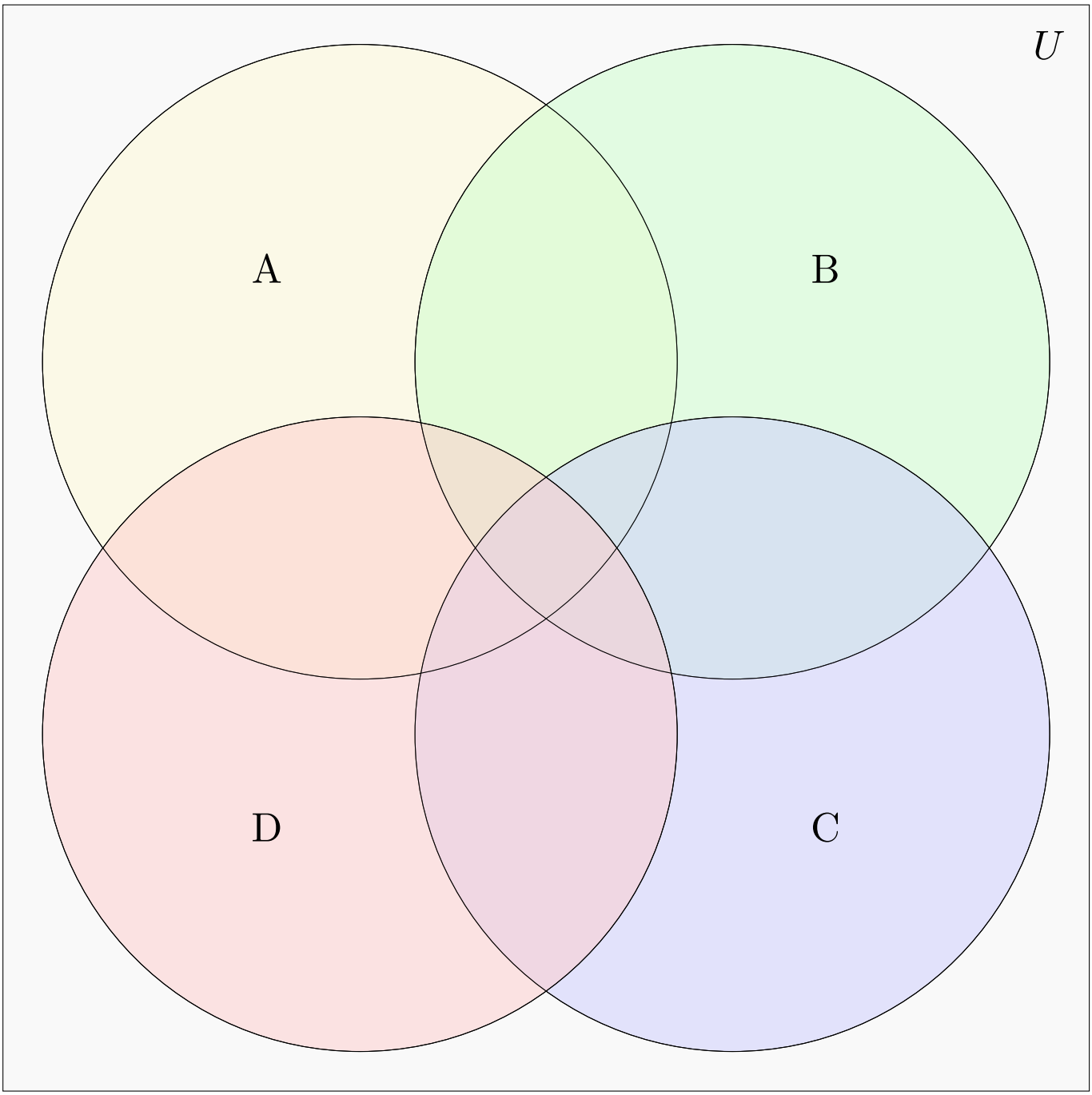


Figure 1: Is this a true Venn Diagram for four sets?

Problem 5 (Is Venn?)

Is this a true Venn Diagram for four sets? If so, confirm that all possible intersections are representable. Otherwise, point out which intersection is not representatble in the regions of the diagram.

Try labeling each region with the intersection it can be used to represent. For example, $A = A \cap B^C \cap C^C \cap D^C$, $AB = A \cap B \cap C^C \cap D^c$, $ABC = A \cap B \cap C \cap D^C$ and so on.

