CS1800 Discrete Structures *Out: October 26, 2026*

Problem Set 6

Strange & Varma *Due: October 31, 2026*

- This assignment is **due on October 31, 2026 at 11:59 PM** via Gradescope. You may submit up to 48 hours late with no penalty.
- Your solutions must be handwritten, with your name on the first page. All your work must be shown even if the problem doesn't specifically ask for it.
- Discussing problems with classmates is allowed but all solutions must be written by yourself, in your own words.
- Copying solutions from a classmate, an online source, or AI tools, is a violation of our academic integrity policy. Do *not* submit anything you cannot explain. Please list all your collaborators and online reference sources in your solution for each problem.
- If you get stuck on a homework problem, come by office hours or post on Campuswire! We recommend you spend about 30 minutes trying to figure out a problem, and then ask for help. We'll be happy to clarify material from class and algorithm concepts, but we will not give out solutions or confirm your answers are correct.

Problem 1 (Hobbit Party) 5+10+6+6+13

When Mr. Bilbo Baggins of Bag End announced that he would shortly be celebrating his eleventy-first birthday with a party of special magnificence, there was much talk and excitement in Hobbitton. For subproblems 2, 3, and 4, reduce your answer to an integer and show your work. For subproblems 1 and 5, a concise mathematical expression suffices.

1. Suppose Bilbo invited 111 guests. In how many ways might the guests arrive at the party if they arrive one at a time?
2. Suppose the guests include 11 Proudfoots (Proudfeet!) consisting of 4 women, 4 men, and 3 children. All the women arrive together and enter one at a time in some order, similarly for the men and the children; however which group arrives in what order is not fixed. How many ways can all 11 of the Proudfeet arrive if they come in this grouped manner?
3. Each table has a flower arrangement with 11 flowers on each table. Bilbo makes the flower arrangement by making a bunch of 11 flowers using some combination of roses, daisies, irises, lilies, and elanor (a golden star-shaped flower that grows in Lothl'orien). How many different flower arrangements are possible?
4. Bilbo has 11 silver knives, 11 silver forks, and 11 silver spoons. The knives are all indistinguishable from each other as are the forks and spoons. In how many ways can Lobelia steal 11 pieces of silverware while ensuring she takes at least 2 knives, 2 forks, and 2 spoons.
5. What if Frodo Baggins and Lobelia Sackville-Baggins are two of the eleven guests at one of the (round) tables and they must NOT be seated next to each other. How many distinct and valid seating arrangements are there for this table? Rotating guests around the table does not count as a distinct seating arrangement.

Problem 2 (Fruit Bowl) 18

A fruit bowl contains apples, bananas, oranges, and pears (at least 14 of each kind).

1. In how many ways can we choose 14 fruits, including at least 5 oranges?
2. In how many ways can we choose 14 fruits, so that there are at most 4 oranges?
3. In how many ways can we choose 14 fruits, so that there are at most 4 oranges and at most 2 pears?

Problem 3 (Pigeonhole Principle) 4+8

1. What is the minimum number of players that must be assigned to 10 different gaming groups such that at least one group has at least 4 members?
2. Suppose a set of 8 numbers are selected from the set $\{1, 2, 3, ..., 13, 14\}$. Show that two of the selected numbers must sum to 15.
   [***Hint**: Think about how many subsets of 2 elements you can form such that the sum of the values of the two elements is 15.*]

Notation: Throughout the next problem, the notation ndk is used to denote the following process: a k-sided dice is rolled n times and the resulting numbers are all added up to get the final result. For example, 3d6 would be 3 rolls of a 6-sided dice and looking at the sum of the face-up numbers (eg. 2, 2, 4) as the final result (eg. 8). Additionally, multiple such dice combinations can be performed together and the results added up, for example: 3d6 + 2d4 + 2 means 3 rolls of a 6-sided dice, 2 rolls of a 4-sided dice, add up all the results and then add a 2 to get the final result.

Problem 4 (Probability of Dice) 40

Consider a set of dice that have a 4-, 6-, 8-, 10-, 12-, and 20-sided die. What combination of dice will make it most likely to roll a nine. That is, what ndk (out of the following) has the highest probability to roll a 9.

1. 3d6
2. 2d10
3. 1d12 + 1d4
4. 1d20

For each of the above dice combinations, i) calculate the size of the sample space, ii) determine the ways the dice can sum to nine (event space size), iii) calculate the probability of rolling nine.

Problem 5 (Chance of Snow) 20

Akshar has lived in Boston for a long time. From experience, he knows that each day has a 5% chance to be snowy. Akshar is looking ahead to the next 10 days.

1. What is the probability none of those days will be snowy?
2. What is the probability at least one of those days will be snowy?
3. What is the probability exactly 3 of those days will be snowy?
4. Akshar decides that, he will wear his heavy coat to work 3 of the 10 days without looking at the weather. What is the probability Akshar will wear his coat on at least one snowy day?