CS1800 Discrete Structures

Solutions to Recitations 3

Strange & Varma *Date: October 8 & 9, 2026*

- This recitation handout contains practice problems. You do *not* need to submit anything.
- We do recommend handwritting your solutions to completion. Do ensure that you are aware of how and why each step in the solution exists.
- For best effects, read through the questions before your recitation section and if possible attempt them on your own. This way you can compare your perspective to the TAs during the recitation.
- If you get stuck on a problem, use your recitation section to get help. Of course, you can also use office hours or post on Campuswire!
- The solutions to this recitation will be posted after Friday. Use those to verify your own attempts and for any clarification.

Problem 1 (Thorin's Company)

Consider the Universe set $U = \{\text{Thorin, Balin, Bifur, Bofur, Bombur, Dori, Dwalin, Fíli, Glóin, Kíli, Nori, Óin, Ori}\}$ containing the names of the dwarves in Thorin's Company. Answer the following questions:

(a) Write the following set in roster notation: $B = \{d \in U \mid d\text{'s name starts with B}\}$.

Solution:

$B = \{\text{Balin, Bifur, Bofur, Bombur}\}$ ■

(b) Write the following set in set builder form: $I = \{\text{Dori, Fíli, Kíli, Nori, Ori}\}$.

Solution:

$I = \{d \in U \mid d\text{'s name end with I}\}$ ■

(c) Write the following set in roster notation:
$E = \{d \in U \mid d\text{'s name end with R}\} \cup \{d \in U \mid d\text{'s name end with N}\}$.

Solution:

$$\begin{aligned} E &= \{\text{Bifur, Bofur, Bombur}\} \cup \{\text{Thorin, Balin, Dwalin, Glóin, Óin}\} \\ &= \{\text{Bifur, Bofur, Bombur, Thorin, Balin, Dwalin, Glóin, Óin}\} \end{aligned}$$

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(d) Write the following set in roster notation:
$T = \{x \in U \mid \exists y \in U \ \wedge$ last three letters of x and y are the same$\}$.

Solution:

These are all those who have rhyming names. Everyone except Thorin and Bombur (although this partially does rhyme with Bifur and Bofur).

$T = \{\underline{\text{Balin, Dwalin}},\ \underline{\text{Bifur, Bofur}},\ \underline{\text{Dori, Nori, Ori}},\ \underline{\text{Fíli, Kíli}},\ \underline{\text{Glóin, Óin}}\}$

$T = U \setminus \{\text{Thorin, Bombur}\}$ ■

(e) Determine if the following are True or False:

Solution:

i. $\text{Thorin} \in E$
True: Thorin ends in an "n".

ii. $\{\text{Balin, Bifur, Dori}\} \subseteq E \cup I$
True: Balin and Bifur are part of E while Dori is in I. So all of them are in $E \cup I$, hence the set containing these 3 dwarves is a subset of $E \cup I$.

iii. Balin $\in B \cap I$
False: Balin needs to be in both B and I for this statement to be true, but Balin doesn't end in "i", so he cannot be in the set I.

iv. $I \subseteq T$
True: All the dwarves in I do belong to the set T since T contains everyone except Thorin and Bombur.

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(f) Write the full cartesian product: {Dori, Nori} × {Bifur, Bofur}.

Solution:

$$\{\text{Dori, Nori}\} \times \{\text{Bifur, Bofur}\} = \{(\text{Dori, Bifur}), (\text{Dori, Bofur}), (\text{Nori, Bifur}), (\text{Nori, Bofur})\}$$

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(g) How many elements will $B \times I$ contain?

Solution:

$|B \times I| = |B| \times |I| = 4 \times 5 = 20$. So this Cartesian product will contain 20 elements. ■

(h) What's the power set of {Thorin, Fíli, Kíli}? That is, what is $\mathfrak{P}\Big(\{\text{Thorin, Fíli, Kíli}\}\Big)$?

Solution:

$$\mathfrak{P}\Big(\{\text{Thorin, Fíli, Kíli}\}\Big) = \Big\{\{\}, \{\text{Thorin}\}, \{\text{Fíli}\}, \{\text{Kíli}\}, \{\text{Thorin, Fíli}\}, \{\text{Thorin, Kíli}\}, \{\text{Fíli, Kíli}\}, \{\text{Thorin, Fíli, Kíli}\}\Big\}$$

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(i) What's the power set of $\mathfrak{P}\big(\{\text{Thorin}\}\big)$? *Note: This is the power set of a power set.*

Solution:

$\mathfrak{P}\big(\{\text{Thorin}\}\big) = \{\{\}, \{\text{Thorin}\}\}$.

Therefore,

$$\mathfrak{P}\Big(\mathfrak{P}\big(\{\text{Thorin}\}\big)\Big) = \mathfrak{P}\Big(\{\{\}, \{\text{Thorin}\}\}\Big) = \Big\{\Big\{\Big\}, \Big\{\{\}\Big\}, \Big\{\{\text{Thorin}\}\Big\}, \Big\{\{\}, \{\text{Thorin}\}\Big\}\Big\}$$

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(j) What's the cardinality of $B \cup \{\text{Bofur, Dori}\}$?