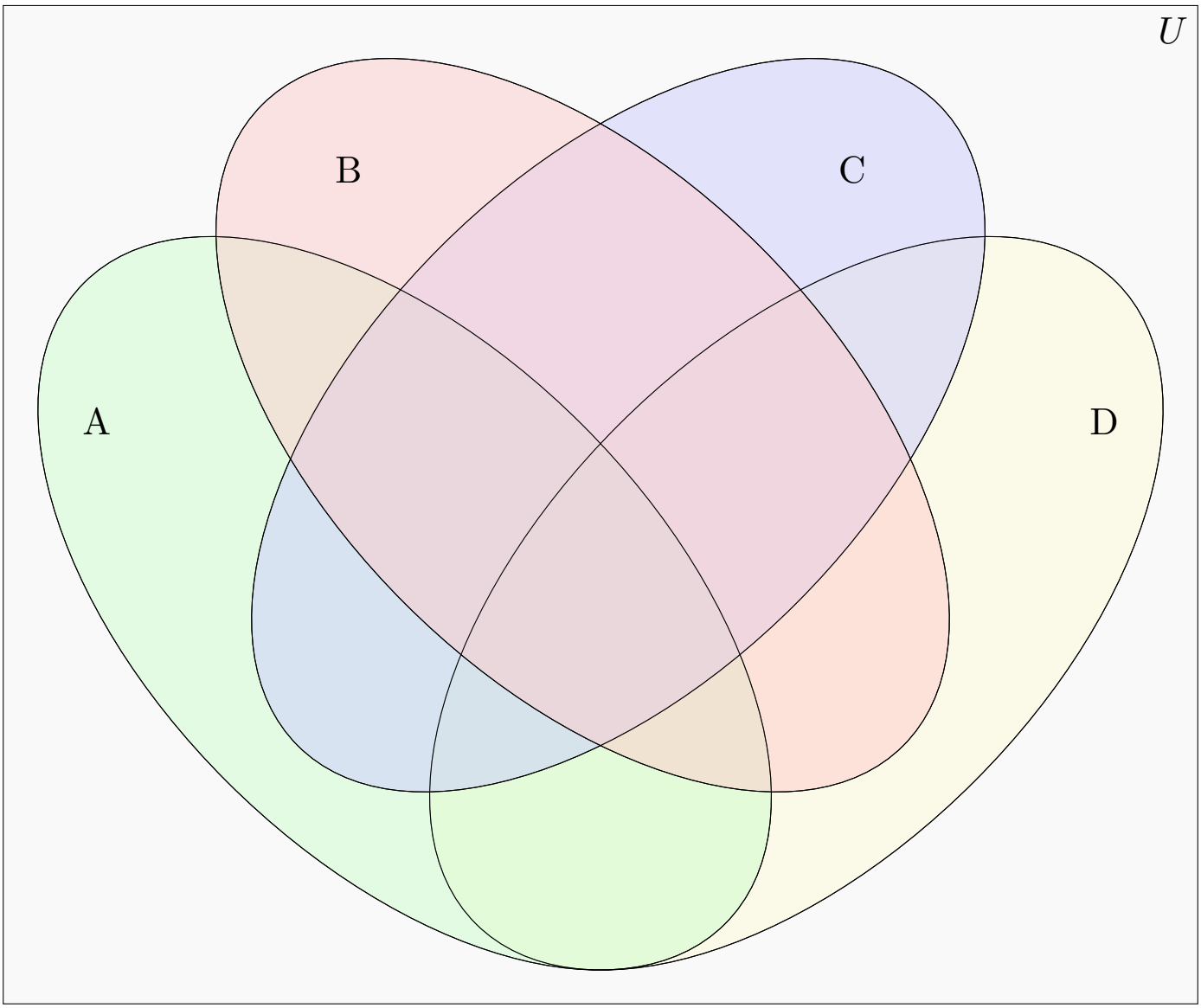


Figure 2: Is this a true Venn Diagram for four sets?

Problem 6 (Factorio)

Let $X = \{n \in \mathbb{N} \mid 0 \leq n \leq 9\}$. Define $P(x, S)$ to be the predicate $\forall f \in S : x \bmod f = 0$. Here the notation $a \bmod b$ is the remainder when the number a is divided by the number b; so $3 \bmod 2 = 1$, $5 \bmod 3 = 2$, $10 \bmod 5 = 0$. So the predicate is testing that x is divisible by all elements f in the set S.

Define the set:

$$F = \Big\{ (x, S) \in X \times \Big(\mathcal{P}(X) \setminus \{\emptyset\}\Big) \;\Big|\; P(x, S) \wedge \forall R \in \Big(\mathcal{P}(X) \setminus \{S\}\Big) : P(x, R) \implies |R| < |S| \Big\}$$

Write out the set F in roster notation and describe in words what the subset S consists of in the pairs $(x, S) \in F$.