CS1800 Discrete Structures *Out: October 17, 2026*

Instructor Solutions to Problem Set 5

Strange & Varma *Due: October 23, 2026*

- This assignment is **due on October 23, 2026 at 11:59 PM** via Gradescope. You may submit up to 48 hours late with no penalty.
- Your solutions must be handwritten, with your name on the first page. All your work must be shown even if the problem doesn't specifically ask for it.
- Discussing problems with classmates is allowed but all solutions must be written by yourself, in your own words.
- Copying solutions from a classmate, an online source, or AI tools, is a violation of our academic integrity policy. Do *not* submit anything you cannot explain. Please list all your collaborators and online reference sources in your solution for each problem.
- If you get stuck on a homework problem, come by office hours or post on Campuswire! We recommend you spend about 30 minutes trying to figure out a problem, and then ask for help. We'll be happy to clarify material from class and algorithm concepts, but we will not give out solutions or confirm your answers are correct.

Problem 1 (Product and Sum rule) 5+10+10+(5+5+5+5)

Indicate which rule is applicable in the following situations: product rule, sum rule, or both. Then calculate the number of combinations for the scenario. *You should leave the solution unsimplified, we care more about the process and approach than the arithmetic to find an exact number.*

1. Shallan owns 5 thick and 3 thin brushes. She also owns 10 different colors. How many different ways can Shallan pair the brushes with the colors? Provide the answers separately if only thick or only thin brushes are used and also if either brush can be used.

   Solution:

   If thick brushes are used, then 5 choices for the brush, 10 for the colors. Product rule: 50 pairs.

   If thin brushes are used, then 3 choices for the brush, 10 for the colors. Product rule: 30 pairs.

   In total, sum rule gives $50 + 30 = \boxed{80}$ possible pairings that can be formed. ■

2. License plates in Elendel starts with four (atomic) symbols, each chosen from one of the eight "internal" metals.[1] This is followed by three digits (from 0–9), e.g. "Fe Cu Al Au 89342". Alternatively, it can also be made of six "internal" metal symbols followed by three digits (from 0–9), e.g. "Cu Cu Al Fe Au Al 275". How many different license plates are possible?

   Solution:

   4 metals-5 letter: Each metal has 8 possibilities and there are 4 digits to be chosen, product rule gives us 8^4, each digit has 10 possibilities and 5 digits to be chosen, product rule gives us 10^5. Product rule on the combined, gives us $8^4 \cdot 10^5$ license plates.

   6 metals-3 digits: Each metal symbol has 8 possibilities and there are 6 symbols to be chosen, product rule gives us 8^6, each digit has 10 possibilities and 3 digits to be chosen, product rule gives us 10^3. Product rule on the combined, gives us $8^6 \cdot 10^3$ license plates.

   Sum rule for the two kinds of license plates gives us a total of $\boxed{8^4 \cdot 10^5 + 8^6 \cdot 10^3} = 8^4 \cdot 10^3(10^2 + 8^2)$ possible license plates. ■

3. How many strings of length 5 can you write, using the letters A, B, and C so that no two consecutive letters are the same? For example, A C B A C is a valid string, but A B C B B is not.

   Solution:

   Product rule will be applied.

   There are 3 possible choice for character 1 (slot 1) of the string. For the slot 2, the previously chosen letter cannot be used, but the other 2 can be, so 2 choices. Similar reasoning for slot 3 gives 2 choices there as well. Continuing, we get 2 choices for all remaining slots.

[1] It doesn't matter which they are, just that there are 8 of them.

Total number of strings: $\boxed{3 \cdot 2 \cdot 2 \cdot 2 \cdot 2}$ strings. ■

4. In how many ways can you answer the TRUE/FALSE questions of a quiz that consists of a total of 20 questions. Give your answer for the following three scenarios.

Solution:

(a) If you decide to answer all 20 questions.

Each question's answer has two choices, product rule across 20 questions gives us $\boxed{2^{20}}$ possible ways.

(b) If you decide to leave some questions blank.
Now each question has 3 choices: True/False/Blank. Product rule across 20 questions gives us $\boxed{3^{20}}$ ways.

