CS1800 Discrete Structures *Out: October 10, 2026*

Instructor Solutions to Problem Set 4

Strange & Varma *Due: October 16, 2026*

- This assignment is **due on October 16, 2026 at 11:59 PM** via Gradescope. You may submit up to 48 hours late with no penalty.
- Your solutions must be handwritten, with your name on the first page. All your work must be shown even if the problem doesn't specifically ask for it.
- Discussing problems with classmates is allowed but all solutions must be written by yourself, in your own words.
- Copying solutions from a classmate, an online source, or AI tools, is a violation of our academic integrity policy. Do *not* submit anything you cannot explain. Please list all your collaborators and online reference sources in your solution for each problem.
- If you get stuck on a homework problem, come by office hours or post on Campuswire! We recommend you spend about 30 minutes trying to figure out a problem, and then ask for help. We'll be happy to clarify material from class and algorithm concepts, but we will not give out solutions or confirm your answers are correct.

Problem 1 (Where's the order in all this chaos?) 1x8 + 2 + 2

We define $(a, b) = \{\{a\}, \{a, b\}\}$. For each of the statements below, determine whether they are True or False and provide a brief reasononing in 1–3 sentences.

Solution:

i. $a \in (a, b)$
False: It is $\{a\}$ that belongs to (a, b), not a.

ii. $\{a\} \in (a, b)$
True. See above.

iii. $(a, a) = \{a\}$
False: $(a, a) = \{\{a\}, \{a\}\} = \{\{a\}\}$. While we can remove duplicates, nesting of sets cannot be undone.

iv. $b \in (a, b)$
False: It is $\{a, b\}$ that's an element of (a, b).

v. $\{b\} \in (a, b)$
False: It is $\{a, b\}$ that's an element of (a, b).

vi. $\{a, b\} \in (a, b)$
True.

vii. $a \in (a, a)$
False: It is {a} that is a member of (a, a).

viii. $\{a\} \in (a, a)$
True. See above.

ix. $(a, b) = (b, a)$
False: The first is $\{\{a\}, \{a, b\}\}$ while the second is $\{\{b\}, \{a, b\}\}$. Although both have $\{a, b\}$ as an element, the other element is $\{a\}$ in the (a, b) while it is $\{b\}$ in (b, a). So the two sets are different.

x. $(a, b) = (b, a) \implies a = b$
True. As mentioned in the previous example, the difference in (a, b) and (b, a) are the elements $\{a\}$ and $\{b\}$. If the two pairs are the same, then these two elements must be the same. If $\{a\} = \{b\}$ then it must be that $a = b$. So this implication is true.

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Problem 2 (Cartesian Products) 4+4+6+8+8

Define the set $X = \{0, 1, 2, \ldots, 9\} = \{n \in \mathbb{N} \mid 0 \leq n \leq 9\}$ = set of first ten natural numbers.

Given one form of a set, translate to the other forms out of: English, roster, and set builder notations. That is, if given set builder notation, provide the English and roster notations and so on. You are allowed to utilize any set operations as required.

(a) $A = \{(x, y) \in X \times X \mid y = x^2\}$

Solution:

English: "Set of pairs of natural numbers with the first element in the pair coming from X, the second element also coming from X and the second being the square of the first."

Roster: $A = \{(0,0), (1,1), (2,4), (3,9)\}$ ■

(b) $B = \{(x, y) \in X \times X \mid 2y = x\}$

Solution:

English: "Set of pairs of natural numbers with the first element in the pair coming from X, the second element also coming from X and the first being twice the second."

Roster: $A = \{(0,0), (2,1), (4,2), (6,3), (8,4)\}$

■

(c) "C is the set of pairs of natural numbers between 0 and 9 (both inclusive) such that the second number in the pair is a positive power of the first number."

Solution:

Set builder: $C = \{(x, y) \in X \times X \mid \exists k \in \mathbb{Z}^+ : y = x^k\}$

Roster:

$$C = \Big\{ (0,0), (1,1), (2,2), (3,3), (4,4), (5,5), (6,6), (7,7), (8,8), (9,9) \qquad n^1$$
$$(2,4), (3,9) \qquad n^2$$
$$(2,8) \Big\} \qquad n^3$$

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(d) $D = \{(0,0,0), (1,1,1), (2,4,8), (3,9,27), (4,16), (5,25), (6,36), (7,49)\}$

Solution:

English: "Set of triples, with elements from X such that all triples have the form (x, x^2, x^3) and x^2 is also in X unioned with the set of pairs (x, x^2) where x comes from X and $x^2 \notin X$ and $x^2 \leq 50$."

$$D = \{(x, y, z) \in X \times X \times \mathbb{N} \mid y = x^2, z = x^3\}$$
$$\cup \{(x, y) \in X \times \mathbb{N} \mid y = x^2, y \notin X, y \leq 50\}$$

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(e) "E is the set of all triples made of natural numbers between 0 and 9 (both inclusive) such that the first number is positive, it is the product of the second and third, the second number is not smaller than the third."

