CS1800 Discrete Structures

Solutions to Recitations 1

Strange & Varma *Date: September 17 & 18, 2026*

- This recitation handout contains practice problems. You do *not* need to submit anything.
- We do recommend handwritting your solutions to completion. Do ensure that you are aware of how and why each step in the solution exists.
- For best effects, read through the questions before your recitation section and if possible attempt them on your own. This way you can compare your perspective to the TAs during the recitation.
- If you get stuck on a problem, use your recitation section to get help. Of course, you can also use office hours or post on Campuswire!
- The solutions to this recitation will be posted after Friday. Use those to verify your own attempts and for any clarification.

Problem 1 (Transforming Circuits I)

TAs will walk you through one of these.

1. For the circuit given in Figure 1, fill up its Truth Table.
2. Directly translate from logic gates to using boolean operators to represent the same circuit as a logical formula.
3. Based on the truth table, what is the simplest logical expression representing the circuit?
4. Using the laws of boolean algebra, simplify the direct translation from circuits to formula to get the same simplification as you got from the truth table.

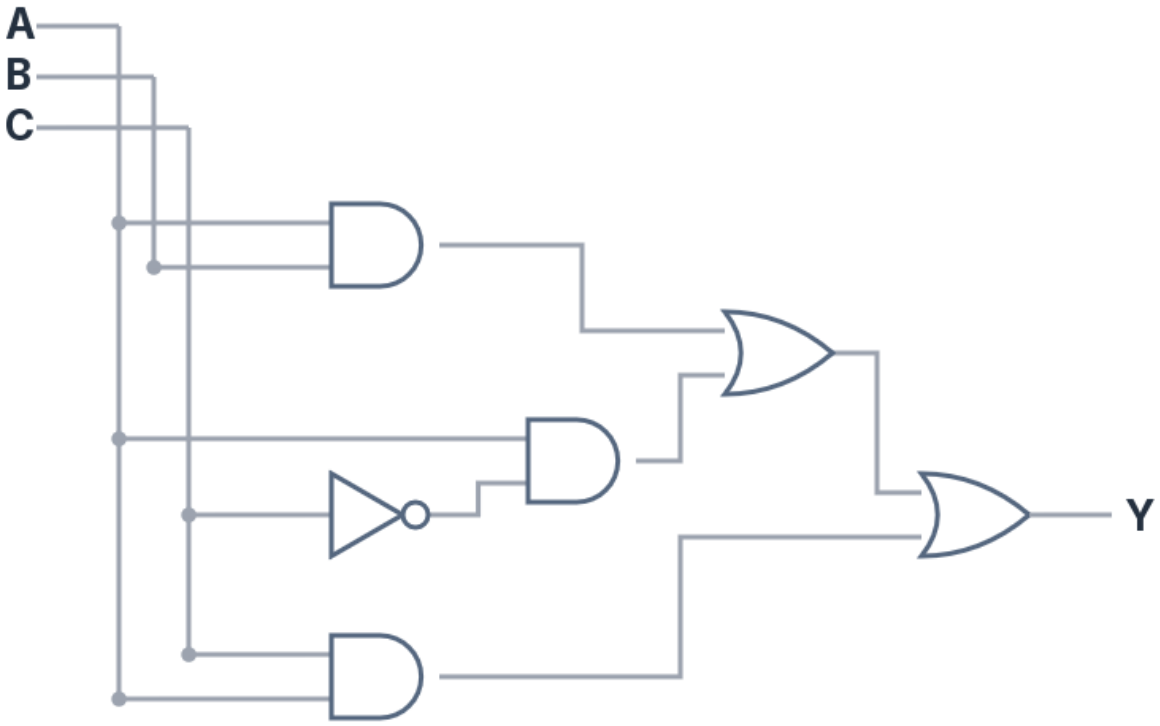


Figure 1: Circuit 1

Solution:

Boolean expression:

$$(A \wedge B) \vee (A \wedge \neg C) \vee (C \wedge A)$$

Using the truth table, the final result seems to simplify to just A.