Solution:

$B \cup \{\text{Bofur, Dori}\} = \{\text{Balin, Bifur, Bofur, Bombur, Dori}\}$. Therefore its cardinality is 5.

Alternatively, by the Principle of Inclusion-Exclusion we have $|A \cup B| = |A| + |B| - |A \cap B|$ for any A and B.

Therefore, we get: $|B \cup \{\text{Bofur, Dori}\}| = |B| + |\{\text{Bofur, Dori}\}| - |B \cap \{\text{Bofur, Dori}\}| = 4 + 2 - 1 = 5$. ■

(k) Draw a Venn Diagram with the sets B, I, E and write each of the dwares in U in their appropriate place inside the Venn Diagram.

Solution:

Draw a classic 3 set Venn diagram. There are no dwarves outside these sets nor are there any dwarves in the region shared by all 3 sets. Further, there are no dwarves that are only inside the B region. Everyone in the set B is actually in the region shared by B and E. Everyone in the set I is in the region that solely belongs to set I. Everyone else is in the region solely belonging to E. ■

Problem 2 (Venn will this end?)

Draw Venn diagrams (separate for each subpart) which shows the Universal set U, along with sets A, B, (and C when present) and the specified elements x and y in their correct place in the Venn Diagram according to the description.

Solution:

1. $x \in A$, $A \subset B$

   A inside B, x inside A. Circle inside circle and point in the inner circle.

2. $x \in A$, $A \cap B = \emptyset$

   x inside A, B is completely separate from A. The circles don't intersect at all.

3. $x \in A$, $x \notin B$, $B \subseteq A$

   B inside A, x in the gap between A and B. Circle contains a circle and a point, separately.

4. $x \in A$, $x \in B$, $A \not\subset B$, $B \not\subset A$

   A and B intersect (classic Venn diagram) and x is in the intersection region.

5. $x \in A$, $A \subset B$, $y \notin A$, $y \in B$

   x inside A which is inside B. y between A and B.

6. $x \notin A$, $B \subseteq A$

   B inside A and the point x outside both.

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Problem 3 (Bits$_{\text{e}}$trings)

A bit string is a string of 1s and 0s. A bit string representation of a set is a bitstring with each bit representing a different elemnet in the universe. The bitstring contains a 1 if the element is in the set and a 0 otherwise. See example for A and B in Table 1 below.

1. Fill the empty rows of the table above using the bit string representation for the corresponding set mentioned in the leftmost column.
2. Tell which logical operator (AND, NOT, XOR, OR) applied to the bit string representations of A, B yield the same bit string representations as the rows of $A \cup B, A \cap B, \overline{A}, A\Delta B$.

Solution:

$A = \{\texttt{halfling}, \texttt{elf}, \texttt{dwarf}\}$ $B = \{\texttt{gnome}, \texttt{dwarf}\}$

U	`elf`	`halfling`	`gnome`	`dwarf`	`tiefling`	Boolean Operation
A	1	1	0	1	0	
B	0	0	1	1	0	
$A \cup B$	1	1	1	1	0	OR: $A \vee B$
$A \cap B$	0	0	0	1	0	AND: $A \wedge B$
$\overline{A}$	0	0	1	0	1	NOT: $\neg A$
$A\Delta B$	1	1	1	0	0	XOR: $A \oplus B$

Table 1: Bitstring representation of Sets

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Problem 4 (What's in the name of sets is going on?)

Determine whether each of these statements is True or False.

Solution:

1. $\emptyset = \{0\}$
   False: the set on the right contains one element, 0. So it is not empty.
2. $\emptyset = \{\emptyset\}$
   False: the set on the right contains one element, the empty set itself. So the set is not empty.
3. $\emptyset \in \{\emptyset\}$
   True: the empty set is indeed an element belonging to the set on the right.

4. $\emptyset \subseteq \{\emptyset\}$
   True: the empty set is a subset of all sets, so it is definitely a subset of the set on the right. However, a more preceise statement would be, $\emptyset \subset \{\emptyset\}$ since we know that the empty set is not equal to the set on the right.

5. $x \in \{x\}$
   True: x is indeed a member of $\{x\}$.

6. $x \subseteq \{x\}$
   False: x is an element, not a set and hence it cannot be a subset of $\{x\}$.

7. $x \in \mathfrak{P}(\{x\})$
   False: $\mathfrak{P}(\{x\}) = \{\{\}, \{x\}\}$ and x is not a member of this set.

8. $x \subseteq \mathfrak{P}(\{x\})$
   False: As above, x is not a set and hence it is not a subset. The correct form would be $\{x\} \in \mathfrak{P}(\{x\})$, which would be True.

9. $\{x\} \in \{x\}$
   False: these are two sets that are equal to each other. One is not an element of the other.

