CS1800 Discrete Structures

Recitations 3

Strange & Varma *Date: October 8 & 9, 2026*

- This recitation handout contains practice problems. You do *not* need to submit anything.
- We do recommend handwritting your solutions to completion. Do ensure that you are aware of how and why each step in the solution exists.
- For best effects, read through the questions before your recitation section and if possible attempt them on your own. This way you can compare your perspective to the TAs during the recitation.
- If you get stuck on a problem, use your recitation section to get help. Of course, you can also use office hours or post on Campuswire!
- The solutions to this recitation will be posted after Friday. Use those to verify your own attempts and for any clarification.

Problem 1 (Thorin's Company)

Consider the Universe set $U = \{\text{Thorin, Balin, Bifur, Bofur, Bombur, Dori, Dwalin, Fíli, Glóin, Kíli, Nori, Óin, Ori}\}$ containing the names of the dwarves in Thorin's Company. Answer the following questions:

(a) Write the following set in roster notation: $B = \{d \in U \mid d\text{'s name starts with B}\}$.

(b) Write the following set in set builder form: $I = \{\text{Dori, Fíli, Kíli, Nori, Ori}\}$.

(c) Write the following set in roster notation:
$E = \{d \in U \mid d\text{'s name end with R}\} \cup \{d \in U \mid d\text{'s name end with N}\}$.

(d) Write the following set in roster notation:
$T = \{x \in U \mid \exists y \in U \ \wedge \ \text{last three letters of } x \text{ and } y \text{ are the same}\}$.

(e) Determine if the following are True or False:

i. $\text{Thorin} \in E$

ii. $\{\text{Balin, Bifur, Dori}\} \subseteq E \cup I$

iii. $\text{Balin} \in B \cap I$

iv. $I \subseteq T$

(f) Write the full cartesian product: $\{\text{Dori, Nori}\} \times \{\text{Bifur, Bofur}\}$.

(g) How many elements will $B \times I$ contain?

(h) What's the power set of $\{\text{Thorin, Fíli, Kíli}\}$? That is, what is $\mathfrak{P}\Big(\{\text{Thorin, Fíli, Kíli}\}\Big)$?

(i) What's the power set of $\mathfrak{P}\big(\{\text{Thorin}\}\big)$? *Note: This is the power set of a power set.*

(j) What's the cardinality of $B \cup \{\text{Bofur, Dori}\}$?

(k) Draw a Venn Diagram with the sets B, I, E and write each of the dwares in U in their appropriate place inside the Venn Diagram.

Problem 2 (Venn will this end?)

Draw Venn diagrams (separate for each subpart) which shows the Universal set U, along with sets A, B, (and C when present) and the specified elements x and y in their correct place in the Venn Diagram according to the description.

1. $x \in A$, $A \subset B$
2. $x \in A$, $A \cap B = \emptyset$
3. $x \in A$, $x \notin B$, $B \subseteq A$
4. $x \in A$, $x \in B$, $A \not\subset B$, $B \not\subset A$
5. $x \in A$, $A \subset B$, $y \notin A$, $y \in B$
6. $x \notin A$, $B \subseteq A$

Problem 3 (Bits$_e$trings)

A bit string is a string of 1s and 0s. A bit string representation of a set is a bitstring with each bit representing a different elemnet in the universe. The bitstring contains a 1 if the element is in the set and a 0 otherwise. See example for A and B in Table 1 below.

$A = \{\texttt{halfling}, \texttt{elf}, \texttt{dwarf}\}$ $B = \{\texttt{gnome}, \texttt{dwarf}\}$

U	`elf`	`halfling`	`gnome`	`dwarf`	`tiefling`	Boolean Operation
A	1	1	0	1	0	—
B	0	0	1	1	0	—
$A \cup B$						
$A \cap B$						
$\overline{A}$						
$A \Delta B$						

Table 1: Bitstring representation of Sets

1. Fill the empty rows of the table above using the bit string representation for the corresponding set mentioned in the leftmost column.
2. Tell which logical operator (AND, NOT, XOR, OR) applied to the bit string representations of A, B yield the same bit string representations as the rows of $A \cup B$, $A \cap B$, $\overline{A}$, $A \Delta B$.

Problem 4 (What's in the name of sets is going on?)

Determine whether each of these statements is True or False.

1. $\emptyset = \{0\}$
2. $\emptyset = \{\emptyset\}$
3. $\emptyset \in \{\emptyset\}$
4. $\emptyset \subseteq \{\emptyset\}$
5. $x \in \{x\}$
6. $x \subseteq \{x\}$
7. $x \in \mathfrak{P}(\{x\})$
8. $x \subseteq \mathfrak{P}(\{x\})$
9. $\{x\} \in \{x\}$
10. $\{x\} \subseteq \{x\}$
11. $\{x\} \in \mathfrak{P}(\{x\})$
12. $\{x\} \subseteq \mathfrak{P}(\{x\})$
13. $|\{x, x\}| = 2$
14. $|\{\emptyset\}| = 0$
15. $|U \cap \emptyset| = 0$

Commutative Laws	$A \cap B \equiv B \cap A$ $A \cup B \equiv B \cup A$
Associative Laws	$(A \cap B) \cap C \equiv A \cap (B \cap C)$ $(A \cup B) \cup C \equiv A \cup (B \cup C)$
Distributive Laws	$A \cap (B \cup C) \equiv (A \cap B) \cup (A \cap C)$ $A \cup (B \cap C) \equiv (A \cup B) \cap (A \cup C)$
Identity Laws	$A \cap U \equiv A$ $A \cup \emptyset \equiv A$
Annihiltor/Domination Laws	$A \cap \emptyset \equiv \emptyset$ $A \cup U \equiv U$
Idempotence Laws	$A \cap A \equiv A$ $A \cup A \equiv A$
Complement/Negation Laws	$A \cap A^C \equiv \emptyset$ $A \cup A^C \equiv U$
Double Negation Law	$\left(A^C\right)^C \equiv A$
Absorption Laws	$A \cap (A \cup B) \equiv A$ $A \cup (A \cap B) \equiv A$
De Morgan's Laws	$(A \cap B)^C \equiv A^C \cup B^C$ $(A \cup B)^C \equiv A^C \cap B^C$

Table 2: Laws of Set Operations. A, B, C are arbitrary sets belonging to the universal set U.

Problem 5 (I am the Law)

Prove the following sets equal using the laws (see Table 2 for reference) of set operations:

a) $B \cup (A \cap \emptyset) = B$

b) $\overline{(A^C \cap U)} = A$

c) $(A \cap B) \cup (A \cap \overline{B}) = A$

d) $(A \cap B) \cup (A \cup \overline{B})^C = B$

e) $A \times (B \cup C) = (A \times B) \cup (A \times C)$
In this case, show that all (x, y) in the set on the left also belongs to the set on the right and vice versa. Laws may be used as part of this process but laws alone will not be sufficient.