Solution:

Set builder: $E = \{(x, y, z) \in \big(X - \{0\}\big) \times X \times X \mid x = y \cdot z, y \geq z\}$

Roster:

$$E = \Big\{(1,1,1), (2,2,1), (3,3,1), (4,4,1), (4,2,2), (5,5,1),$$
$$(6,6,1), (6,3,2), (7,7,1), (8,8,1), (8,4,2), (9,9,1), (9,3,3)\Big\}$$

Note: Without the requirement that $x > 0$, we would have about 50 terms which say $(0, y, z)$ with essentially all possible y and z values. Similarly, without the requirement that $y \geq z$, we would have doubled the number of terms since if (x, y, z) is in E then so is (x, z, y) and these are different since the order matters in tuples. ■

Problem 3 (PowerSet−− and PowerSet++) 4+5+9

In this problem, $\mathcal{P}(X)$ is the power set (set of all subsets) of the set X.

(a) Consider the set $X = \{a, b, c\}$. Write out the set Y in both roster notation and describe the set in English, as consicely as possible: $Y = \mathcal{P}(X) \setminus \{X\}$.

Solution:

The set Y is the power set of X without the subset $X \subseteq X$. In English: "Y is the set of all *proper* or *strict* subsets of the set X".

In roster notation:

$$Y = \{\{\}, \{a\}, \{b\}, \{b\}, \{a, b\}, \{b, c\}, \{c, a\}\}$$

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(b) We have the set H consisting of 4 hobbits "Frodo", "Samwise", "Merry", and "Pippin". Write the set $B = \{S \mid S \subset H, |S| > 1\}$ in roster notation as well as in words.

Solution:

$B = \{$\{Frodo, Samwise\}, \{Samwise, Merry\}, \{Merry, Pippin\}, \{Pippin, Frodo\},
\{Frodo, Samwise, Merry\}, \{Frodo, Samwise, Pippin\},
\{Frodo, Merry, Pippin\}, \{Samwise, Merry, Pippin\}$\}$

B is the set of *proper* or *strict* subsets of H which contain more than 1 element. ■

(c) Let $X = \{2, 3, 5, 7\}$.

Recall the bitstring representation of sets and subsets. All subsets of X can be written as a 4-bit string "xxxx" where the first bit being 1 means 2 is in the subset and 0 means 2 isn't. Similarly,

the second bit represents the element 3, the third bit 5 and the fourth bit 7. We will use $bstr(S)$ to denote the bitstring of the subset $S \subseteq X$. For example $bstr(\{2,5\}) = 1010$ and $bstr(\{3\}) = 0100$.

The set Y is described in set builder notation below, write out the full set in roster notation and explain in English what this set consists of.

$$Y = \left\{ \left(S, \frac{p}{q}\right) \mid S \in \mathcal{P}(X),\ \text{“abcd”} = bstr(S),\ p = 2^a \cdot 3^b \cdot 5^c \cdot 7^d,\ q = \frac{210}{p} \right\}$$

Solution:

Since “abcd”$= bstr(S)$ is a bitstring, $p = 2^a \cdot 3^b \cdot 5^c \cdot 7^d$ just denotes the product of all the elements in the subset S. For example, if $S = \{3,7\}$, then $bstr(\{3,7\}) =$ “0101” and $p = 2^0 \cdot 3^1 \cdot 5^0 \cdot 7^1 = 3 \cdot 7 = 21$.

Also, note that $210 = 2 \cdot 3 \cdot 5 \cdot 7$. So $q = \frac{210}{p}$ is actually the product of all elements *not* in the subset S. Hence, the set is just the pairs $(S, \frac{p}{q})$ where S is a subset of X and $\frac{p}{q}$ is the product of elements in S divided by product of elements not in S.

In roster notation:

$$\begin{aligned} Y = \Bigg\{ & \left(\{\}, \frac{1}{210}\right), \\ & \left(\{2\}, \frac{2}{105}\right), \left(\{3\}, \frac{3}{70}\right), \left(\{5\}, \frac{5}{42}\right), \left(\{7\}, \frac{7}{30}\right), \\ & \left(\{2,3\}, \frac{6}{35}\right), \left(\{3,5\}, \frac{15}{14}\right), \left(\{5,7\}, \frac{35}{6}\right), \\ & \left(\{7,2\}, \frac{14}{15}\right), \left(\{2,5\}, \frac{10}{21}\right), \left(\{3,7\}, \frac{21}{10}\right), \\ & \left(\{2,3,5\}, \frac{30}{7}\right), \left(\{3,5,7\}, \frac{105}{2}\right), \left(\{5,7,2\}, \frac{70}{3}\right), \left(\{2,3,7\}, \frac{42}{5}\right), \\ & \left(\{2,3,5,7\}, \frac{210}{1}\right) \Bigg\} \end{aligned}$$

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Problem 4 (Share of the PIE) 6

Among the 37 people Akshar knows who are interested in Dungeons and Dragons, 15 only play themselves and do not spectate live play games of others. Among the others, 14 like to watch only Critical Role (CR) while there are 2 who watch both CR and Dimension 20 (D20).

