CS1800 Discrete Structures *Out: October 26, 2026*

Instructor Solutions to Problem Set 6

Strange & Varma *Due: October 31, 2026*

- This assignment is **due on October 31, 2026 at 11:59 PM** via Gradescope. You may submit up to 48 hours late with no penalty.
- Your solutions must be handwritten, with your name on the first page. All your work must be shown even if the problem doesn't specifically ask for it.
- Discussing problems with classmates is allowed but all solutions must be written by yourself, in your own words.
- Copying solutions from a classmate, an online source, or AI tools, is a violation of our academic integrity policy. Do *not* submit anything you cannot explain. Please list all your collaborators and online reference sources in your solution for each problem.
- If you get stuck on a homework problem, come by office hours or post on Campuswire! We recommend you spend about 30 minutes trying to figure out a problem, and then ask for help. We'll be happy to clarify material from class and algorithm concepts, but we will not give out solutions or confirm your answers are correct.

Problem 1 (Hobbit Party) 5+10+6+6+13

When Mr. Bilbo Baggins of Bag End announced that he would shortly be celebrating his eleventy-first birthday with a party of special magnificence, there was much talk and excitement in Hobbitton. For subproblems 2, 3, and 4, reduce your answer to an integer and show your work. For subproblems 1 and 5, a concise mathematical expression suffices.

1. Suppose Bilbo invited 111 guests. In how many ways might the guests arrive at the party if they arrive one at a time?

   Solution:

   $$\begin{aligned}111! =&1,762,952,551,090,244,663,872,161,047,107,075,788,761,409,536,\\ &026,565,516,041,574,063,347,346,955,087,248,316,436,555,574,598,\\ &462,315,773,196,047,662,837,978,913,145,847,497,199,871,623,320,\\ &096,254,145,331,200,000,000,000,000,000,000,000,000,000\end{aligned}$$

   ■

2. Suppose the guests include 11 Proudfoots (Proudfeet!) consisting of 4 women, 4 men, and 3 children. All the women arrive together and enter one at a time in some order, similarly for the men and the children; however which group arrives in what order is not fixed. How many ways can all 11 of the Proudfeet arrive if they come in this grouped manner?

   Solution:

   The four women can enter in 4! ways, the men in 4! ways and the children in 3! ways. The groups among themselves can be ordered in 3! ways. In total this is $(4!3!)^2 = 20,736$ ■

3. Each table has a flower arrangement with 11 flowers on each table. Bilbo makes the flower arrangement by making a bunch of 11 flowers using some combination of roses, daisies, irises, lilies, and elanor (a golden star-shaped flower that grows in Lothl'orien). How many different flower arrangements are possible?

   Solution:

   Stars and bars: There are 4 bars to separate the 5 types of flowers and 11 stars need to be arranged along with the 4 bars. This can be done in $\binom{11+4}{4} = \binom{15}{4} = 1,365$ ways. ■

4. Bilbo has 11 silver knives, 11 silver forks, and 11 silver spoons. The knives are all indistinguishable from each other as are the forks and spoons. In how many ways can Lobelia steal 11 pieces of silverware while ensuring she takes at least 2 knives, 2 forks, and 2 spoons.

   Solution:

   Lobelia first ensures she gets 2 knives, forks and spoons for a total of 6. Then she picks $11-6=5$ cutlery and she has 3 kinds of cutlery she can pick. Via bins and balls (3 bins one for each kind

of cutlery and 5 balls to throw) or stars and bars (2 bars to separate the 3 kinds of cutlery and 5 stars to be placed alongside) we get that the number of ways Lobelia can complete her theft is $\binom{5+2}{2} = \binom{7}{2} = \frac{7 \cdot 6}{2} = 21$. ■

5. What if Frodo Baggins and Lobelia Sackville-Baggins are two of the eleven guests at one of the (round) tables and they must NOT be seated next to each other. How many distinct and valid seating arrangements are there for this table? Rotating guests around the table does not count as a distinct seating arrangement.

Solution:

Fixed perspective approach: Since they are sitting on a round table, rotations do not matter. So let us fix Frodo's seat and then place all the others around. Lobelia Sackville-Baggins cannot sit on either side of Frodo, so she has $11 - 1 - 2 = 8$ spots available. The remaining 9 guests can be seated anywhere so that provides 9! ways. In total there are $8 \cdot 9! = 2,903,040$ seating arrangements.

