CS1800 Discrete Structures

Solutions to Recitations 2

Strange & Varma *Date: September 24 & 25, 2026*

- This recitation handout contains practice problems. You do *not* need to submit anything.
- We do recommend handwritting your solutions to completion. Do ensure that you are aware of how and why each step in the solution exists.
- For best effects, read through the questions before your recitation section and if possible attempt them on your own. This way you can compare your perspective to the TAs during the recitation.
- If you get stuck on a problem, use your recitation section to get help. Of course, you can also use office hours or post on Campuswire!
- The solutions to this recitation will be posted after Friday. Use those to verify your own attempts and for any clarification.

Problem 1 (English to/from Logical expression)

For the first part of the question, let s = "stocks are increasing" and i = "interest rates are steady" and b = "it is a bullish market". Rewrite the English statements using $\wedge, \vee, \neg$ and the boolean variables s and i and translate the boolean formulae into English statements.

i. Stocks are increasing but interest rates are steady.

Solution:

Rewriting sentence: Stocks are increasing *and* interest rates are steady.

$s \wedge i$ ■

ii. Neither are stocks increasing nor are interest rates steady.

Solution:

Rewriting sentence: Stocks are *not* increasing and are interest rates are *not* steady.

$\neg s \wedge \neg i = \neg(s \vee i)$ ■

iii. If the interest rate is not steady then the stock will not increase.
Simplify the logic expression using contrapositive and then rewrite that boolean formula back into an English sentence.

Solution:

Logical expression is: $\neg i \implies \neg s$.

Taking contrapositive we get: $s \implies i$.

In English this translates to: If the stocks are increasing then the interest rate must be steady. ■

iv. $b \implies (s \vee \neg i)$

Solution:

If the market is bullish then the stocks are increasing or the interest rate is not steady. ■

v. $\neg(i \wedge \neg s) \implies b$. How is this implication statement related to the previous implication statement (contrapositive, inverse, converse).

Solution:

If it is not the case that the interest rates are steady and the stock are not increasing, then it must be that the market is bullish.

This is the converse of the previous implication. However, De Morgan's (and commutative) law has also been applied since $(s \vee \neg i) \equiv \neg(i \wedge \neg s)$. ■

Problem 2 (Quantifying the shapes)

Refer to Figure 1 to answer the following questions for each of the statements provided.

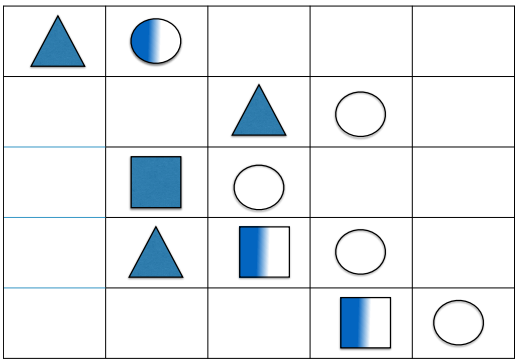


Figure 1: Figure for problem 4. A Tarski World, a tool for understanding logical statements invented by Jon Barwise and John Etchemendy, named after Alfred Tarski

- Express it using logical symbols $(\forall, \exists, \rightarrow, \wedge, \vee, \neg)$ as well as the predicates: Circle, Square, Triangle, Fully Shaded, Half Shaded, Clear.
  You can use notation $Row(shape), Column(shape)$ to denote the row number and column number of shape. Use these when rewriting the statements as logical expression. A statement of the form x is above y refers to the fact that the shape x is on a row above shape y irrespective of column; that is, $Row(x) > Row(y)$. Similarly for left/right/below and so on.
- Tell if it is true or false
- If a statement is universally quantified (with a $\forall$ symbol) and false, then provide a counter-example
- If a statement is existentially quantified (with a $\exists$ symbol) and true, then provide an example

Note, you may only use the negation symbol $\neg$ in front of a predicate.

Solution:

1. For every triangle x, there is a square y, where x is above y.

Symbolic: $\forall\, Triangle(x), \exists\, Square(y) :\ Row(x) > Row(y)$.

Truthfulness: True

Proof: For every triangle, the square on the last row can be used. Every triangle is above this square.

2. There is a circle x, such that for all squares y, x is above y.

Symbolic: $\exists\, Circle(x), \forall\, Square(y) :\ Row(x) > Row(y)$.

Truthfulness: True.

Example: The circle in row 1 is on a higher row than every square.

3. There is a triangle x and a circle y such that x and y have the same shading.

Symbolic: $\exists\, Triangle(x), Circle(y)$ such that $\Big(Fully-Shaded(x) \land Fully-Shaded(y)\Big) \lor \Big(Half-Shaded(x) \land Half-Shaded(y)\Big) \lor \Big(Clear(x) \land Clear(y)\Big)$

Truthfulness: False

Proof: All triangles are fully shaded but no circle is fully shaded.

4. For all circles x, there exist a square y, such that y is to the right of x.

Symbolic: $\forall\, Circle(x), \exists\, Square(y) :\ Column(y) > Column(x)$.

Truthfulness: False

Example: The circle in the rightmost column has no square to the right of it.

5. For all triangles x, and for all squares y, if x is above y, then x is to the right of y.

Symbolic: $\forall\, Triangle(x), \forall\, Square(y) :\ Row(x) > Row(y) \implies Column(x) > Column(y)$.

Truthfulness: False.

Counter-example: For the square in row 3 and triangle in row 1, the triangle is above the square but is to the left of the square.

