CS1800 Discrete Structures *Out: November 8, 2026*

Problem Set 7

Strange & Varma *Due: November 13, 2026*

- This assignment is **due on November 13, 2026 at 11:59 PM** via Gradescope. You may submit up to 48 hours late with no penalty.
- Your solutions must be handwritten, with your name on the first page. All your work must be shown even if the problem doesn't specifically ask for it.
- Discussing problems with classmates is allowed but all solutions must be written by yourself, in your own words.
- Copying solutions from a classmate, an online source, or AI tools, is a violation of our academic integrity policy. Do *not* submit anything you cannot explain. Please list all your collaborators and online reference sources in your solution for each problem.
- If you get stuck on a homework problem, come by office hours or post on Campuswire! We recommend you spend about 30 minutes trying to figure out a problem, and then ask for help. We'll be happy to clarify material from class and algorithm concepts, but we will not give out solutions or confirm your answers are correct.

[***Hint:***
For some of these problems which require calculating the expected value, you would want to utilize Linearity of Expectation*, which says, for random variables X, Y and constants a, that:*

- $E[X + Y] = E[X] + E[Y]$
- $E[aX] = a \cdot E[X]$

]

Problem 1 (Expect to Dice) 2x8 + 2+2

The notation dn is used to represent an n-sided fair[1] dice. A common way to represent chance in RPGs (role-playing games) is to use dice rolls with some modifiers.

The most common example whenever any action is attempted by a player is to roll a $d20$, add a bonus b and compare this to some threshold t. Mathematically, this can be denoted as checking whether $d20 + b \geq t$ is true or not. If the inequality is true, then the action is successful and if the inequality is false then the action is unsuccessful.

More complex numbers can be represented by adding more dice rolls. For example, $d4 + d6$ represents rolling a 4-sided dice and a 6-sided dice and then adding their results. A common shorthand is to write kdn when an n-sided dice must be rolled k times and their results added. For example, in combat, damage to game characters environment and so on can be represented as $8d6$. Thus, a 6-sided dice is rolled 8 times and the resulting numbers are added up.

One common place both kinds of rolls show up is during combat. An attack roll is compared to a threshold to see if the attack hits or not. Then, a damage roll is performed to see how much damage that attack caused.

(a) For each of the attack and damage rolls below, find the expected damage of an attack[2] assuming that the defense threshold is 15.

  i. Attack roll is $d20 + 8$ and damage roll is $3d8 + 3$.

  ii. Attack roll is $2 \times (d10) + 8$ and damage roll is $4d6 + 3$.[3]

(b) Which attack roll is more likely to hit? What if the second attack roll was $2 \times (d10) + 7$ instead?

(c) Which combination, on average, deals most damage? Which is deals least damage?

Problem 2 (Be right about left) 4

We are going to look at the three sections of CS1800, let's call them sections A, B, and C. Let's assume that section A has 100 students, section B has 105 students, and section C has 110 students. In section A, there are 4 left handed students; in section B, there are 10 left handed students; in section C, there are 20 left handed students. Given that I randomly pick one of the (100+105+110) students in the course, and get a left handed student, what is the probability that this student is from section A?

[1] Each of the numbers from 1 to n are equally likely to show up.

[2] The expected damage of a hit is just the expectation of the damage roll. The expected damage of an attack needs to consider whether the attack hits or misses.

[3] In this case $2 \times (d10)$ means: roll a 10-sided dice, and then double the result.

Problem 3 (A wizard's staff) 6

There are two kinds of wizards, those from middle earth and those from Hogwarts. All middle earth wizards use staffs for magic while only 2 in every 100 wizards from Hogwarts use staffs. There are a total of 5 wizards in middle earth while Hogwarts has a 1000 wizards in total.

You randomly choose a wizard in such a way that there is a probability of 0.2 that a wizard from middle earth is chosen. If the wizard you chose happens to use a staff, determine the probability that this wizard came from Hogwarts.

Problem 4 (Roses are red, violets are blue) 3x2 + 8 + 6

There are 3 red balls and 5 blue balls in an urn. You cannot see inside the urn and since the balls are identical except for their color, you cannot distinguish the balls by feel. When you draw a ball from the earn, each of them is equally likely.

There are two ways to draw multiple balls from the bin. The first is "without replacement", so you draw a ball, keep it with you, draw the next ball, and so on. The second is "with replacement", where you draw a ball, note its color *replace* the ball into the bin and then draw a ball again, and so on.

(a) What is the probability of each of the following events happening? You should separately calculate probabilities for the two kinds of ways: "without replacement" and "with replacement".

   i. The first ball is red and the second ball is blue.

   ii. No red ball was drawn out of the total of two balls that are drawn.

   iii. There are two blue balls and one red (drawn in any order).

(b) You draw two balls out of the urn "without replacement". Let the count of blue balls drawn be b and that of the red balls drawn be r. Compute the expectation of $b^3 - r^2 + br$.

(c) Your friend is telling you the colors of the balls being drawn and based on their speed, you think that there is a $8/10$ probability that your friend is drawing balls "with replacement". If the colors your friend mentions (in order) are (Blue, Red), then what is the probability that your friend is actually drawing the balls "with replacement".

[***Hint**: In some problems, you might have to find the probability of event A. However, you cannot find this probability without knowing whether event B happened or not. In such situations, $P(A)$ can be rewritten as: $P(A) = P(B) \cdot P(A|B) + P(\overline{B}) \cdot P(A|\overline{B})$.*]