Accounting for rotations, total-minus-unwanted approach: The unwanted situation is when Frodo and Lobelia sit together; this can be thought of as them being one single guest and there being 9 other guests. In total that gives us 10! arrangements. However, in a round table there is no "first" spot and any ten of the spots could've been the "first" spot. To account for this, we divide by 10 to get 9!. Finally, since Frodo could be to the left or right of Lobelia, there are two ways to make them into a "single" guest giving us $2 \cdot 9!$ total (unwanted, circular) arrangements. The total arrangements ignoring the requirement for Frodo and Lobelia to be separated is placing 11 people on a circle, which is $\frac{11!}{11} = 10!$.

Total minus unwanted gives us $10! - 2 \cdot 9! = 10 \cdot 9! - 2 \cdot 9! = (10 - 2) \cdot 9! = 8 \cdot 9! = 2,903,040$ valid seating arrangements. ■

Problem 2 (Fruit Bowl) 18

A fruit bowl contains apples, bananas, oranges, and pears (at least 14 of each kind).

1. In how many ways can we choose 14 fruits, including at least 5 oranges?

Solution:

If we force 5 oranges to be selected then we have 9 more fruits to pick and the rest is similar to the earlier situation. We can do it in $\binom{9+4-1}{4-1} = \binom{12}{3} = 220$ ways. ■

2. In how many ways can we choose 14 fruits, so that there are at most 4 oranges?

Solution:

Direct approach: At most 4 oranges means, the number of oranges can be one of $\{0, 1, 2, 3, 4\}$. For each of these, we can count the number of ways we can pick from the other kinds of fruits. This

gives us:

$$
\begin{aligned}
&= \binom{14+3-1}{3-1} + \binom{13+3-1}{3-1} + \binom{12+3-1}{3-1} + \binom{11+3-1}{3-1} + \binom{10+3-1}{3-1} \\
&= \binom{16}{2} + \binom{15}{2} + \binom{14}{2} + \binom{13}{2} + \binom{12}{2} \\
&= 460
\end{aligned}
$$

Total-minus-unwanted approach: The total number of ways of picking fruits is $\binom{17}{3}$ as found in the first part. The opposite of at most 4 oranges is at least 5 oranges, which we found in the second subpart of this problem as $\binom{12}{3}$. Subtracting gives us: $\binom{17}{3} - \binom{12}{3} = 460$. ■

3. In how many ways can we choose 14 fruits, so that there are at most 4 oranges and at most 2 pears?

Solution:

While we could attempt the direct approach, it will require adding too many terms. So, in the grand tradition of the greatest mathematicians and computer scientists, we'll be lazy and use the quicker approach.

Total-minus-unwanted approach: The total number of ways of picking the fruits is $\binom{17}{3} = 680$ as found in the first subpart of this problem.

Let A be the set of choices which have at most 4 oranges and B be the set of choices which have at most 2 pears. What we want is $|A \cap B| = |U| - |\overline{A} \cap \overline{B}| = |U| - |\overline{A} \cup \overline{B}| = |U| - \left[|\overline{A}| + |\overline{B}| - |\overline{A} \cup \overline{B}|\right]$.

- $|A|$ is the number of ways when at least 5 oranges are selected, which can be done in: $\binom{14-5+4-1}{4-1} = \binom{12}{3} = 220$ ways.
- $|B|$ is the number of ways when at least 3 pears are selected, which can be done in: $\binom{14-3+4-1}{4-1} = \binom{14}{3} = 364$ ways.
- $|\overline{A} \cup \overline{B}|$ is the number of ways when at least 5 oranges are selected *and* at least 3 pears are selected, which can be done in $\binom{(14-5-3)+4-1}{4-1} = \binom{9}{3} = 84$ ways.

Substituting these values back in, we get: $|A \cap B| = 680 - [220 + 364 - 84] = 180$ ways. ■

Problem 3 (Pigeonhole Principle) 4+8

1. What is the minimum number of players that must be assigned to 10 different gaming groups such that at least one group has at least 4 members?

Solution:

We have 10 pigeonholes (the groups) and we want that at least one of them has 4 members. Using the more general form of the pigeonhole principle, we want the ratio of number of players to the number of pigeonholes to be more than 3 so that taking its ceiling gives us 4.