6. **(****)** It is not true that: For all triangles x, and for all squares y, if x is above y, then x is to the right of y.

Symbolic:

$$\begin{aligned}
&\neg\Big(\forall\, Triangle(x), \forall\, Square(y) :\ Row(x) > Row(y) \implies Column(x) > Column(y)\Big) \\
&\equiv \exists\, Triangle(x), \exists\, Square(y) :\ \neg\Big(Row(x) > Row(y) \implies Column(x) > Column(y)\Big) \\
&\equiv \exists\, Triangle(x), \exists\, Square(y) :\ \neg\Big(Row(x) \leq Row(y) \lor Column(x) > Column(y)\Big) \\
&\equiv \exists\, Triangle(x), \exists\, Square(y) :\ \Big(Row(x) > Row(y) \land Column(x) \leq Column(y)\Big)
\end{aligned}$$

Truthfulness: True.

Proof: For the square in row 3 and triangle in row 1, the triangle is above the square but is not to the right of the square.

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Problem 3 (Logical expression simplification)

Write beside each step the rule being used in the following proof of equivalence or fill in the missing step given the rule.

Solution:

$$
\begin{aligned}
& (\neg c \vee (c \vee a)) \wedge \neg(\neg b \wedge \neg b) && \\
\equiv\ & ((\neg c \vee c) \vee a) \wedge \neg(\neg b \wedge \neg b) && \text{Associative law} \\
\equiv\ & ((\neg c \vee c) \vee a) \wedge (\neg\neg b \vee \neg\neg b) && \text{De Morgan's law} \\
\equiv\ & ((\neg c \vee c) \vee a) \wedge (b \vee b) && \text{Double Negation law} \\
\equiv\ & ((\neg c \vee c) \vee a) \wedge b && \text{Idempotence law} \\
\equiv\ & (T \vee a) \wedge b && \text{Complement law} \\
\equiv\ & T \wedge b && \text{Annihilator law} \\
\equiv\ & b && \text{Identity law}
\end{aligned}
$$

■

Problem 4 (Convert to decimal)

NOTE: If you haven't done this in class yet, skip for now and attempt after the lecture.

Convert the following non-decimal numbers into decimal. Recall that the subscript tells you the base used:

(a) 1323_4

(b) 162_8

(c) 10101000011_2

(d) $AB3_{16}$ (A and B are the next "digits" after 8, 9 equaling values of 10 and 11 respectively.)

Solution:

(a) $1323_4 = 1 \cdot 4^3 + 3 \cdot 4^2 + 2 \cdot 4^1 + 3 \cdot 4^0 = 1 \cdot 64 + 3 \cdot 16 + 2 \cdot 4 + 3 \cdot 1 = 123$

(b) $162_8 = 1 \cdot 64 + 6 \cdot 8 + 2 \cdot 1 = 114$

(c) $10101000011_2 = 1 \cdot 2^9 + 0 \cdot 2^8 + 1 \cdot 2^7 + 0 \cdot 2^6 + 1 \cdot 2^5 + 0 \cdot 2^4 + 0 \cdot 2 + 0 \cdot 2^3 + 0 \cdot 2^2 + 1 \cdot 2^1 + 1 \cdot 2^0 = 512 + 128 + 32 + 2 + 1 = 675$

(d) $AB3_{16} = A \cdot 16^2 + B \cdot 16^1 + 3 \cdot 16^0 = 10 \cdot 256 + 11 \cdot 16 + 3 = 2739.$

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Problem 5 (Convert from decimal)

NOTE: If you haven't done this in class yet, skip for now and attempt after the lecture.

Convert these decimal numbers to the specified base:

(a) 133 to base 2

(b) 162 to base 9

(c) 121 to base 4

(d) 729 to base 2

Solution:

(a) 133 to base 2

Dividing by 2 and tracking the quotient and remainder:

$$\begin{aligned} 133 &\to (66, 1) \\ &\to (33, 0) \\ &\to (16, 1) \\ &\to (8, 0) \\ &\to (4, 0) \\ &\to (2, 0) \\ &\to (1, 0) \\ &\to (0, 1) \end{aligned}$$

Reading remainders and placing them right to left gives the binary number: 10000101_2

(b) 162 to base 9 Dividing by 2 and tracking the quotient and remainder:

$$\begin{aligned} 162 &\to (18, 0) \\ &\to (2, 0) \\ &\to (0, 2) \end{aligned}$$

Reading remainders and placing them right to left gives the nonary number: 200_9. Easy to verify since this says the number is twice the square of 9: $2 \cdot 81 = 162$.

(c) 121 to base 4 Dividing by 4 and tracking the quotient and remainder:

$$\begin{aligned} 121 &\to (30, 1) \\ &\to (7, 2) \\ &\to (1, 3) \\ &\to (0, 1) \end{aligned}$$

Reading remainders and placing them right to left gives the base-4 number: 1321_4

(d) 729 to base 2 Dividing by 2 and tracking the quotient and remainder:

$$\begin{aligned} 729 &\to (364, 1) \\ &\to (182, 0) \\ &\to (91, 0) \\ &\to (45, 1) \\ &\to (22, 1) \\ &\to (11, 0) \\ &\to (5, 1) \\ &\to (2, 1) \\ &\to (1, 0) \\ &\to (0, 1) \end{aligned}$$

Reading remainders and placing them right to left gives the binary number: 1011011001_2

■