10. $\{x\} \subseteq \{x\}$
    True: every set is a subset of itself, since these two sets are equal, they are subsets of each other.

11. $\{x\} \in \mathfrak{P}(\{x\})$
    True: $\mathfrak{P}(\{x\}) = \{\{\}, \{x\}\}$ and $\{x\}$ is indeed an element of this set.

12. $\{x\} \subseteq \mathfrak{P}(\{x\})$
    False: x is not an element of $\mathfrak{P}(\{x\})$, therefore $\{x\}$ cannot be a subset of it.

13. $|\{x, x\}| = 2$
    False: $\{x, x\} = \{x\}$ which only contains a single element, so its cardinality is 1.

14. $|\{\emptyset\}| = 0$
    False: $\{\emptyset\}$ is a set that contains 1 element, the empty set. Therefore its cardinalty is 1, not 0. It would be correct to say $|\emptyset| = 0$.

15. $|U \cap \emptyset| = 0$
    True: By the Annihilator/Domination law (see Table 2) $U \cap \emptyset = \emptyset$. This has no elements, so the cardinality of 0 is correct.

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Commutative Laws	$A \cap B \equiv B \cap A$ $A \cup B \equiv B \cup A$
Associative Laws	$(A \cap B) \cap C \equiv A \cap (B \cap C)$ $(A \cup B) \cup C \equiv A \cup (B \cup C)$
Distributive Laws	$A \cap (B \cup C) \equiv (A \cap B) \cup (A \cap C)$ $A \cup (B \cap C) \equiv (A \cup B) \cap (A \cup C)$
Identity Laws	$A \cap U \equiv A$ $A \cup \emptyset \equiv A$
Annihiltor/Domination Laws	$A \cap \emptyset \equiv \emptyset$ $A \cup U \equiv U$
Idempotence Laws	$A \cap A \equiv A$ $A \cup A \equiv A$
Complement/Negation Laws	$A \cap A^C \equiv \emptyset$ $A \cup A^C \equiv U$
Double Negation Law	$\left(A^C\right)^C \equiv A$
Absorption Laws	$A \cap (A \cup B) \equiv A$ $A \cup (A \cap B) \equiv A$
De Morgan's Laws	$(A \cap B)^C \equiv A^C \cup B^C$ $(A \cup B)^C \equiv A^C \cap B^C$

Table 2: Laws of Set Operations. A, B, C are arbitrary sets belonging to the universal set U.

Problem 5 (I am the Law)

Prove the following sets equal using the laws (see Table 2 for reference) of set operations:

a) $B \cup (A \cap \emptyset) = B$

Solution:

$$\begin{aligned} B \cup (A \cap \emptyset) &= B \cup \emptyset && \text{Annihilator/Domination} \\ &= B && \text{Identity} \end{aligned}$$

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b) $\overline{(A^C \cap U)} = A$

Solution:

$$\begin{aligned} \overline{(A^C \cap U)} &= \overline{A^C} \cup \overline{U} && \text{De Morgan's} \\ &= A \cup \emptyset && \text{Double Negation + Definition} \\ &= A && \text{Identity} \end{aligned}$$

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c) $(A \cap B) \cup (A \cap \overline{B}) = A$

Solution:

$$\begin{aligned}
(A \cap B) \cup (A \cap \overline{B}) &= A \cap (B \cup \overline{B}) && \text{Distributive} \\
&= A \cap U && \text{Complement} \\
&= A && \text{Identity}
\end{aligned}$$

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d) $(A \cap B) \cup (A \cup \overline{B})^C = B$

Solution:

$$\begin{aligned}
(A \cap B) \cup (A \cup \overline{B})^C &= (A \cap B) \cup (A^C \cap \overline{B^C}) && \text{De Morgan} \\
&= (A \cap B) \cup (A^C \cap B) && \text{Double Negation} \\
&= (A \cup A^C) \cap B && \text{Distributive} \\
&= U \cap B && \text{Complement} \\
&= B && \text{Identity}
\end{aligned}$$

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e) $A \times (B \cup C) = (A \times B) \cup (A \times C)$
In this case, show that all (x, y) in the set on the left also belongs to the set on the right and vice versa. Laws may be used as part of this process but laws alone will not be sufficient.

Solution:

$$\begin{aligned}
(A \times B) \cup (A \times C) &= \{(x, y) \mid x \in A \wedge y \in B\} \\
&\quad \cup \{(x, y) \mid x \in A \wedge y \in C\} && \text{Cartesian Product Definition} \\
&= \{(x, y) \mid (x \in A \wedge y \in B) \vee (x \in A \wedge y \in C)\} && \text{Union definition} \\
&= \{(x, y) \mid x \in A \wedge (y \in B \vee y \in C)\} && \text{Logic Distributive law} \\
&= \{(x, y) \mid x \in A \wedge (y \in B \cup C)\} && \text{Union definition} \\
&= A \times (B \cup C) && \text{Cartesian Product definition}
\end{aligned}$$

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