As an equation, that becomes $n/10 > 3 \implies n > 30$. Thus, 31 players will ensure that at least one of the groups gets 4 members. ■

2. Suppose a set of 8 numbers are selected from the set $\{1, 2, 3, ..., 13, 14\}$. Show that two of the selected numbers must sum to 15.
[***Hint**: Think about how many subsets of 2 elements you can form such that the sum of the values of the two elements is 15.*]

Solution:

The following 2 element subsets all sum up to 15: $\{1, 14\}, \{2, 13\}, \{3, 12\}, \{4, 11\}, \{5, 10\}, \{6, 9\}, \{7, 8\}$. There are seven of these so 7 pigeonholes, each containing pairs of distinct numbers. Since we are picking an 8 element subset there are 8 pigeons. By the pigeonhole principle, two pigeons end up in the same hole, which is to say that there must be two numbers which add up to 15.

Alternatively, 7 numbers can be picked up without any two of them summing to 15, by picking all elements ≤ 7 or all elements ≥ 8. However, since we need to pick another element, we will have to pick a number whose pair is already picked up. So as soon as the 8th element is picked, there will be a subset which adds up to 8. ■

Notation: Throughout, the notation ndk is used to denote the following process: a k-sided dice is rolled n times and the resulting numbers are all added up to get the final result. For example, 3d6 would be 3 rolls of a 6-sided dice and looking at the sum of the face-up numbers (eg. 2, 2, 4) as the final result (eg. 8). Additionally, multiple such dice combinations can be performed together and the results added up, for example: 3d6 + 2d4 + 2 means 3 rolls of a 6-sided dice, 2 rolls of a 4-sided dice, add up all the results and then add a 2 to get the final result.

Problem 4 (Probability of Dice) 40

Consider a set of dice that have a 4-, 6-, 8-, 10-, 12-, and 20-sided die. What combination of dice will make it most likely to roll a nine. That is, what ndk (out of the following) has the highest probability to roll a 9.

1. 3d6
2. 2d10
3. 1d12 + 1d4
4. 1d20

For each of the above dice combinations, i) calculate the size of the sample space, ii) determine the ways the dice can sum to nine (event space size), iii) calculate the probability of rolling nine.

Solution:

1. 3d6
   - Sample space size is 216.
   - To count this, consider each die as a unique bin and 9 indistinguishable units of 1. The number of 1's in each bin corresponds to the value of the die. Since no die can roll a 0, there is already 1 unit in each bin. That leaves 6 units for 3 bins.

     $$\binom{6+3-1}{6} = \frac{8!}{6!2!} = \frac{8*7}{2} = 28$$

     However, notice that there are 3 illegal combinations, different permutations of (1,1,7) since you cannot roll a 7 on a six-sided die. Therefore, there are 25 ways to sum to 9.
   - The probability is $\frac{25}{216} \approx 11.57\%$
2. 2d10
   - Sample space has $10 * 10 = 100$ elements.
   - There are 8 ways to sum to 9.

Die 1	1	2	3	4	5	6	7	8
Die 2	8	7	6	5	4	3	2	1

   - Probability is $\frac{8}{100} = 8\%$.
3. 1d12 + 1d4
   - Sample space has $12 * 4 = 48$ elements.
   - There are 4 ways to sum to 9.

12-side	8	7	6	5
4-side	1	2	3	4

- Probability is $\frac{4}{48} = \frac{1}{12} \approx 8.33\%$.

4. 1d20
   - Sample space size is 20.
   - There is only 1 way to sum to 9.
   - The probability is $\frac{1}{20} = 5\%$

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Problem 5 (Chance of Snow) 20

Akshar has lived in Boston for a long time. From experience, he knows that each day has a 5% chance to be snowy. Akshar is looking ahead to the next 10 days.

1. What is the probability none of those days will be snowy?

   Solution:

   The probability is $(1 - 0.05)^{10} \approx 0.5987 = 59.87\%$. ■

2. What is the probability at least one of those days will be snowy?

   Solution:

   The complement of this event is that none are snowy. So, the probability of this event is $1 - (0.95)^{10} \approx 0.4013 = 40.13\%$. ■

3. What is the probability exactly 3 of those days will be snowy?

   Solution:

   We can choose 3 days and make them snowy. The remaining days must not be snowy. The probability is $\binom{10}{3}(0.05)^3(0.95)^{10} \approx .0105 = 1.05\%$ ■

4. Akshar decides that, he will wear his heavy coat to work 3 of the 10 days without looking at the weather. What is the probability Akshar will wear his coat on at least one snowy day?

   Solution:

   The complement of this event is the event where Akshar chooses 3 days to wear his coat and none of them are snowy. The probability is $1 - (0.95)^3 \approx 0.1426 = 14.26\%$. ■