(c) If you decide that you'll answer all questions using True/False except questions which are numbered multiples of three or five. These questions you might decide to leave blank instead of choosing an answer. But you also decide that question 15 must be left blank because you are feeling particularly chaotic.
The multiples of 3 or 5 are the set $\{3, 5, 6, 9, 10, 12, 15, 18, 20\}$. Excluding 15, this has 8 questions. So, out of the 20 questions, the answer for the 15^{th} is left blank, 8 of the questions are answered using one of 3 choices, the remaining 11 are answered using one of two choices. Using both the product rule, the number of ways to answer are:

$$2^{11} \cdot 3^8$$

(d) If you decide to answer one-half of the questions True and one-half False, in how many ways can you answer the questions?
Product rule approach: There are a total of 20 slots to fill. That's 20! ways. However, all 10 Trues are interchangeable so permutations among the Trues get overcounted, so we need to divide by 10!. Similarly, we need to divide by another 10! for the Falses. In total this gives us:

$$\frac{20!}{10! \cdot 10!}$$

Using combinations: Out of the twenty questions, once we choose the half (10 of them) that will be answered True, the others are fixed. Using combinations, the number of ways is

$$\binom{20}{10}$$

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Problem 2 (Password Troubles) 30

You are generating random numeric 7-digit passwords in order to send them to the members of your website as part of their first time login credentials. Your website is very successful, and you know that

you will have at least 5 million people interested in becoming a member.[2] You also require that each member should be given a unique password; no two members can have the same password.

1. Assuming that you do not want to use "0" in your passwords, will you be able to generate enough passwords for your members? Why or why not? *You can use a calculator to figure out the exact value, but we need to see the approach/process for full credit.*

   Solution:

   Without "0", we have 9 possible choices for each of the 7 digits, which gives us $\boxed{9^7 = 4,782,969}$ possible passwords. This is smaller than 5 million, so we cannot generate unique passwords for every member. ■

2. A bug due to vibe coding this causes digits 3 and 8 cannot be distinguished anymore, so "1328" looks the same as "1823" which is the same as "1323" and "1828". Of course, 0 is still not used. How many different passwords would you be able to generate in this case?

   Solution:

   Now there are only 8 possible choices which is 0-9 without the 0 and withotu the 8 (since they will all look like 3s anyway). So $\boxed{8^7}$ distinct passwords are possible, which is worse than the previous 9^7 so we're still short of the 5 million. ■

3. Unfortunately, the programmer who was supposed to fix the previous bug has introduced a new one. For some reason, the new code generates all possible passwords which have a "3" or "8" on exactly 3 of the 7 places and nowhere else. 0 is still excluded, but all other digits can appear. How many passwords does their program generate?

   Solution:

   We first choose the 3 slots out of 7 which get set to a "3" or "8" (in $\binom{7}{3}$ ways), and then choose which are "3" and which are "8" (in 2^3 ways). In the other 4 slots, we can have any of the remaining 7 digits except "0", "3" and "8" (in 7^4 ways). Via product rule, the number of different passwords is then:

   $$\binom{7}{3} \cdot 2^3 \cdot 7^4$$

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Problem 3 (Words and Numbers) 39

1. A palindrome is a word (or a number) which does not change when read from left or right. How many 7-letter or 8-digit palindromes are there which consist of any letters of the alphabet (for the 7-letter) or any digit (for the 8-digit) palindromes? Note that the obtained 7-letter words do not have to have a meaning.

[2] They are surely, definitely, absolutely, not AI bots.

Solution:

For a 7-letter word to be a palindrome, the first and last letters must be the same, letters at places 2 and 6 must match, letters at places 3 and 5 must match and the fourth letter can be anything. Thus, we only have a choice for the first four places which completely fix the full 7 letter word. Since there are 26 characters, we get 26^4 palindromic 7-letter words.

For a 8-digit number, the reasoning is similar, but the pairings are slot 1 = slot 8, slot 2 = slot 7, slot 3 = slot 6, and slot 4 = slot 5. Each slot (except the first) has 10 choices and product rule gives us 10^3 for the remaining slots. The first slot can have any of the digits from 1–9, so it has 9 possibilities. In total, this gives $9 \cdot 10^3$ palindromic 8-digit numbers.

Sum rule gives the total of: $\boxed{26^4 + 9 \cdot 10^3}$ palindromes of the required type. ■

2. How many 9-digit numbers are there whose all digits are prime and which is not even?

Solution:

There are four prime digits: $\{2, 3, 5, 7\}$. We have 9 slots, each slot except the last can have any of the 4 prime digits and the last can have any of the 3 odd prime digits (number would be even if last (rightmost) digit is 2). Product rule gives us: $\boxed{4^8 \cdot 3}$ such numbers. ■

3. How many distinct words (they do not need to mean anything) can be formed from the word MASSACHUSETTS that use all of the 13 letters?

Solution:

There are a total of 13 characters in MASSACHUSETTS, which can be rearranged in $13! = 6,227,020,800$ ways. However there are 4 copies of 'S', 2 copies of 'A' and 'T' each. The 13! count of rearrangements considers each of these copies as distinct. Thus, each of the 4! rearrangements of the 4 'S' copies were being considered distinct words which they are not. Similarly for the copies of 'A' and 'T'. Accounting for this, the number of distinct words is:

$$\frac{13!}{4!2!2!}$$

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Problem 4 (Permutations and Combinations) 5+10+15+15

1. How many strings can you write using 7 As and 4 Bs, so that no two Bs are consecutive?
[***Hint**: Insert the Bs between the As*]

Solution:

There are 8 spaces between the A's and we need to choose 4 for the B's:

$$\binom{8}{4}$$

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2. In how many ways can you arrange the symbols

$$\spadesuit, \blacksquare, \blacktriangle, \blacktriangledown, \blacklozenge, \bigstar$$

in a row under the restriction that the symbol ♠ cannot appear at the beginning or at the end of the sequence.