Draw a Venn diagram with the universe of 37 people and the sets for those who watch CR and D20. Label *each region* of this diagram with the number of people who belong to precisely that region. Show all your work.

Solution:

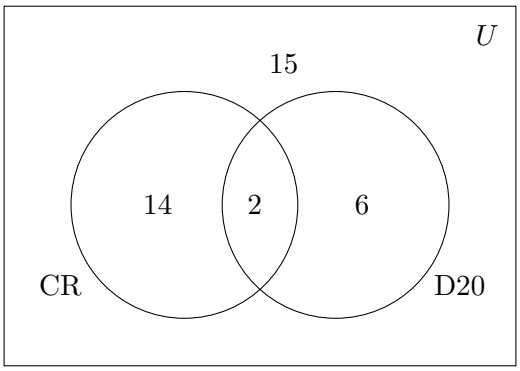


Figure 1: Venn diagram for Dungeons and Dragons. Since there are 15 outside either set, there must be 22 inside. 14 are only in the CR set while 2 are in the intersection. That leaves 6 who must be solely in the D20 set. That accounts for all 37 people.

Without the Venn diagram, we could have used the inclusion-exclusion principle and said $|CR \cup D20| = |CR| + |D20| - |CR \cap D20|$. The left region is 14, which is $|CR \setminus D20|$, so $|CR| = |CR \setminus D20| + |CR \cap D20| = 14+2$. The only unknown remaining in this equation is $|D20|$ so we get $22 = 16+|D20|-2$ solving which gives us $|D20| = 8$. The right region is $|D20| - |D20 \cap CR| = 8 - 2 = 6$. ■

Problem 5 (Building Worlds) 16

We asked 108 enthusiasts of high fantasy and world building whether they liked Lord of the Rings ($LOTR$), A Song of Ice and Fire ($ASOIAF$), or Cosmere (Cos). Here's the data[1] about interest in these fictional settings.

- 28 of those surveyed didn't like any of these.
- 7 liked all three.
- Everyone who liked $ASOIAF$ also liked Cos.
- 15 enthusiasts liked both $LOTR$ and Cos (and maybe $ASOIAF$ as well!)
- 40 enthusiasts indicated they like exactly one of these settings.

Solution:

(a) Draw a Venn diagram with regions filled in with numbers (of people belonging to that region) in whatever regions possible based on the data above. If there are unknown regions, clearly mark those and explain why they are unknown. Explain your steps in figuring out the numbers from the given data.

[1] All totally made up just for this problem.

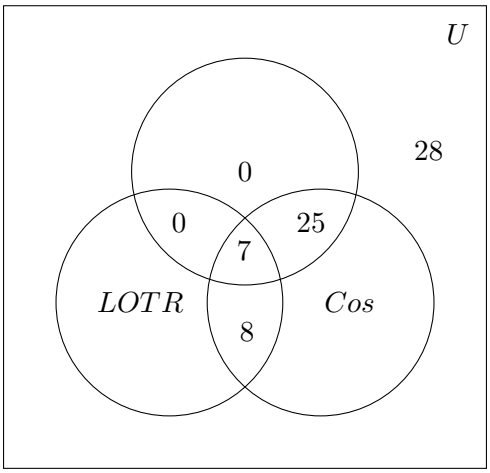


Figure 2: Venn diagram breakdown based on the data. We also know that the sum of values who only like $LOTR$ (bottom left section) or who only like Cos (bottom right section) is 40 but we do not know how they are distributed.

(b) You can either use the Venn diagram or the Principle of Inclusion-Exclusion formula[2] to answer the following question. In either approach, show all your steps clearly.
How many of those surveyed indicated they like both $ASOIAF$ and Cos
(Note, these individuals might have also liked $LOTR$!)

$$
\begin{aligned}
|U| &= 108 && \text{Given} \\
|\overline{LOTR \cup ASOIAF \cup Cos}| &= 28 && \text{Given} \\
|LOTR \cup ASOIAF \cup Cos| &= 108 - 28 = 80 && \text{Disjoint sets} \\
|A| = |LOTR \cap ASOIAF \cap Cos| &= 7 && \text{Given} \\
|ASOIAF \setminus Cos| &= 0 && \text{Given} \\
|(LOTR \cap Cos) \setminus A| &= 15 - 7 = 8 && \text{Splitting 15 into regions} \\
|(ASOIAF \cap Cos) \setminus A| &= 80 - 40 - 7 - 8 - 0 - 0 = 25 && \text{40 like exactly one} \\
|ASOIAF \cap Cos| &= 25 + 7 = 32 && \text{Combining everything}
\end{aligned}
$$

Note that we do not have sufficient information to figure out $|LOTR|$ or $|Cos|$ but we do know that the sum of these two is 40.

■

[2]For three sets X, Y, Z, we have: $|X + Y + Z| = |X| + |Y| + |Z| - |X \cap Y| - |Y \cap Z| - |Z \cap X| + |X \cap Y \cap Z|$.