CS1800 Discrete Structures *Out: October 10, 2026*

Problem Set 4

Strange & Varma *Due: October 16, 2026*

- This assignment is **due on October 16, 2026 at 11:59 PM** via Gradescope. You may submit up to 48 hours late with no penalty.
- Your solutions must be handwritten, with your name on the first page. All your work must be shown even if the problem doesn't specifically ask for it.
- Discussing problems with classmates is allowed but all solutions must be written by yourself, in your own words.
- Copying solutions from a classmate, an online source, or AI tools, is a violation of our academic integrity policy. Do *not* submit anything you cannot explain. Please list all your collaborators and online reference sources in your solution for each problem.
- If you get stuck on a homework problem, come by office hours or post on Campuswire! We recommend you spend about 30 minutes trying to figure out a problem, and then ask for help. We'll be happy to clarify material from class and algorithm concepts, but we will not give out solutions or confirm your answers are correct.

Problem 1 (Where's the order in all this chaos?) 1x8 + 2 + 2

We define $(a, b) = \{\{a\}, \{a, b\}\}$. For each of the statements below, determine whether they are True or False and provide a brief reasononing in 1–3 sentences.

i. $a \in (a, b)$

ii. $\{a\} \in (a, b)$

iii. $(a, a) = \{a\}$

iv. $b \in (a, b)$

v. $\{b\} \in (a, b)$

vi. $\{a, b\} \in (a, b)$

vii. $a \in (a, a)$

viii. $\{a\} \in (a, a)$

ix. $(a, b) = (b, a)$

x. $(a, b) = (b, a) \implies a = b$

Problem 2 (Cartesian Products) 4+4+6+8+8

Define the set $X = \{0, 1, 2, \ldots, 9\} = \{n \in \mathbb{N} \mid 0 \leq n \leq 9\}$ = set of first ten natural numbers.

Given one form of a set, translate to the other forms out of: English, roster, and set builder notations. That is, if given set builder notation, provide the English and roster notations and so on. You are allowed to utilize any set operations as required.

(a) $A = \{(x, y) \in X \times X \mid y = x^2\}$

(b) $B = \{(x, y) \in X \times X \mid 2y = x\}$

(c) "C is the set of pairs of natural numbers between 0 and 9 (both inclusive) such that the second number in the pair is a positive power of the first number."

(d) $D = \{(0, 0, 0), (1, 1, 1), (2, 4, 8), (3, 9, 27), (4, 16), (5, 25), (6, 36), (7, 49)\}$

(e) "E is the set of all triples made of natural numbers between 0 and 9 (both inclusive) such that the first number is positive, it is the product of the second and third, the second number is not smaller than the third."

Problem 3 (PowerSet−− and PowerSet++) 4+5+9

In this problem, $\mathcal{P}(X)$ is the power set (set of all subsets) of the set X.

(a) Consider the set $X = \{a, b, c\}$. Write out the set $Y = \mathcal{P}(X) \setminus \{X\}$ in both roster notation and describe the set (as consicely as possible) in English:

(b) We have the set H consisting of 4 hobbits "Frodo", "Samwise", "Merry", and "Pippin". Write the set $B = \{S \mid S \subset H, |S| > 1\}$ in roster notation as well as in words.

(c) Let $X = \{2, 3, 5, 7\}$.

Recall the bitstring representation of sets and subsets. All subsets of X can be written as a 4-bit string "xxxx" where the first bit being 1 means 2 is in the subset and 0 means 2 isn't. Similarly, the second bit represents the element 3, the third bit 5 and the fourth bit 7. We will use $bstr(S)$ to denote the bitstring of the subset $S \subseteq X$. For example $bstr(\{2, 5\}) = 1010$ and $bstr(\{3\}) = 0100$.

The set Y is described in set builder notation below, write out the full set in roster notation and explain in English what this set consists of.

$$Y = \left\{ \left(S, \frac{p}{q} \right) \mid S \in \mathcal{P}(X), \ \text{"abcd"} = bstr(S), \ p = 2^a \cdot 3^b \cdot 5^c \cdot 7^d, \ q = \frac{210}{p} \right\}$$

Problem 4 (Share of the PIE) 6

Among the 37 people Akshar knows who are interested in Dungeons and Dragons, 15 only play themselves and do not spectate live play games of others. Among the others, 14 like to watch only Critical Role (CR) while there are 2 who watch both CR and Dimension 20 (D20).

Draw a Venn diagram for the 37 people and the sets for those who watch CR and D20. Label *each region* of the diagram with the number of people in that region. Show all your work.

Problem 5 (Building Worlds) 16

We asked 108 enthusiasts of high fantasy and world building whether they liked Lord of the Rings ($LOTR$), A Song of Ice and Fire ($ASOIAF$), or Cosmere (Cos). Here's the data[1] about interest in these fictional settings.

- 28 of those surveyed didn't like any of these.
- 7 liked all three.
- Everyone who liked $ASOIAF$ also liked Cos.
- 15 enthusiasts liked both $LOTR$ and Cos (and maybe $ASOIAF$ as well!)
- 40 enthusiasts indicated they like exactly one of these settings.

(a) Draw a Venn diagram with regions filled in with numbers (of people belonging to that region) in whatever regions possible based on the data above. If there are unknown regions, clearly mark those and explain why they are unknown. Explain your steps in figuring out the numbers from the given data.

(b) You can either use the Venn diagram or the Principle of Inclusion-Exclusion formula[2] to answer the following question. In either approach, show all your steps clearly.
How many of those surveyed indicated they like both $ASOIAF$ and Cos?
(Note, these individuals might have also liked $LOTR$!)

[1] All totally made up just for this problem.
[2] For three sets X, Y, Z, we have: $|X + Y + Z| = |X| + |Y| + |Z| - |X \cap Y| - |Y \cap Z| - |Z \cap X| + |X \cap Y \cap Z|$.