Solution:

Choose-then-arrange solution: Since the ♠ is not allowed in the two extreme spots, choose one out of the remaining 4 spots to put the ♠ in. This can be done in 4 ways. Then arrange the remaining 5 symbols in 5! ways to get a total of $\boxed{4 \cdot 5!}$ ways.

Alternate solution: Total arrangements of all symbols is 6!, the number of arrangements in which the ♠ is at the first spot is 5! which is also the number of arrangements in which the ♠ is at the last spot. Number of "valid" arrangements is total arrangements minus the "invalid" arrangements: $\boxed{6! - 2 \cdot 5!} = 6 \cdot 5! - 2 \cdot 5! = (6-2) \cdot 5! = 4 \cdot 5!$ which is the same as the earlier solution. ■

3. Akshar likes 15 K-pop songs, 13 progressive instrumental post-rock songs, and 6 Mongolian throat singing metal fusion songs. How many playlists[3] can be formed that consist of 6 K-pop, 4 post-rock, and 2 fusion-metal songs if:

   (a) Repetitions are allowed (the same song may be played multiple times)

   Solution:

   There are $6+4+2 = 12$ songs to be listed in some order to determine our playlist. For each artist, we choose the spots their songs occupy: $\binom{12}{6} \cdot \binom{6}{4} \cdot \binom{2}{2}$. Once the spots are selected, we select 6 K-pop songs out of the 15 we have in 15^6 ways, select 4 post-rock songs out of the 13 we have in 13^4 ways and finally select 2 fusion-metal songs out of the 6 that we have in 6^2 ways. Since all of this needs to be done to determine a playlist, by product rule the total number of ways of doing this is:

$$\binom{12}{6} \cdot 15^6 \cdot \binom{6}{4} \cdot 13^4 \cdot \binom{2}{2} \cdot 6^2$$

   ■

   (b) Repetitions arent allowed.

[3] A playlist is decided once you specify the order in which to play the selected songs.

Solution:

Everything that we did earlier holds, except that we cannot re-use an already selected song which affects the number of ways we can choose the specific songs. Changing from the n^r form for repetition to the $n \cdot (n-1) \cdots$ form for no repetition gives us:

$$\binom{12}{6} \cdot (15 \cdot 14 \cdot 13 \cdot 12 \cdot 11 \cdot 10) \cdot \binom{6}{4} \cdot (13 \cdot 12 \cdot 11 \cdot 10) \cdot \binom{2}{2} \cdot (6 \cdot 5)$$

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Problem 5 (Games) 15+20

1. The gaming group has four different board games, *Catan* which is a 4-player game, *Set* which is a 3-player game, *Monopoly* which is a 4-player game, and *Solitaire* which is a single player game. How many different game assignments can be made up of the 20 members? Note that since there is only one copy of each game, some of the members won't be assigned games and will not be able to play.

   Solution:

   First $4 + 3 + 4 + 1 = 12$ players need to be chosen out of the 20 members, the rest will not play. Then out of the 11, we need to choose 4 who will play Catan, from the remaining 8 we need to choose 3 who will play Set, from the remaining 5 we need to choose 4 who will play Monopoly leaving one final person who plays Solitaire. The number of game assignments are thus:

   $$\binom{20}{12} \cdot \binom{12}{4} \cdot \binom{8}{3} \cdot \binom{5}{4}$$

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2. In the second round, the groups wants to change things and wish to reassign members. The person who was playing solitaire shouldn't continue to play that. One specific person from the *Catan* group, the *Set* group, and the *Monopoly* group should continue to play the same game. The others can be moved anywhere else. How many ways can the members be rearranged?

   Solution:

   Choose 1 out of 4 to keep playing *Catan*, 1 out of 3 for *Set* and 1 out of 4 for *Monopoly*. Choose the new *Solitaire* player from the $12 - 1 - 1 - 1 - 1 = 8$ players. These handle the requirements for the rearrangement. Everyone else can be filled in any manner as long as player counts are maintained for the games. So choose 3 more (out of the 8 remaining) for Catan, 2 more out of the 5 that'll remain after picking for Catan and then pick 3 out of the last 3.

   $$\binom{4}{1} \cdot \binom{3}{1} \cdot \binom{4}{1} \cdot \binom{8}{1} \cdot \binom{8}{3} \cdot \binom{5}{2} \cdot \binom{3}{3}$$

   ■