Table 1: Truth Table for circuit in 1

A	B	C	$A \wedge B$	$\neg C$	$A \wedge \neg C$	$C \wedge A$	$(A \wedge B) \vee (A \wedge \neg C) \vee (C \wedge A)$
0	0	0	0	1	0	0	0
0	0	1	0	0	0	0	0
0	1	0	0	1	0	0	0
0	1	1	0	0	0	0	0
1	0	0	0	1	1	0	1
1	0	1	0	0	0	1	1
1	1	0	1	1	1	0	1
1	1	1	1	0	0	1	1

Simplifying using laws:

$$
\begin{aligned}
& (A \wedge B) \vee (A \wedge \neg C) \vee (C \wedge A) && \\
= \; & A \wedge \left(B \vee \neg C \vee C \right) && \text{Distributive} \\
= \; & A \wedge \left(B \vee \mathrm{T} \right) && \text{Complement} \\
= \; & A \wedge \mathrm{T} && \text{Domination} \\
= \; & A && \text{Identity}
\end{aligned}
$$

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Problem 2 (Transforming Circuits II)

ATTEMPT THIS ON YOUR OWN.

1. For the circuit given in Figure 2, fill up its Truth Table.
2. Directly translate from logic gates to using boolean operators to represent the same circuit as a logical formula.
3. Based on the truth table, what is the simplest logical expression representing the circuit?
4. Using the laws of boolean algebra, simplify the direct translation from circuits to formula to get the same simplification as you got from the truth table.

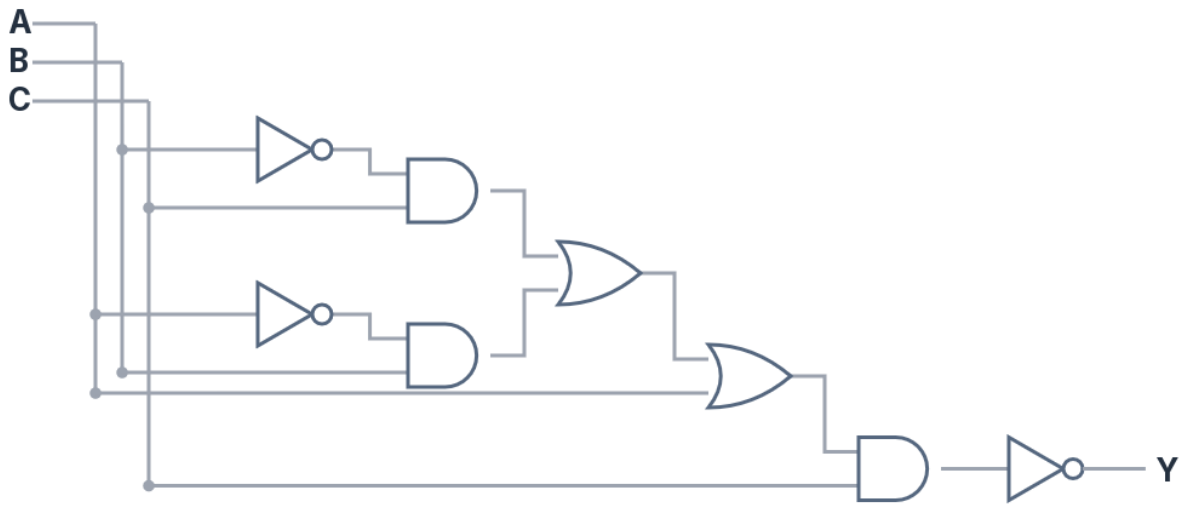


Figure 2: Circuit 2

Solution:

A	B	C	$\neg B$	$\neg B \wedge C$	$\neg A$	$\neg A \wedge B$	OR gates	AND with C	final NOT gate
0	0	0	1	0	1	0	0	0	1
0	0	1	1	1	1	0	1	1	0
0	1	0	0	0	1	1	1	0	1
0	1	1	0	0	1	1	1	1	0
1	0	0	1	0	0	0	1	0	1
1	0	1	1	1	0	0	1	1	0
1	1	0	0	0	0	0	1	0	1
1	1	1	0	0	0	0	1	1	0

Table 2: Truth Table for circuit in 2

Boolean expression:

$$\neg\left(((\neg B \wedge C) \vee (\neg A \wedge B) \vee A) \wedge C\right)$$

Using the truth table, the final result seems to simplify to just $\neg C$.

Simplifying using laws:

$$
\begin{aligned}
& \neg\Big(\big((\neg B \wedge C) \vee \boxed{(\neg A \wedge B)} \vee \boxed{A}\big) \wedge C\Big) \\
= \neg & \Big(\big((\neg B \wedge C) \vee \boxed{\big(\boxed{(A \vee \neg A)} \wedge (A \vee B)\big)}\big) \wedge C\Big) && \text{Distributive} \\
= \neg & \Big(\boxed{\big((\neg B \wedge C) \vee \big(\boxed{\mathrm{T}} \wedge (A \vee B)\big)\big) \wedge C}\Big) && \text{Complement} \\
= \neg & \Big(\boxed{(C \wedge \neg B \wedge C) \vee (C \wedge \mathrm{T} \wedge (A \vee B))}\Big) && \text{Distributive} \\
= \neg & \Big(\boxed{C \wedge \big(\neg B \vee \boxed{(\mathrm{T} \wedge (A \vee B))}\big)}\Big) && \text{Distributive} \\
= \neg & \Big(C \wedge \boxed{\big(\neg B \vee \boxed{A} \vee \boxed{B}\big)}\Big) && \text{Identity} \\
= \neg & \Big(C \wedge \boxed{(A \vee \mathrm{T})}\Big) && \text{Complement} \\
= \neg & \Big(\boxed{C \wedge \boxed{\mathrm{T}}}\Big) && \text{Domination} \\
= \neg & \boxed{C} && \text{Identity}
\end{aligned}
$$

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Problem 3 (Converting to Logic)

TAs will guide through this, one by one.

Consider the following pieces of identification a person might have in order to apply for a credit card:

- B = "Applicant presents a birth certificate."
- D = "Applicant presents a drivers license."
- M = "Applicant presents a marriage license."

Write a logical expression for the requirements under the following conditions:

1. The applicant must present either a birth certificate, a drivers license, or a marriage license.
2. The applicant must not present a marriage license and a drivers license, and must present a birth certificate.
3. The applicant must present either a birth certificate or both a drivers license and a marriage license.

Solution:

1. $B \vee D \vee M$
2. $\neg(M \wedge D) \wedge B$
3. $B \vee (D \wedge M)$

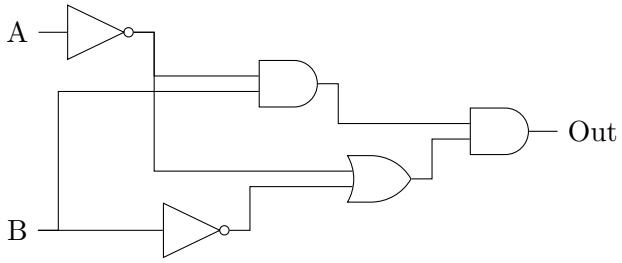
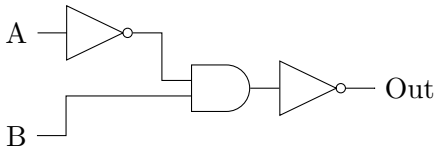
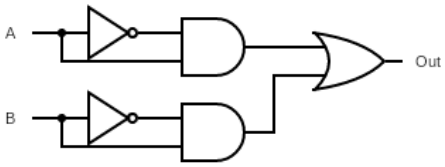
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Problem 4 (Truth Tables ↔ Boolean Formulae ↔ Circuits)

ATTEMPT THIS ON YOUR OWN.

Fill in the following table with the missing truth tables, Boolean formulae, and circuits:

Solution:

Truth Table	Boolean Formula	Circuit
A | B | Out 0 | 0 | 0 0 | 1 | 1 1 | 0 | 0 1 | 1 | 0	$(\neg A \vee \neg B) \wedge (B \wedge \neg A)$	
A | B | Out 0 | 0 | 1 0 | 1 | 0 1 | 0 | 1 1 | 1 | 1	$\neg(\neg A \wedge B)$	
A | B | Out 0 | 0 | 0 0 | 1 | 0 1 | 0 | 0 1 | 1 | 0	F	

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Problem 5 (Simplify)

These are more practice for you.

Simplify each of the following boolean formulas:

1. $B \vee B \vee C \vee A \vee C \vee C \vee (C \wedge B)$

Solution:

$$
\begin{aligned}
& \boxed{B \vee B \vee C \vee A \vee C \vee C \vee (C \wedge B)} && \\
= \; & \boxed{B \vee B \vee C \vee A \vee C \vee C} && \text{Absorption} \\
= \; & \boxed{B \vee C \vee A \vee C \vee C} && \text{Idempotent} \\
= \; & \boxed{B \vee C \vee A \vee C} && \text{Idempotent} \\
= \; & \boxed{B \vee C \vee A} && \text{Idempotent}
\end{aligned}
$$

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2. $((C \vee \neg C) \wedge B \wedge (B \vee \neg B)) \vee A \vee C \vee \neg\neg C$

Solution:

$$
\begin{aligned}
& \boxed{((C \vee \neg C) \wedge B \wedge (B \vee \neg B))} \vee A \vee C \vee \neg\neg C && \\
= & \boxed{\left(\boxed{(C \vee \neg C)} \wedge B\right)} \vee A \vee C \vee \neg\neg C && \text{Absorption} \\
= & \left(\boxed{\mathrm{T}} \wedge B\right) \vee A \vee C \vee \boxed{\neg\neg C} && \text{Complement} \\
= & \boxed{(\mathrm{T} \wedge B) \vee A \vee C \vee \boxed{C}} && \text{Double Negation} \\
= & \boxed{\boxed{(\mathrm{T} \wedge B)} \vee A \vee C} && \text{Idempotent} \\
= & \boxed{B} \vee A \vee C && \text{Identity}
\end{aligned}
$$

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3. $\neg\neg C \vee \neg\left((A \wedge C) \vee B \vee C \vee A\right)$

Solution:

$$
\begin{aligned}
& \boxed{\neg\neg C} \vee \neg\left((A \wedge C) \vee B \vee C \vee A\right) && \\
= \; & \boxed{C} \vee \neg\left(\boxed{(A \wedge C) \vee B \vee C \vee A}\right) && \text{Double Negation} \\
= \; & C \vee \boxed{\neg\left(\boxed{B \vee C \vee A}\right)} && \text{Absorption} \\
= \; & C \vee \boxed{(\neg B \wedge \neg C \wedge \neg A)} && \text{DeMorgan's} \\
= \; & \boxed{C \vee \boxed{(\neg(B \vee A) \wedge \neg C)}} && \text{DeMorgan's} \\
= \; & \boxed{(C \vee \neg(B \vee A)) \wedge \boxed{(C \vee \neg C)}} && \text{Distributive} \\
= \; & \boxed{(C \vee \neg(B \vee A)) \wedge \boxed{\mathrm{T}}} && \text{Complement} \\
= \; & \boxed{C \vee \neg(B \vee A)} && \text{Identity}
\end{aligned}
$$

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4. $(\neg(\neg C \wedge \neg A) \wedge (B \vee C \vee C \vee B \vee B) \wedge (\neg C \vee B \vee B)) \vee B$

Solution:

$$\left(\neg(\neg C \wedge \neg A) \wedge \boxed{(B \vee C \vee C \vee B \vee B)} \wedge \boxed{(\neg C \vee B \vee B)}\right) \vee B$$

$$= \left(\neg\left(\boxed{\neg C \wedge \neg A}\right) \wedge \boxed{(B \vee ((C \vee C) \wedge \neg C))}\right) \vee B \qquad \text{Distributive}$$

$$= \left(\boxed{\neg \boxed{\neg(C \vee A)}} \wedge (B \vee ((C \vee C) \wedge \neg C))\right) \vee B \qquad \text{DeMorgan's}$$

$$= \left(\boxed{(C \vee A)} \wedge \left(B \vee \left(\boxed{(C \vee C)} \wedge \neg C\right)\right)\right) \vee B \qquad \text{Double Negation}$$

$$= \left((C \vee A) \wedge \left(B \vee \boxed{\left(\boxed{C} \wedge \neg C\right)}\right)\right) \vee B \qquad \text{Idempotent}$$

$$= \left((C \vee A) \wedge \boxed{\left(B \vee \boxed{\mathrm{F}}\right)}\right) \vee B \qquad \text{Complement}$$

$$= \boxed{\left((C \vee A) \wedge \boxed{B}\right) \vee B} \qquad \text{Identity}$$

$$= \boxed{B} \qquad \text{Absorption}$$

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